CP Violation in B Decays

Hitoshi Yamamoto

University of Hawaii

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1. CP Violation and CKM Matrix
2. CP Violation in Mixing
3. CP Violation by Mixing-Decay Interference
4. CP Violation in decay

CPV and CKM Martrix

General left-handed quark-W Interaction

$$L_{\text{int}}(t) = \int d^3x \left(\mathcal{L}_{qW}(x) + \mathcal{L}^{\dagger}_{qW}(x)\right)$$

$$\mathcal{L}_{qW}(x) = \frac{g}{\sqrt{8}} \sum_{i,j=1,3} V_{ij}\, \bar{U}_i\, \gamma_\mu (1-\gamma_5) D_j\, W^\mu$$

$$U_i(x) \equiv \begin{pmatrix} u(x) \\ c(x) \\ t(x) \end{pmatrix}, \quad D_j(x) \equiv \begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \text{(CKM matrix)}$$

Experimentally, V has a hierarchical structure.
Approximately,

$$|V_{ij}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda \sim 0.22$$

Transformation of L_{int} under CP

CP: exchanges particle $(n) \leftrightarrow$ antiparticle $(\bar{n})$
flips momentum sign $(\vec{p} \leftrightarrow -\vec{p})$ (a)
keeps the spin z-component (σ) the same

Such CP operator in Hilbert space is not unique:

$$\mathcal{C}\mathcal{P} a^{\dagger}_{n,\vec{p},\sigma} \mathcal{P}^{\dagger}\mathcal{C}^{\dagger} = \eta_n a^{\dagger}_{\bar{n},-\vec{p},\sigma}$$

η_n: 'CP phase': arbitrary, depends on n
(for antiparticle: $\eta_{\bar{n}} = (-)^{2J}\eta_n^*$, J =spin)

The choice of η_n amounts to choosing a specific operator in Hilbert space among those satisfying (a).

Then, a pure algebra leads to

$$\mathcal{C}\mathcal{P}\ \bar{u}(x)\gamma_\mu(1-\gamma_5)d(x)W^\mu(x)\ \mathcal{P}^{\dagger}\mathcal{C}^{\dagger} = \eta_u \eta_d^* \eta_W^* \Big(\bar{u}(x')\gamma^\mu(1-\gamma_5)d(x')W_\mu(x')\Big)^{\dagger}$$

$$x' \equiv (t, -\vec{x})$$

$\mathcal{L}_{qW}$ transforms as (taking $\eta_W = 1$)

$$\mathcal{CP}\ \mathcal{L}_{qW}(x)\ \mathcal{P}^\dagger\mathcal{C}^\dagger = \frac{g}{\sqrt{8}} \sum_{i,\,j=1,3} \eta_{U_i}\eta^*_{D_j} V_{i\,j} \left(\bar{U}_i(x')\ \gamma^\mu(1-\gamma_5) D_j(x') W_\mu(x')\right)^\dagger$$

IF $\eta_{U_i}\eta^*_{D_j}$ can be chosen s.t.

$$\eta_{U_i}\eta^*_{D_j} V_{i\,j} = V^*_{i\,j} \quad (2)\,,$$

then, $L_{\text{int}}(t)$ becomes invariant under CP:

$$\mathcal{CP}\ \mathcal{L}_{qW}(x)\ \mathcal{P}^\dagger\mathcal{C}^\dagger = \mathcal{L}^\dagger_{qW}(x') \quad (x' = (t, -\vec{x}))$$

$$\begin{aligned} \rightarrow \mathcal{CP}\ L_{\text{int}}(t)\ \mathcal{P}^\dagger\mathcal{C}^\dagger & \\ &= \int d^3x\, \mathcal{CP}\big[\mathcal{L}_{qW}(x) + \mathcal{L}^\dagger_{qW}(x)\big]\mathcal{P}^\dagger\mathcal{C}^\dagger \\ &= \int d^3x\ \big[\mathcal{L}^\dagger_{qW}(x') + \mathcal{L}_{qW}(x')\big] \\ &= L_{\text{int}}(t) \end{aligned}$$

$\rightarrow S$ operator is invariant under CP
(through Dyson series)

Condition for CP Invariance

Rewrite the condition (2):

$$\frac{\eta_{D_j}}{\eta_{U_i}} = 2 \arg V_{i,j}$$

Thus, for a given matrix $V_{i,j}$, if the CP phases η's can be chosen so that the phase difference between η_{D_j} and η_{U_i} is twice the arbitrary phase of $V_{i,j}$, then the physics is invariant under CP.

This is equivalent to rotate the quark phases to make $V_{i,j}$ all real.

In general, there are 5 phase differences for 6 quarks $\rightarrow$ 5 elements of V can be set to real always.

For example.,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$V_{i,j}$: real
$V_{i,j}$: complex

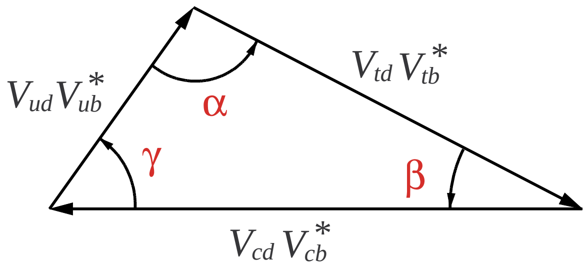
(No unitarity condition imposed)

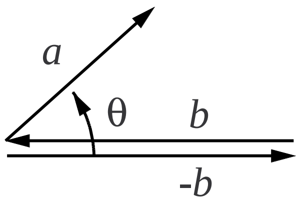
Any of the four red elements is not real
$\rightarrow$ CP violation

A Main Question of the CPV Study in B: 'Is V unitary?'

e.g: orthogonality of d-column and b-column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$





$$\theta = \arg \frac{a}{-b}$$

$$\alpha \equiv \arg\left(\frac{V_{td}V_{tb}^*}{-V_{ud}V_{ub}^*}\right), \quad \beta \equiv \arg\left(\frac{V_{cd}V_{cb}^*}{-V_{td}V_{tb}^*}\right), \quad \gamma \equiv \arg\left(\frac{V_{ud}V_{ub}^*}{-V_{cd}V_{cb}^*}\right)$$

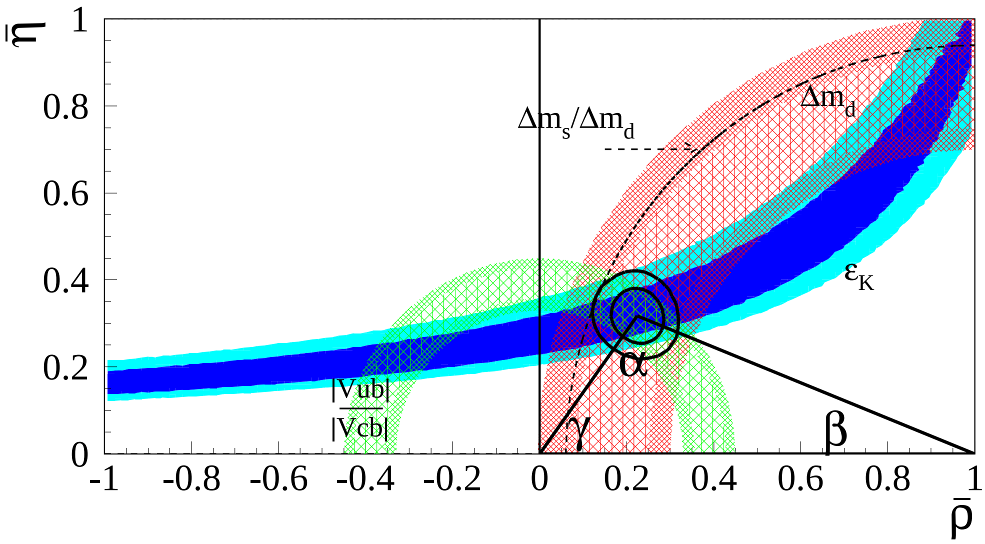
(Another notation: $\alpha \equiv \phi_2$, $\beta \equiv \phi_1$, $\gamma \equiv \phi_3$)

Fit of the CKM unitarity triangle

Experimental inputs:

1. $|V_{ub}/V_{cb}|$
2. B_d mixing $(\delta m_d) \rightarrow |V_{td}|$
3. ϵ_K
4. B_s mixing $\rightarrow \delta m_s/\delta m_s \rightarrow |V_{ts}/V_{td}|$.
   $|V_{ts}|$ known from unitarity of CKM $\rightarrow |V_{td}|$

Many people have performed a fit.
One recent example: Ciuchini et.al.:



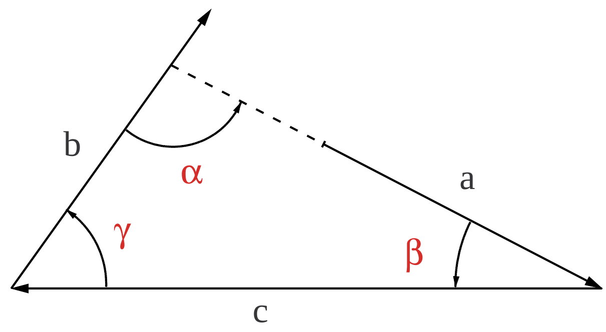
Normalized to the bottom length of the triangle.
(two bands for each are 68% and 95% c.l.)

Three bands cross at one point
$\rightarrow$ already a triumph of the standard model.

For **any** complex numbers a, b, c, trivially

$$\alpha + \beta + \gamma = \pi \pmod{2\pi}$$

$$\alpha \equiv \arg\left(\frac{a}{-b}\right), \quad \beta \equiv \arg\left(\frac{b}{-c}\right), \quad \gamma \equiv \arg\left(\frac{c}{-a}\right).$$



→ The condition $\alpha + \beta + \gamma = \pi$ (mod2π) holds even if the triangle does not close. It does **not** test the unitarity of V_{CKM}.

It simply tests if the angles measured are as defined in (3) in terms of V_{CKM}.

→ It is critical to measure the length of the sides.

3 types of CPV in B decays

1. CPV in mixing. (neutral B)
   Particle-antiparticle imbalance in physical neutral B states ($B_{a,b}$):

   $$|\langle B^0|B_{a,b}\rangle|^2 \neq |\langle \bar{B}^0|B_{a,b}\rangle|^2$$

2. CPV by mixing-decay interference. (neutral B)
   When both B^0&$\bar{B}^0$ can decay to the same final state f:

   $$\begin{array}{ccccc} & & \bar{B}^0 & & \\ & \nearrow & & \searrow & \\ B^0 & & \longrightarrow & & f \end{array} \qquad \begin{array}{ccccc} & & B^0 & & \\ & \nearrow & & \searrow & \\ \bar{B}^0 & & \longrightarrow & & \bar{f} \end{array}$$

   the inteference results in

   $$\Gamma_{B^0\to f}(t) \neq \Gamma_{\bar{B}^0\to \bar{f}}(t)\,.$$

   ($\Gamma_{B^0\to f}(t)$: pure B^0 at $t=0$, decaying to f at t.)

3. CPV in decay. (neutral and charged B)
   Partial decay rate asymmetries.

   $$|Amp(B\to f)| \neq |Amp(\bar{B}\to \bar{f})|$$

   ($Amp(B^0\to f)$: instantaneous decay amplitude.)

CPV in mixing

Eigenstates of mass & decay rate (assume CPT):

$$\begin{cases} B_a = pB^0 + q\overline{B}^0 \\ B_b = pB^0 - q\overline{B}^0 \end{cases},$$

B_a (mass: m_a, decay rate: γ_a)
B_b (mass: m_b, decay rate: γ_b)

$\longrightarrow$ Particle-antiparticle asymmetry in $B_{a,b}$:

$$\delta \equiv \frac{|\langle B^0|B_{a,b}\rangle|^2 - |\langle \overline{B}^0|B_{a,b}\rangle|^2}{|\langle B^0|B_{a,b}\rangle|^2 + |\langle \overline{B}^0|B_{a,b}\rangle|^2} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}$$

$CPT \to B_a$ and B_b have the same δ (incl. sign)

Use $B^0 \to \ell^+$, $\bar{B}^0 \to \ell^-$ to distinguish B^0 and $\bar{B}^0$.

For the neutral K system

$$\delta_K \equiv \frac{Br(K_L \to \pi^- \ell^+ \nu) - Br(K_L \to \pi^+ \ell^- \nu)}{Br(K_L \to \pi^- \ell^+ \nu) + Br(K_L \to \pi^+ \ell^- \nu)} = (3.27 \pm 0.12) \times 10^{-3}$$

$\gamma_a \sim \gamma_b \to B_a$ and B_b cannot be separated easily.
Measure same-sign di-lepton asymmetry in
$\Upsilon 4S \to B^0 \bar{B}^0$ (Okun,Zakharov,Pontecorvo,1975):

$$A_{\ell\ell} \equiv \frac{N(\ell^+\ell^+) - N(\ell^-\ell^-)}{N(\ell^+\ell^+) + N(\ell^-\ell^-)} = 2\delta$$

CLEO 1993 (by $A_{\ell\ell}$ on $\Upsilon 4S$)

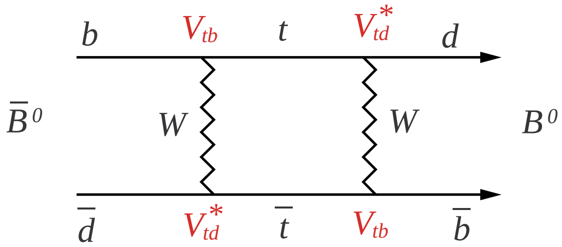
$$\delta = 0.015 \pm 0.048 \pm 0.016$$

OPAL 1997 (by fitting the time dependence of tagged semileptonic decays of B's on Z^0)

$$\delta = -0.004 \pm 0.014 \pm 0.006$$

Standard Model prediction for $\delta(= A_{\ell\ell}/2)$

The dominant diagram for mixing:



$$\rightarrow \begin{cases} p = \frac{1}{\sqrt{2}} e^{i\phi} \\ q = \frac{1}{\sqrt{2}} e^{-i\phi} \end{cases}, \qquad \phi = \arg(V_{tb} V_{td}^*)$$

This does not result in $|p| \neq |q|$ (or $A_{\ell\ell} \neq 0$).

The interference of the above diagram with the same one with t replaced by c gives

$$A_{\ell\ell} \sim -4\pi \frac{m_c^2}{m_t^2} \Im \left(\frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \right) \sim 10^{-3}$$

Long-distance effects may dominate
(hadronic intermediate states)
(Altomari, Wolfenstein, Bjorken, 1988):

$$B^0 \leftrightarrow \begin{pmatrix} D^0 \bar{D}^0 \\ D^+ D^- \\ \text{etc.} \end{pmatrix} \leftrightarrow \bar{B}^0$$

$$|A_{\ell\ell}| = 10^{-3} \sim 10^{-2}.$$

Large theoretical uncertainty.

$\longrightarrow$ Cannot determine CKM phases from $A_{\ell\ell}$.

$\delta(= A_{\ell\ell}/2)$ of 10^{-2} or larger signals **new physics**.

(Also, $\delta = 0$ assumed in most calculations.
$\rightarrow$ engineering value.)

Progress expected in the near future

There is also CP asymmetry in single lepton yield, (assuming leptons from $B^{\pm}$ cannot be separated)

$$A_\ell \equiv \frac{N_{\Upsilon(4S)\to\ell^+} - N_{\Upsilon(4S)\to\ell^-}}{N_{\Upsilon(4S)\to\ell^+} + N_{\Upsilon(4S)\to\ell^-}} = \chi\,\delta$$

$\chi \equiv Br(B^0$ decays as $\bar{B}^0) \sim 0.17$

Time measurement increases sensitibity.

B-factories: $N(B^0, \bar{B}^0) \sim 4 \times 10^7$ already

$\sigma_\delta(\ell + \ell\ell) \sim 0.1\%$ (B-factories now)

Quite possible that leptonic CP asymmetry will be observed in near future.

CPV by Mixing-Decay Interference

$\Gamma_{B(\overline{B})\to f}(t)$: the probability that a pure $B^0(\overline{B}^0)$ at $t = 0$ decays to a final state f at t is

$$\Gamma_{B(\overline{B})\to f}(t) = |pA|^2 e^{-\gamma t}\left[1 \pm \Im\left(\frac{q\overline{A}}{pA}\right)\sin \delta m\, t\right]$$

(for $|q\overline{A}/pA| = 1$, or f: CP eigenstate):

$$\begin{cases} B_a = pB^0 + q\overline{B}^0 \\ B_b = pB^0 - q\overline{B}^0 \end{cases},$$

$$\begin{cases} A \equiv Amp(B^0 \to f) \\ \overline{A} \equiv Amp(\overline{B}^0 \to f) \end{cases}, \qquad \begin{cases} \gamma_a = \gamma_b \equiv \gamma \\ \delta m \equiv m_a - m_b \end{cases}$$

Time-integrated asymmetry:

$$A_f \equiv \frac{\Gamma_{B\to f} - \Gamma_{\bar{B}\to f}}{\Gamma_{B\to f} + \Gamma_{\bar{B}\to f}} = \frac{x}{1+x^2}\Im\left(\frac{q\overline{A}}{pA}\right)$$

$$x \equiv \frac{\delta m}{\gamma} \sim 0.71 \pm 0.06 \quad \to \quad \frac{x}{1+x^2} \sim \frac{1}{2}$$

On $\Upsilon 4S \to B^0 \bar{B}^0$

Tag 'the other side' by a lepton:

$$\ell^{\pm} X(t_{tag}) \leftarrow (B^0 \bar{B}^0) \to f(t_{sig})$$

$B^0 \bar{B}^0$ created in a coherent $L = 1$ state.
Quantum correlation:

ℓ^+ tag at t $\longrightarrow$ Signal side is $\bar{B}^0$ at t
ℓ^- tag at t $\longrightarrow$ Signal side is B^0 at t

The decay time distribution is nearly identical to the single B case with

$$t \quad \longrightarrow \quad t_- \equiv t_{sig} - t_{tag}$$

(in fact, esactly identical for $t_- > 0$)

$$\Gamma_{4S \to \ell^{\mp} f}(t_-) \propto e^{-\gamma |t_-|} \left[1 \pm \Im \left(\frac{q\overline{A}}{pA} \right) \sin \delta m \, t_- \right]$$

(f: CP eigenstate):

Gold-plated mode $B \to \Psi K_S$

What phases of V_{CKM} do we measure?

Recall $$\begin{cases} p = \frac{1}{\sqrt{2}} e^{i\phi} \\ q = \frac{1}{\sqrt{2}} e^{-i\phi} \end{cases}, \qquad \phi = \arg(V_{tb} V_{td}^*)$$

Actually, we need to include the CP phase of B^0:

$$\frac{q}{p} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \eta_B \,, \qquad (CP|B\rangle = \eta_B |\bar{B}\rangle) \,,$$

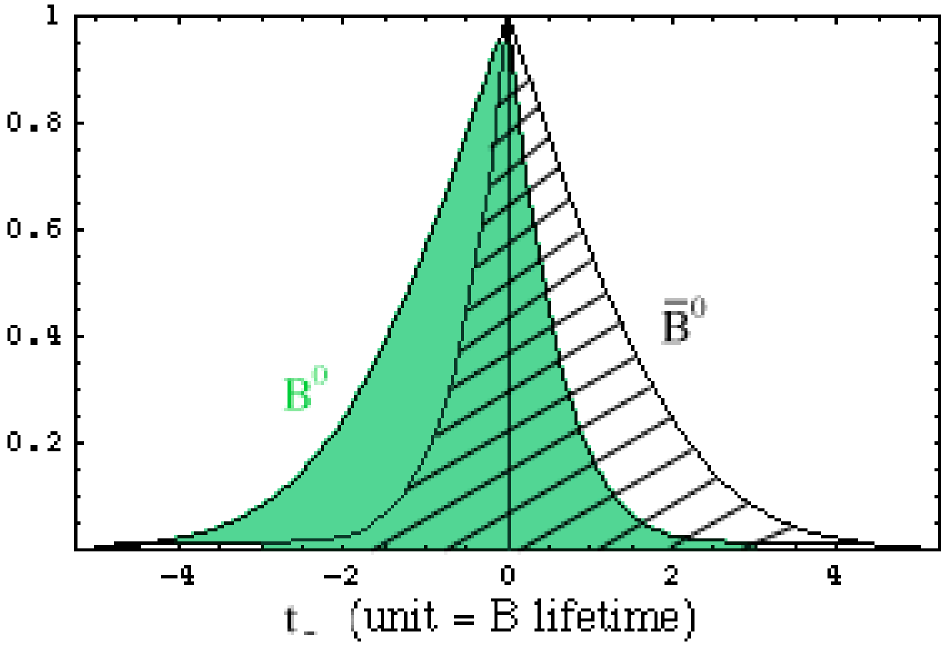
$$\frac{\bar{A}}{A} = \frac{\langle Ks|\bar{K}\rangle}{\langle Ks|K\rangle} \frac{\langle \Psi \bar{K}|H|\bar{B}\rangle}{\langle \Psi K|H|B\rangle}$$
$$= \left[\frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \eta_K^* \right] \left[(-)^{L_{\Psi K}} \eta_\Psi \eta_K \frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}} \eta_B^* \right]$$

$$(CP|K\rangle = \eta_K |\bar{K}\rangle \,, \qquad CP|\Psi\rangle = \eta_\Psi |\Psi\rangle) \,,$$

$$\eta_\Psi = +1, L_{\Psi K} = 1 \to \frac{q\bar{A}}{pA} = \left(\frac{V_{cd} V_{cb}^*}{-V_{td} V_{tb}^*} \right)^* \Big/ \left(\frac{V_{cd} V_{cb}^*}{-V_{td} V_{tb}^*} \right)$$

$$\Rightarrow \quad \Im \left(\frac{q\bar{A}}{pA} \right) = -\sin 2\beta \quad (\Psi K_S)$$

$\Gamma_{4S\to\ell^{\mp}f}(t_-) \quad f = \Psi K_S$



$B^0 \equiv \ell^-$ tag, $\quad \bar{B}^0 \equiv \ell^+$ tag,

Total rate asymmetry $= 0$
$\rightarrow$ need to measure t_-
($\Rightarrow$ Asymmetric B-factory)

[At CLEO, $B^0\bar{B}^0$ are nearly at rest]

Measurements of $\sin 2\phi_1/\sin 2\beta$ at B-factories

ICHEP2000 (Osaka)

Summer, 2000

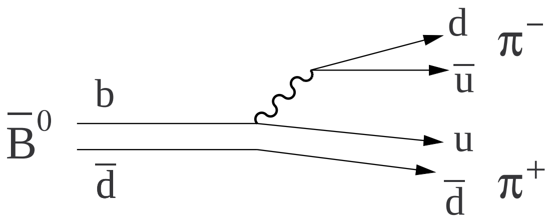
- Belle (KEK) 6.2fb^{-1}
  $\sin 2\phi_1 = 0.45^{+0.44}_{-0.45}$(stat+sys)
- BaBar (SLAC) 9.0fb^{-1}
  $\sin 2\beta = 0.12 \pm 0.37$(stat) ± 0.09(sys)

BCP4 (Ise, Japan)

End of Feb, 2001

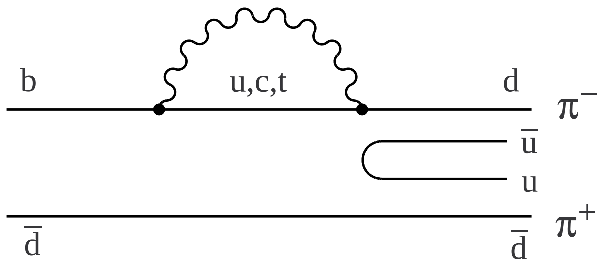
- Belle (KEK) 10.4fb^{-1}
  $\sigma \sin 2\phi_1 \sim 0.34$
- BaBar (SLAC) 26fb^{-1}
  $\sin 2\beta \sim 0.22$

$B \to \pi^+\pi^-$: measurement of α



$$\frac{q}{p}\frac{\bar{A}}{A} = \left(-\frac{V_{tb}^{*}V_{td}}{V_{tb}V_{td}^{*}}\eta_B \; \frac{V_{ub}V_{ud}^{*}}{V_{ub}^{*}V_{ud}}\eta_B^{*}\right)$$
$$= -\left(\frac{V_{tb}^{*}V_{td}}{-V_{ub}V_{ud}^{*}}\right) \Big/ \left(\frac{V_{tb}^{*}V_{td}}{-V_{ub}V_{ud}^{*}}\right)^{*}$$
$$= -e^{2i\alpha}$$

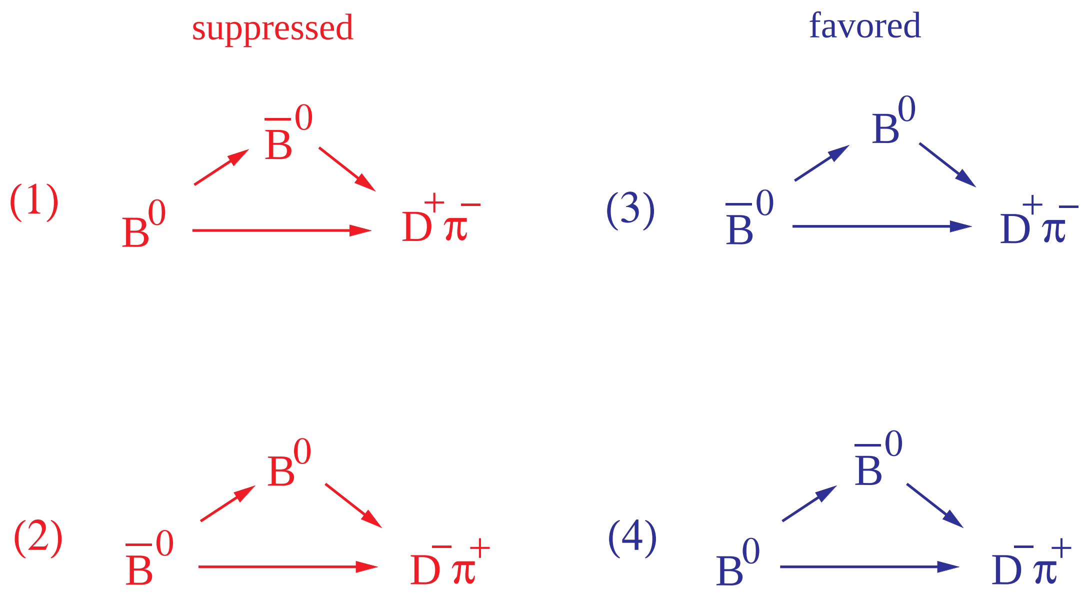
Penguin contamination:

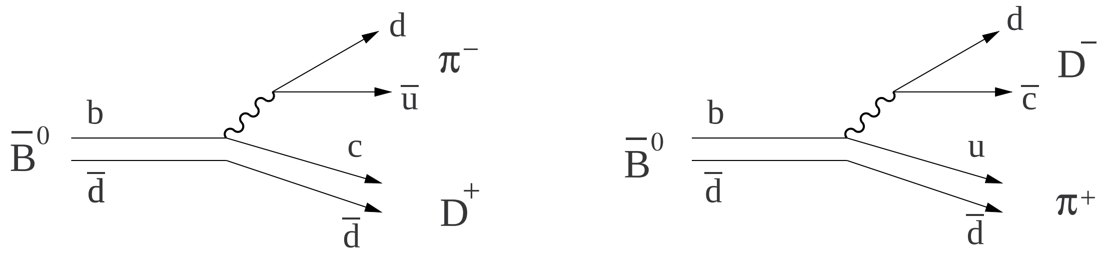


No penguin contribution to I=2.
Extract I=2 contribution by isospin analysis.
Requires $B \to \pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0$.

$B \to D^{(*)+}\pi^-$: Mixing $\to$ non-CP

Sachs (1985), Dunietz, Rosner PRD34 (1986) 1404.





$V_{cb}V_{ud}^*$ $\qquad$ $V_{ub}V_{cd}^*$

$$|\text{Amplitude ratio}|\ r \sim \left|\frac{V_{ub}V_{cd}^*}{V_{cb}V_{ud}^*}\right| \sim 0.4\lambda^2 \sim 0.02$$

Strong phase difference $= \delta$

Assume $\gamma_a = \gamma_b$, $|p/q| = 1$,
(In unit of $|A(B^0 \to D^-\pi^+)A(B^0 \to \ell^+)|^2$)

$$(1)\ \Gamma(D^+\pi^-, \ell^-) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) - (1-r^2)\mathsf{c}_{\delta m t_-} - 2r\,\xi\,\mathsf{s}_{\delta m t_-}\Big]$$

$$(2)\ \Gamma(D^-\pi^+, \ell^+) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) - (1-r^2)\mathsf{c}_{\delta m t_-} + 2r\,\xi'\,\mathsf{s}_{\delta m t_-}\Big]$$

$$(3)\ \Gamma(D^+\pi^-, \ell^+) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) + (1-r^2)\mathsf{c}_{\delta m t_-} + 2r\,\xi\,\mathsf{s}_{\delta m t_-}\Big]$$

$$(4)\ \Gamma(D^-\pi^+, \ell^-) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) + (1-r^2)\mathsf{c}_{\delta m t_-} - 2r\,\xi'\,\mathsf{s}_{\delta m t_-}\Big]$$

$$t_- \equiv t_{\text{sig}} - t_{\text{tag}}, \quad r \sim 0.02$$

$$\xi \equiv \sin(2\beta + \gamma + \delta)\,, \quad \xi' \equiv \sin(2\beta + \gamma - \delta)$$

Asymmetry in the suppressed modes (1) $\leftrightarrow$ (2)

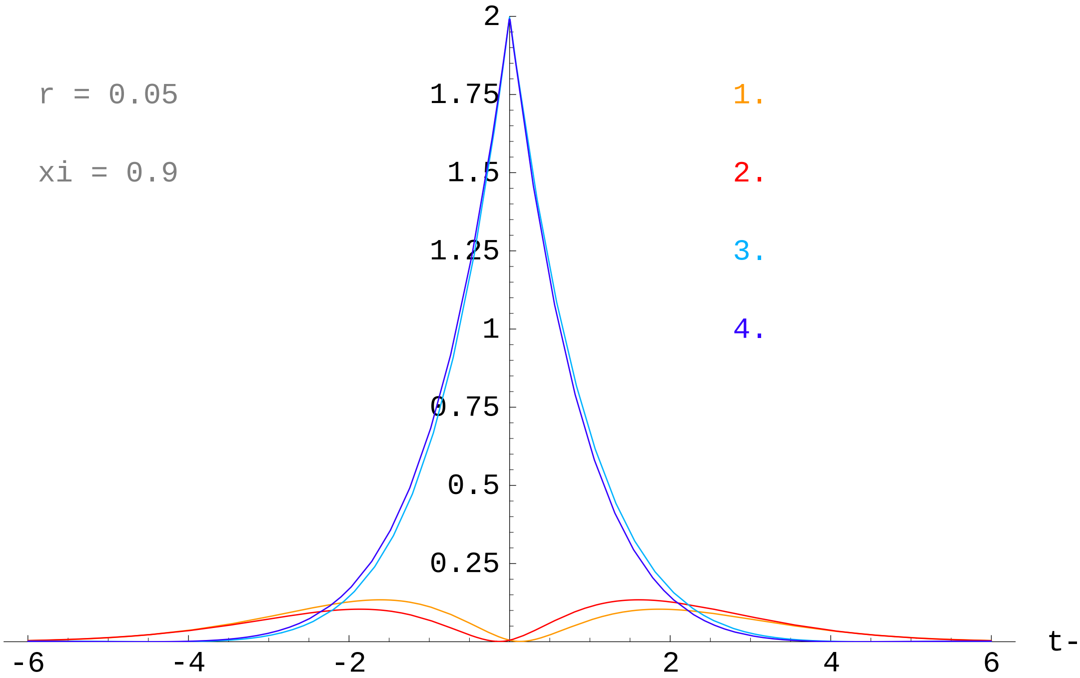
Smaller asymmetry in the favored modes (3) $\leftrightarrow$ (4)

Asymmetry is essentially rate asymmetries:
(1), (2) have similar shapes
(3), (4) have similar shapes

Some gain in $\#\sigma$ by fitting t_-.

t_- distributions (unit $= \tau_B$)

($\delta = 0$ for simplicity)



Asymmetry in the suppressed ('mixed') modes:
($r = 0.02$, $x = \delta m/\gamma = 0.71$)

$$A_s \equiv \frac{(1)-(2)}{(1)+(2)} \sim -\frac{2r}{x}\,\xi \sim -0.057\,\xi$$

Asymmetry in the favored ('unmixed') modes:

$$A_f \equiv \frac{(3)-(4)}{(3)+(4)} \sim \frac{2rx}{2+x^2}\,\xi \sim 0.011\,\xi$$

The favored modes has 5 times stat, but 5 times less asym. $\rightarrow \sqrt{5}$ times less in $\#\sigma$.

Most of the info is in the suppressed modes.

Statistics needed for $D^{(*)}\pi$

$$\sigma_\xi = 0.1 \to \sigma_{A_s} = 0.0057 \to N_s = 30K$$

(suppressed modes)

We need $6 \times 30K = 180K$ total tagged $D\pi$'s.

Belle preliminary:
3.7 $\text{fb}^{-1} \to 282 \pm 25$ lepton-tagged $D^*\pi$'s
(partial reconstruction)

No-bkg equivalent: $\left(\frac{282}{25}\right)^2 \sim 127$

300 $\text{fb}^{-1} \to 10K$ to be compared with 180K needed.

- Need to improve background.
- Need to improve tagging efficiency.
- Add various modes (exclusive and partial).
  (strong phases?)

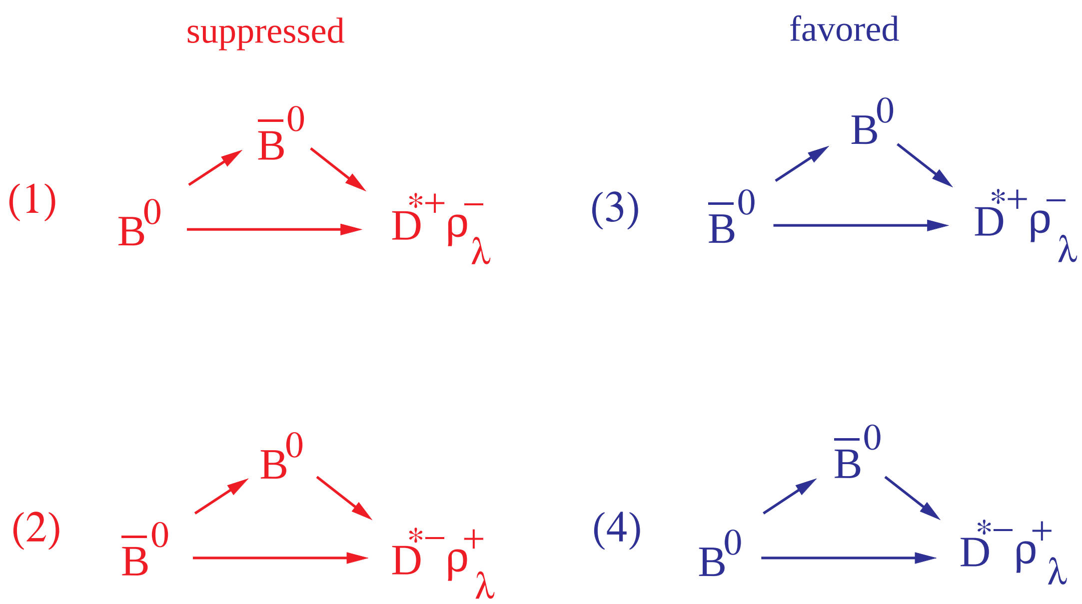
$$\sigma_{\sin(2\beta+\gamma)} \sim (4 \text{ to } 5) \times \sigma_{\sin 2\beta}$$

$B \to D^{*+}\rho^-$

Mixing $\to$ non-CP eigenstate + angular correlation

London, Sinha, Sinha, hep-ph/0005248.

Similar to $B \to D\pi$ (needs to be flavor-tagged):
(Measures $2\beta + \gamma$)



Repeats for each helicity final state.

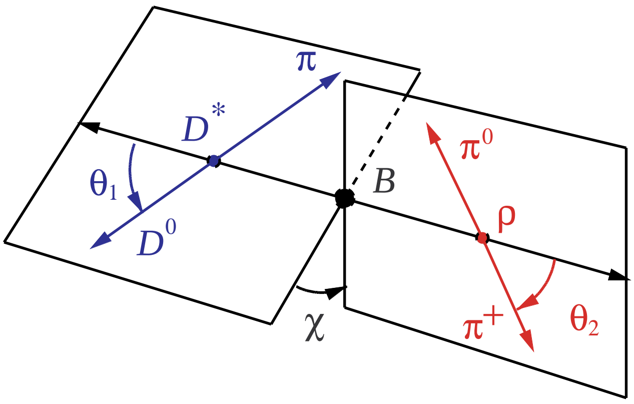
$$\lambda = \begin{cases} +, -, 0 \text{ (helicity basis)}, & \text{or} \\ ||, \perp, 0 \text{ (tranversity basis)} & \end{cases}$$

|Amplitude ratio| $r \sim 0.02$

$\to$ asymmetry in each $\lambda \sim 0.02$

Angular correlation in $B \to D^*\rho$

(helicity basis)



$$\frac{1}{\Gamma}\frac{d^3\Gamma}{dc_{\theta_1}dc_{\theta_2}d\chi} =$$

$$\frac{9}{32\pi}\Big\{4|H_0|^2 c_{\theta_1}^2 c_{\theta_2}^2 + (|H_+|^2 + |H_-|^2) s_{\theta_1}^2 s_{\theta_2}^2$$

$$+[\Re(H_+^* H_-) c_{2\chi} + \Im(H_+^* H_-) s_{2\chi}] 2 s_{\theta_1}^2 s_{\theta_2}^2$$

$$+[\Re(H_+^* H_0 + H_-^* H_0) c_\chi + \Im(H_+^* H_0 - H_-^* H_0) s_\chi] s_{2\theta_1} s_{2\theta_2}\Big\}$$

$(c_x \equiv \cos x\,, \quad s_x \equiv \sin x)$

New ingredients in $D^*\rho$:

Interference between different polarization states
$(\lambda = \parallel, 0, \perp)$

$$\Gamma(B^0 \to D^{*+}\rho^-) = e^{-\gamma t} \sum_{\lambda \le \lambda'} \Big[\Lambda_{\lambda\lambda'} + \Sigma_{\lambda\lambda'} \mathsf{c}_{\delta m t} - \rho_{\lambda\lambda'} \mathsf{s}_{\delta m t} \Big] g_\lambda g_{\lambda'}$$

(g_λ : real functions of angles)

The term with $\lambda = \lambda'$ corresponds to the CP vilating terms we have seen in $D\pi$:

$$\rho_{\lambda\lambda} = \Im\left(\frac{q}{p}(A^*(B^0 \to D^{*+}\rho^-_\lambda)A(\bar{B}^0 \to D^{*+}\rho^-_\lambda)\right)$$

The interference term of ρ have similar size: $(\lambda \ne \lambda')$

$$\rho_{\lambda\lambda'} = \Im\Big(\frac{q}{p}(A^*(B^0 \to D^{*+}\rho^-_\lambda)A(\bar{B}^0 \to D^{*+}\rho^-_{\lambda'}) + A^*(B^0 \to D^{*+}\rho^-_{\lambda'})A(\bar{B}^0 \to D^{*+}\rho^-_\lambda))\Big)$$

$\to$ If similar stat as $D\pi$, simiar sensitivity to $2\phi_1 + \phi_1$.
But has more degrees of freedom to measure.
(more powerful resolving ambiguities.
but more sys. study needed)

Statistics for $D^*\rho$

CLEO: 3.1 fb^{-1} $\rightarrow$ 197 ± 15 signal events.

300 fb^{-1} $\rightarrow$ 19K events. With the high-p_t lepton tag efficiency of 12%, we have 2.3K tagged $D^*\rho$.

This is compared with 10K (bkg-free equivalent for 300 fb^{-1}) of $D^*\pi$ partial reconstruction analysis. Or compared with 180K needed for $\sigma_\xi = 0.1$.

$\rightarrow$ Number of events is $\sim \frac{1}{4}$ of $D^*\pi$, but more paramters to measure.

Comments:

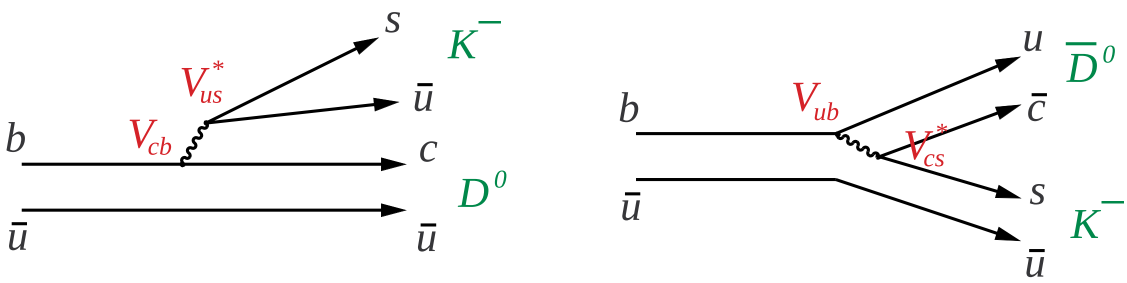
- Partical reconstruction cannot be used. This may not be too big a problem since partial reconstruction efficiency is not that good.
- Need to tackle with the systematics of non-resonant component of ρ.
- Also check the sys. of ρ mass dependence of amplitudes.

CPV in Decay

$$B^- \to D^0_{CP} K^-$$

D^0_{CP} : CP eigenstate. e.g. $K_S\pi^0, K^+K^- \cdots$

Both D^0 and $\bar{D}^0$ decay to a CP eigenstate.
$\to$ 2 diagrams



$a \equiv Amp(B^- \to D^0 K^-)$	$b \equiv Amp(B^- \to \bar{D}^0 K^-)$
$\lambda_c \equiv V_{cb}V^*_{us}$	$\lambda_u \equiv V_{ub}V^*_{cs}$
Color-favored	Color-suppressed
$(a_1 + a_2 \sim 1.24)$	$(a_2 \sim 0.24)$
$\bar{a} \equiv Amp(B^+ \to \bar{D}^0 K^+)$	$\bar{b} \equiv Amp(B^+ \to D^0 K^+)$
$\bar{a} = a^*$	$\bar{b} = b^*$

$(\lambda_c : \lambda_u \sim 1 : 0.4)$

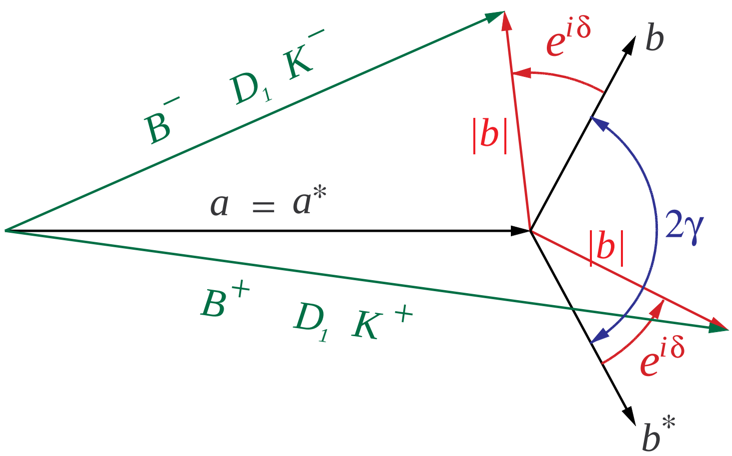
Strong final-state-interaction phase:
b relative to a : $e^{i\delta}$ (δ could be complex)

Phase convention: $a = a^*$

$$D_{1,2} = \frac{1}{\sqrt{2}}(D^0 \pm \bar{D}^0) \quad (CP\pm)\,,$$

$$A(B^- \to D_1 K^-) = \frac{1}{\sqrt{2}}(a + b\, e^{i\delta})$$

$$A(B^+ \to D_1 K^+) = \frac{1}{\sqrt{2}}(a^* + b^* e^{i\delta})$$



$$\left(\text{arg}\,\frac{b}{a} = \text{arg}\,\frac{\lambda_u}{\lambda_c} = \text{arg}\,\frac{V_{ub}V_{cs}^*}{V_{cb}V_{us}^*} \sim -\gamma\right)$$

$\Gamma(B^- \to D_1 K^-) \neq \Gamma(B^+ \to D_1 K^+)$: direct CPV

CP asymmetry expected:

$$a_{cp} \equiv \frac{\Gamma[B^- \to D^0_{CP}K^-] - \Gamma[B^+ \to D^0_{CP}K^+]}{\Gamma[B^- \to D^0_{CP}K^-] + \Gamma[B^+ \to D^0_{CP}K^+]}$$

$$\frac{|b|}{|a|} \sim \underbrace{\text{(color factor)}}_{\frac{a_2}{a_1+a_2} \sim 0.2} \underbrace{\text{(CKM factor)}}_{\frac{\lambda_u}{\lambda_c} \sim 0.4} \sim 0.08$$

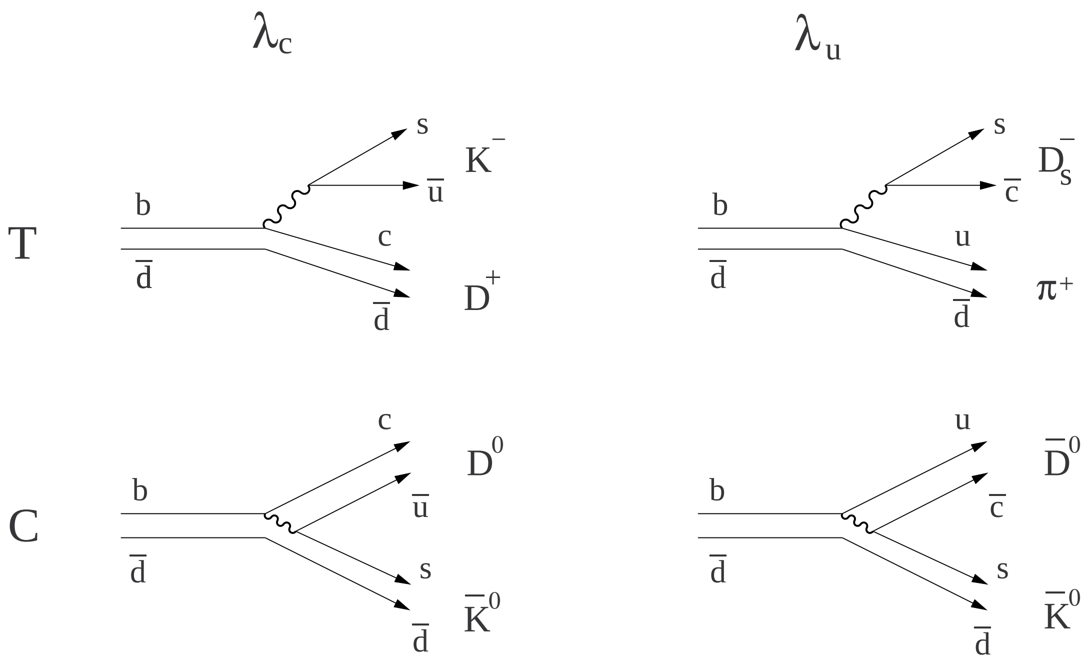
$\rightarrow a_{cp}$ is of order 10%.

Relevant D^0 decay modes:

CP eigenstates	$K_S\,\pi^0$	$1.06 \pm 0.11\%$	$CP-$
	$K_S\,\rho^0$	$0.60 \pm 0.09\%$	$CP-$
	$K_S\,\phi$	$0.84 \pm 0.10\%$	$CP-$
	K^+K^-	$0.43 \pm 0.03\%$	$CP+$
	$\pi^+\pi^-$	$0.15 \pm 0.01\%$	$CP+$
calibration	$K^-\pi^+$	$3.83 \pm 0.12\%$	

D^0 decay FSI phase does not contribute.
$\rightarrow$ can be combined.

Classification of $\bar{B}^0 \to DK$

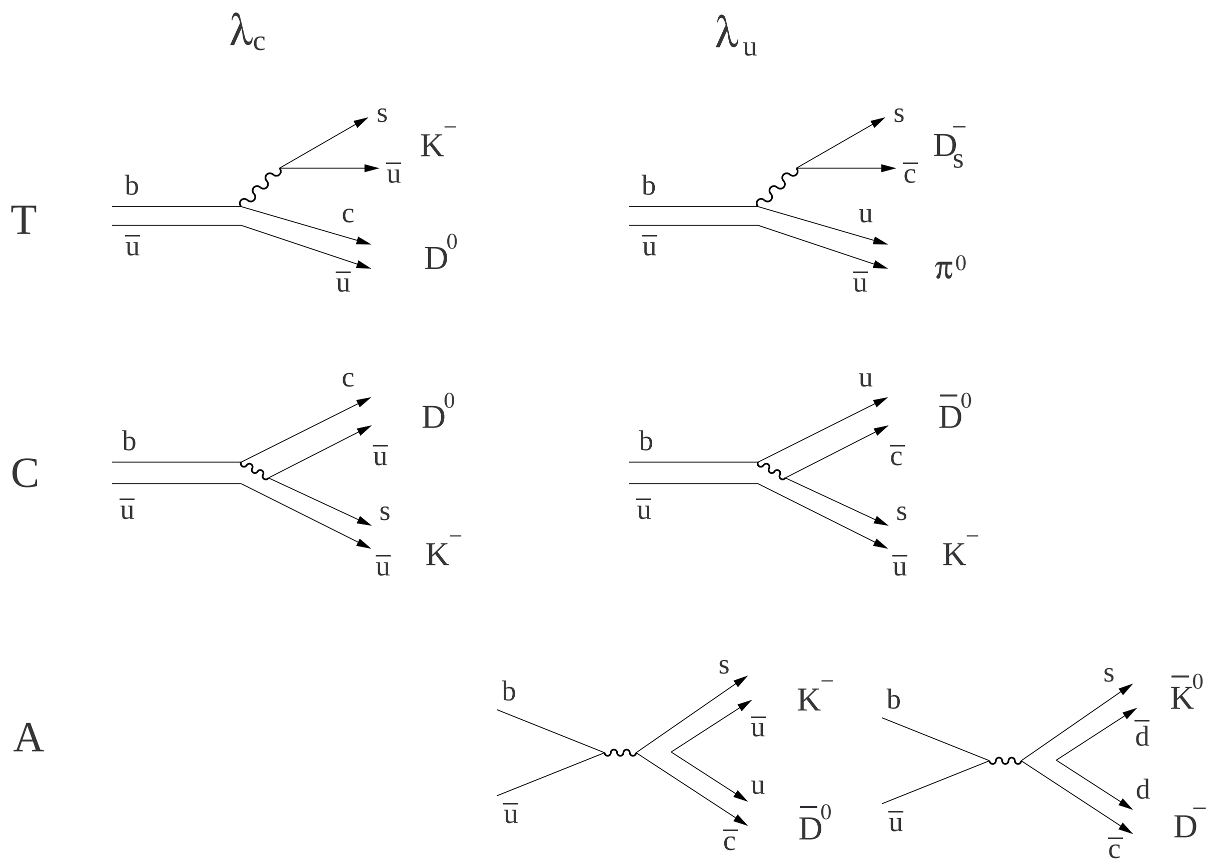


T: tree, C: color-suppressed
(T, C: depends on $b \to c$ or $b \to u$)

$$\lambda_c = V_{cb}V_{cs}^*, \quad \lambda_u = V_{ub}V_{us}^*.$$

$$\begin{aligned} Amp(\bar{B}^0 \to D^+K^-) &= \lambda_c T_c \\ Amp(\bar{B}^0 \to D^0\bar{K}^0) &= \lambda_c C_c \\ Amp(\bar{B}^0 \to \bar{D}^0\bar{K}^0) &= \lambda_u C_u \\ Amp(\bar{B}^0 \to D_s^-\pi^+) &= \lambda_u T_u \end{aligned} \quad (4)$$

Classification of $B^- \to DK$

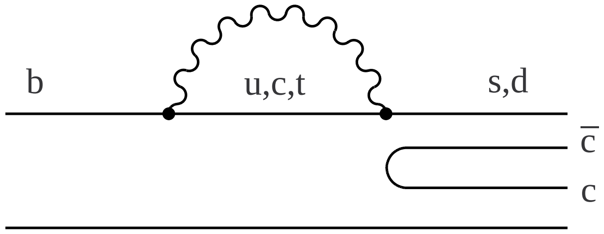


$$Amp(B^- \to D^0 K^-) = \lambda_c T_c + \lambda_c C_c \qquad (5a)$$
$$Amp(B^- \to \bar{D}^0 K^-) = \lambda_u C_u + \lambda_u A \qquad (5b)$$
$$Amp(B^- \to D^- \bar{K}^0) = \lambda_u A \qquad (5c)$$
$$Amp(B^- \to D_s^- \pi^0) = \frac{1}{\sqrt{2}} \lambda_u T_u \qquad (5d)$$

$B \to DK$ **Modes**

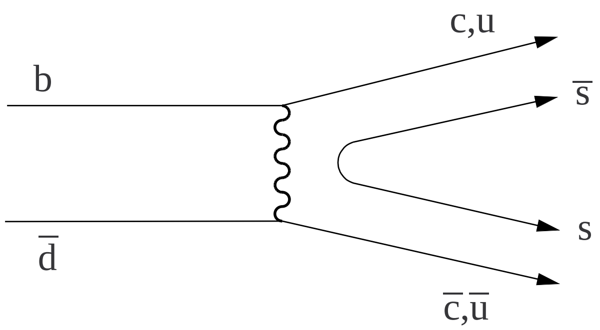
Final state: one charm, one strange.

- **No penguine contaminations**



Penguine should have even number of charms.
(True for charged and neutral B)

- **Neutral B has no annihilations**



Annihilations should have even number of stranges.

- **All tree diagrams (no complications by loops)**

Final-state Rescatterings

Final-state rescattering can occur:

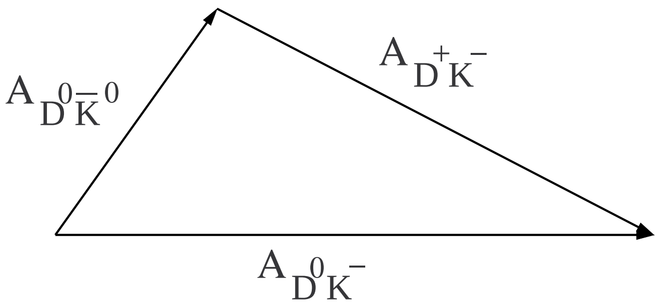
$$\bar{B}^0 \to D^+K^-(T_c) \to D^0\bar{K}^0(C_c)$$
$$\bar{B}^0 \to D_s^-\pi^+(T_u) \to \bar{D}^0\bar{K}^0(C_u)$$

We **define** T_c, C_c, T_u, C_u **by (4)** including rescattering effects.

Then, is (5a) still true?

$$\begin{aligned} Amp(B^- \to D^0K^-) &= \lambda_c T_c + \lambda_c C_c \\ &= Amp(\bar{B}^0 \to D^+K^-) + Amp(\bar{B}^0 \to D^0\bar{K}^0) \end{aligned}$$

which is nothing but the isospin relation for H_{eff} having $|1/2, -1/2\rangle$ structure: (good to all orders as long as $m_u = m_d$)

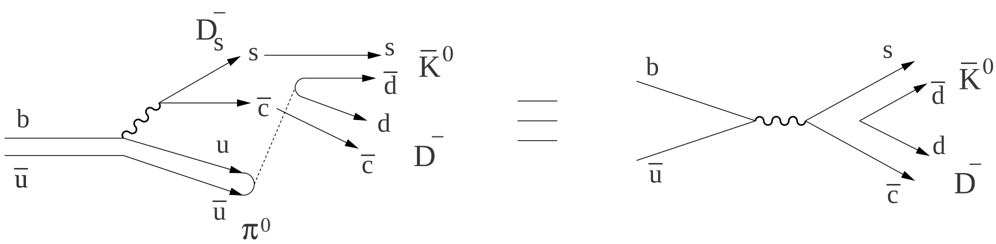


Final-state Rescatterings - annihilation

Final-state $D^- \bar{K}^0$ can be reached by

$$B^- \to D_s^- \pi^0 \to D^- \bar{K}^0$$

This is a 'long-distance' annihilation:

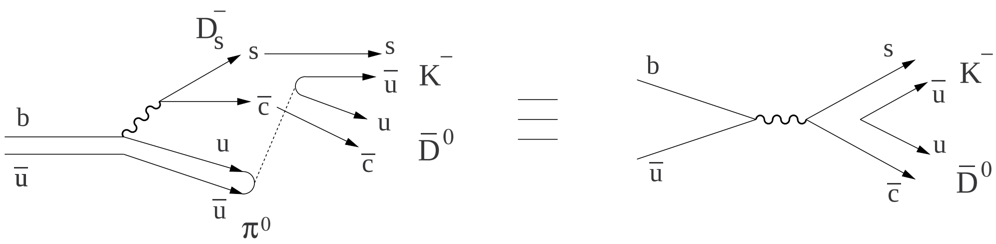


We thus **define** A by

$$Amp(B^- \to D^- \bar{K}^0) = \lambda_u A \quad (5c)$$

including the rescattering effect.

Then, the annihilation in $B^- \to \bar{D}^0 K^-$ (5b) has exactly the same rescattering contribution:



Gronau-London-Wyler (GLW) method

$$a \equiv A(B^- \to D^0 K^-) = \lambda_c(T_c + C_c)$$
$$b \equiv A(B^- \to \bar{D}^0 K^-) = \lambda_u(C_u + A)$$

Measure $|a|$, $|b|$, $A(B^- \to D_1 K^-)$, and $A(B^+ \to D_1 K^+)$.
Reconstruct the two triangles $\to \gamma$.

Problem:

How to measure $B = Amp(B^- \to \bar{D}^0 K^-)$?

$$B^- \xrightarrow{b} \bar{D}^0\, K^- \quad \text{but also} \quad B^- \xrightarrow{a} D^0\, K^-$$
$$\hookrightarrow K^+\pi^- \qquad\qquad\qquad \hookrightarrow K^+\pi^- \; (DCSD)$$

The ratio of the two amplitudes (r_{DCSD}):

$$r_{DCSD} = \underbrace{\frac{A}{B}}_{\sim \frac{1}{0.08}} \underbrace{\frac{Amp(D^0 \to K^+\pi^-)}{Amp(D^0 \to K^-\pi^+)}}_{0.088 \pm 0.020} \sim 1$$

(CLEO 94)

Phase of r_{DCSD} not known $\to$ difficult to measure $|b|$.
(Difficult to detect $D^0 \to X_s^- \ell^+ \bar{\nu}$)

The interference of DCSD and B-amplitude causes CP asymmetry of **order unity** in the wrong-sign $K\pi$ modes:

ADS method to extract ϕ_3/γ

Measure $B^- \to DK^-$ in two decay modes of D:
wrong-sign flavor-specific modes or CP eigenstates, say $K^+\pi^-$ and $K_S\,\pi^0$ (and their conjugate modes).

$$\Gamma[B^- \to (K^+\pi^-)K^-] \quad \Gamma[B^+ \to (K^-\pi^+)K^+]$$
$$\Gamma[B^- \to (K_S\,\pi^0)K^-] \quad \Gamma[B^+ \to (K_S\,\pi^0)K^+]$$

Assume we know $|A|$ and D branching fractions
$\longrightarrow$ 4 unknowns:

$$\phi_3\,, \quad \delta_{K^-\pi^+}\,, \quad \delta_{K_S\pi^0}\,, \quad \frac{|B|}{|A|}$$

$\longrightarrow$ can be solved.

Statistics: Possible at B-factories
(300 fb^{-1} needed for $\sigma_{\phi_3} \sim 0.3$ rad.)

Avoid using wrong-sign $B^+ \to D^0 K^+$

External input (experiment, theory):

$$r = \left|\frac{B}{A}\right| = \left|\frac{\bar{B}}{\bar{A}}\right| \sim 0.08$$

Measure

$$\begin{aligned}
\Gamma(B^- \to D_1 K^-) &= 1 + r^2 + 2r\cos(\phi_3 + \delta) \\
\Gamma(B^- \to D_2 K^-) &= 1 + r^2 - 2r\cos(\phi_3 + \delta) \\
\Gamma(B^+ \to D_1 K^+) &= 1 + r^2 + 2r\cos(\phi_3 - \delta) \\
\Gamma(B^+ \to D_2 K^+) &= 1 + r^2 - 2r\cos(\phi_3 - \delta)
\end{aligned}$$

in unit of $\Gamma(B^- \to D^0 K^-)$.

$\to$ fit for ϕ_3 and δ.

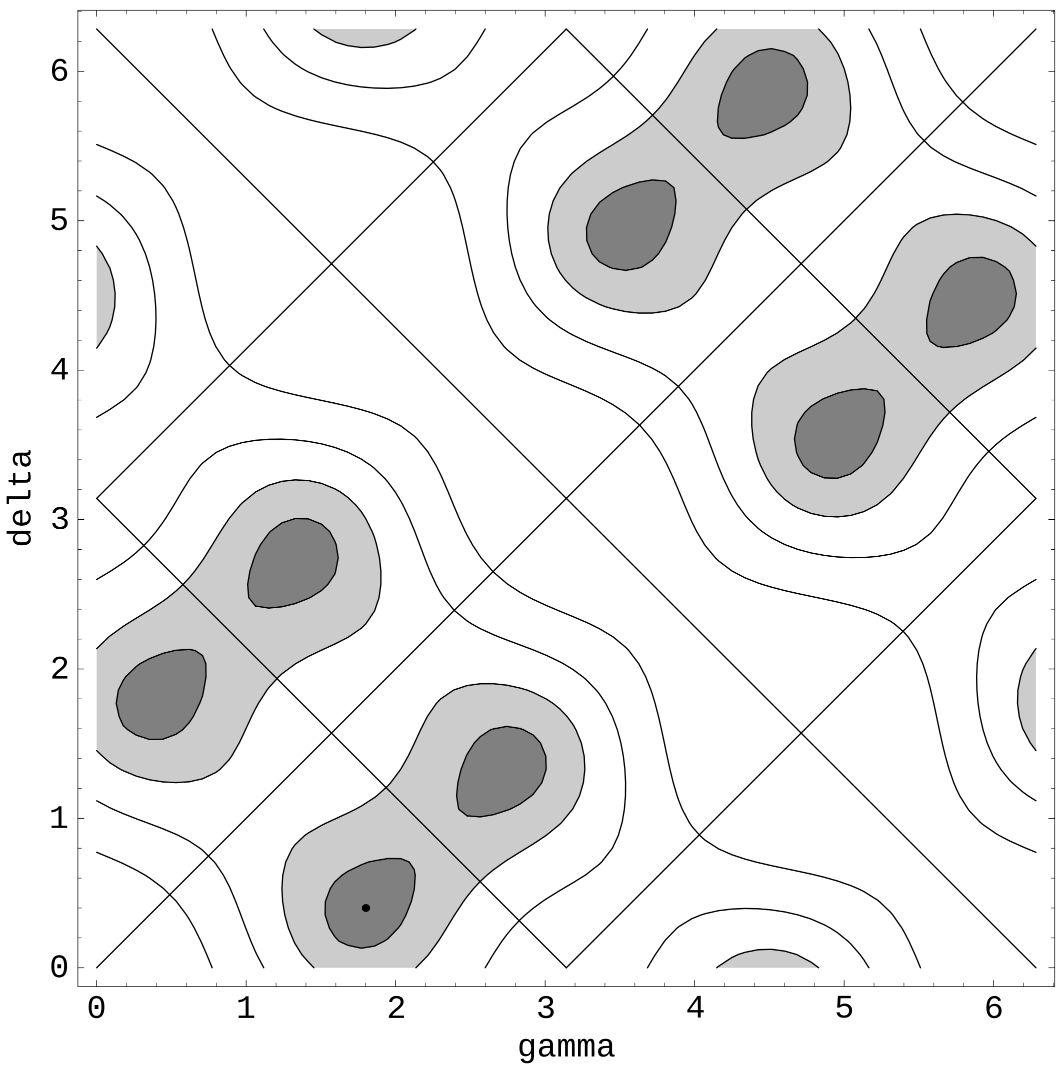
Ambiguity: the equations are symmetric under

$$\begin{cases} \phi_3 \to & n\pi + \delta \\ \delta \to & -n\pi + \gamma \end{cases} \text{or} \begin{cases} \phi_3 \to n\pi - \delta \\ \delta \to n\pi - \phi_3 \end{cases} \qquad (n : \text{integer})$$

Fit result for ϕ_3 and δ

Input:

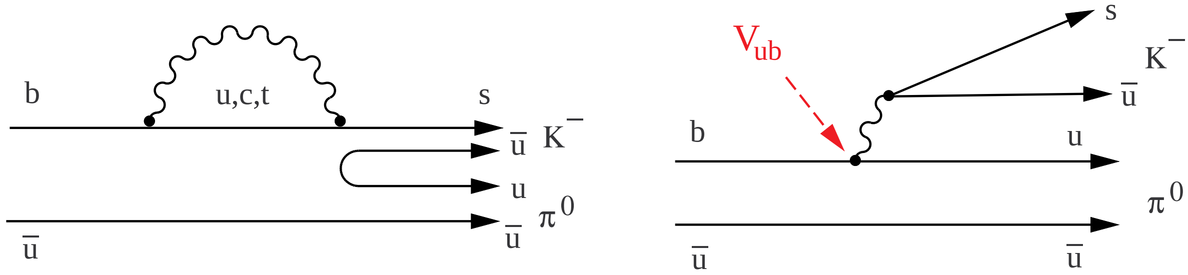
$\phi_3 = 1.8\,,\, \delta = 0.4$
$\sigma(\Gamma' s) = 10\%$ (100 events each)
(300fb^{-1})



Using $B \to K\pi, \pi\pi$

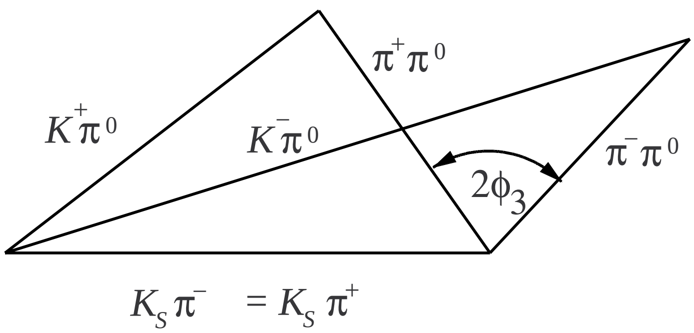
Tree-penguin interference
$\to$ large direct CP asymmetries expected.

For example: $B^- \to K^-\pi^0$



Interference $\to$ asymmetry $B^- \to K^-\pi^0$ vs $B^+ \to K^+\pi^0$
(infromation on arg$V_{ub} = -\phi_3/\gamma$.)

Need to remove unknown strong FSI phase.
One historical method:



- Charged B modes $\rightarrow$ self-tagging.
- SU(3) breaking effect are reasonably under control. Complication by EW penguins which breaks the isospin.
- Requires substantial development in theory.
  $\rightarrow$ QCD factorization formalism:
  Benecke, Buchalla, Neubert, Sachrajda hep-ph/0006124.

Probably the way to approach is to take theorist's predictions of branching ratios (ratios of branching ratios) for various modes and perform a global fit.

Summary

- Test of SM involves sizes as well as phases of CKM elements.
  $\rightarrow$ Enough efforts needed for measurements of $|V_{ij}|$'s.

- Lepton asymmetry (CPV in mixing) sensitivity is already $\sigma_\delta \sim 0.1$. It is quite possible that non-zero δ is measured soon.

- β/ϕ_1: in good shape both theoretically and experimentally.
  $\sigma_{\sin 2\beta} \sim 0.1$ with 150 fb^{-1} (in a few years).

- α/ϕ_2: $\pi^+\pi^-$ mode - $\sigma_{\sin 2\alpha} \sim 3\sigma_{\sin_{2\beta}}$ (stat only)

- γ/ϕ_3: DK, $D^*\pi$, $D^*\rho$ have similar sensitivities.
  $\sigma_{\gamma/phi_3} \sim 20^\circ$ at 300 fb^{-1} each.
  $K\pi$, $\pi\pi$ have more statistical power, but requires substantial theoretical development.