Top electroweak couplings study using di-leptonic state at $\sqrt{s}$ = 500 GeV, ILC with the Matrix Element Method

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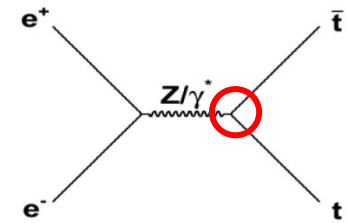
Introduction

- Top quark is the heaviest particle in the SM. Its large mass implies that it is strongly coupled to the mechanism of electroweak symmetry breaking (EWSB)

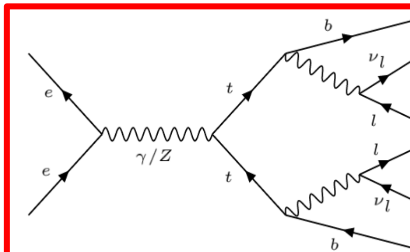
→ **Top EW couplings are good probes for New physics behind EWSB**

$$\mathcal{L}_{\rm int} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t} \gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$



- Di-leptonic state of top-pair production has rich observables, so one can get higher intrinsic sensitivity and do multi-parameters fit.

→ **Out target is the di-leptonic state**



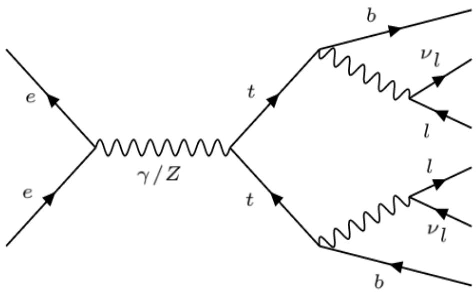
- Use the Matrix Element method to handle many observables and many parameters simultaneously.

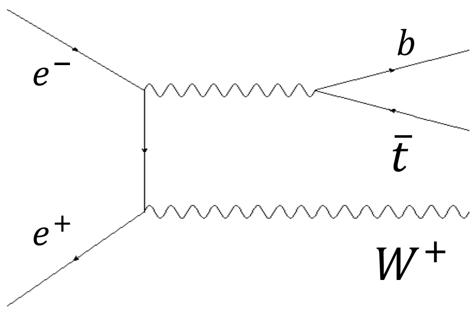
Set Up

$\sqrt{s}$	500 GeV
Polarization (P_{e^-}, P_{e^+})	(-0.8, +0.3) “Left” / (+0.8, -0.3) “Right”
Integrated luminosity	500 fb^{-1} (250 fb^{-1} for each polarization)
Generator	Whizard (including ISR/BS, $\gamma\gamma \rightarrow \mathrm{hadrons}$)
Detector model	ILD_01_v05 (TDR version)

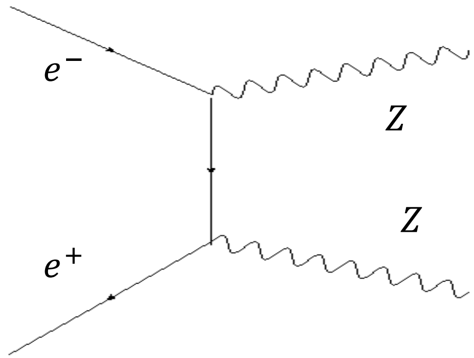
Sample of events

- Signal : $ee \rightarrow bb\mu\mu\nu\nu$
  - We focus on only the ***di-muonic state*** which is the most accurate to be reconstructed in the di-leptonic state.
  - This includes **top pair production**, single top production and so on.





- Main background
  - $ee \rightarrow bbll\nu\nu$ (except for $bb\mu\mu\nu\nu$)
  - $ee \rightarrow qqll$ (mainly ZZ)
  - $ee \rightarrow bbl\nu qq$ (mainly top pair production)



Process of study

Event Reconstruction

- Isolated muon reconstruction
- $\gamma\gamma \rightarrow$ hadrons suppression
- b-jet reconstruction
- ***Kinematical reconstruction*** ***Today's topic***

Analysis and Discussion

- Helicity angles computation
- ***Analysis with Matrix Element Method*** ***Today's topic***
- ***Optimal variables computation*** ***Today's topic***
- Assessment of goodness of fit

Kinematical Reconstruction : Strategy

Neutrinos and photon of ISR cannot be reconstructed by detectors.

There are **7 unknowns** in di-muonic state of top pair production.

$$P_{x,\nu}, P_{y,\nu}, P_{z,\nu}, P_{x,\bar{\nu}}, P_{y,\bar{\nu}}, P_{z,\bar{\nu}}, P_{z,\gamma_{\mathrm{ISR}}}$$

To recover them, we impose **8 constraints**,

- Initial state constraints : $E_{total} = 500\ \mathrm{GeV}, \vec{P}_{\mathrm{total}} = \vec{0}$
- Mass constraints : $m_t = m_{\bar{t}} = 174\ \mathrm{GeV}, m_{W^+} = m_{W^-} = 80.4\ \mathrm{GeV}$

There are enough constraints to determine the missing variables.

Kinematical Reconstruction : Algorithm

Introduce ***4 free parameters*** **:** $\boldsymbol{P_{x,\nu}, P_{y,\nu}, P_{z,\nu}, P_{z,\gamma_{ISR}}}$

Other missing variables are defined as follows;

$$P_{x,\bar{\nu}} = -\left(P_{x,\text{Visible}} + P_{x,\nu}\right), P_{y,\bar{\nu}} = -\left(P_{y,\text{Visible}} + P_{y,\nu}\right), P_{z,\bar{\nu}} = -\left(P_{z,\text{Visible}} + P_{z,\nu} + P_{z,\gamma_{\text{ISR}}}\right)$$

(All physics variables also can be computed using these parameters.)

Define the likelihood function;

$$\boldsymbol{L_0 = BW(m_t, 174)BW(m_{\bar{t}}, 174)BW(m_{W^+}, 80.4)BW(m_{W^-}, 80.4)Gaus(E_{total}, 500)}$$

(BW : Breit-Wigner function, $Gaus$: Gaussian function, other parameters are written in backup)

To correct the energy resolution of b-jets reconstruction, we add ***2 parameters,***

$\boldsymbol{E_b, E_{\bar{b}}}$, and resolution functions, R, to the likelihood function.

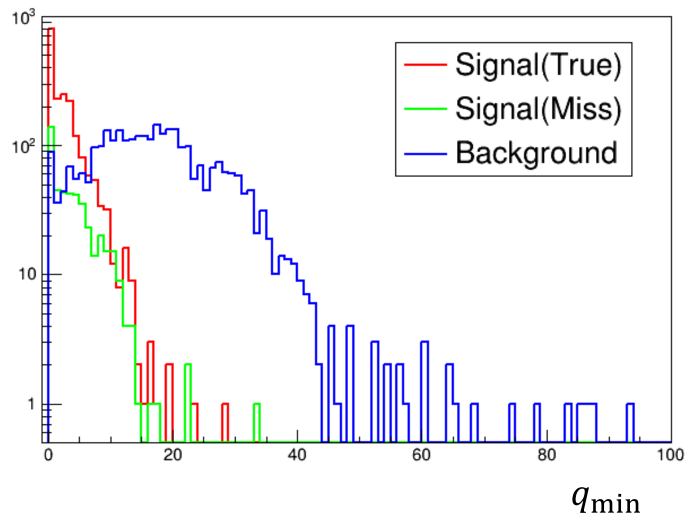
$$\boldsymbol{L = L_0 * R\left(E_b, E_b^{\text{reconstructed}}\right)R\left(E_{\bar{b}}, E_{\bar{b}}^{reconstructed}\right)}$$

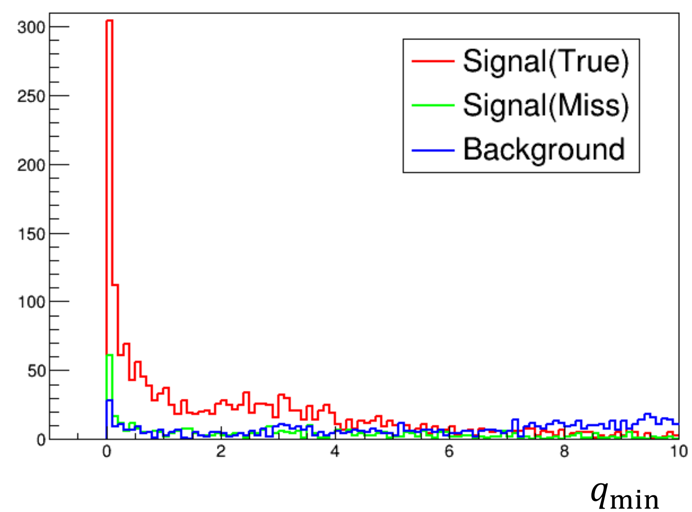
Kinematical Reconstruction

For simplicity, we define $\boldsymbol{q = -2\log L + C}$ (scaled as the minimum value becomes 0)

There are two possibilities for combination of b-jet and muon.

→ Define ***the best candidate*** as a candidate having **smaller** $\boldsymbol{q}$ and $\boldsymbol{q_{\mathrm{min}}}$ as q of the best candidate. One can check that it is true or miss combination by generator information.

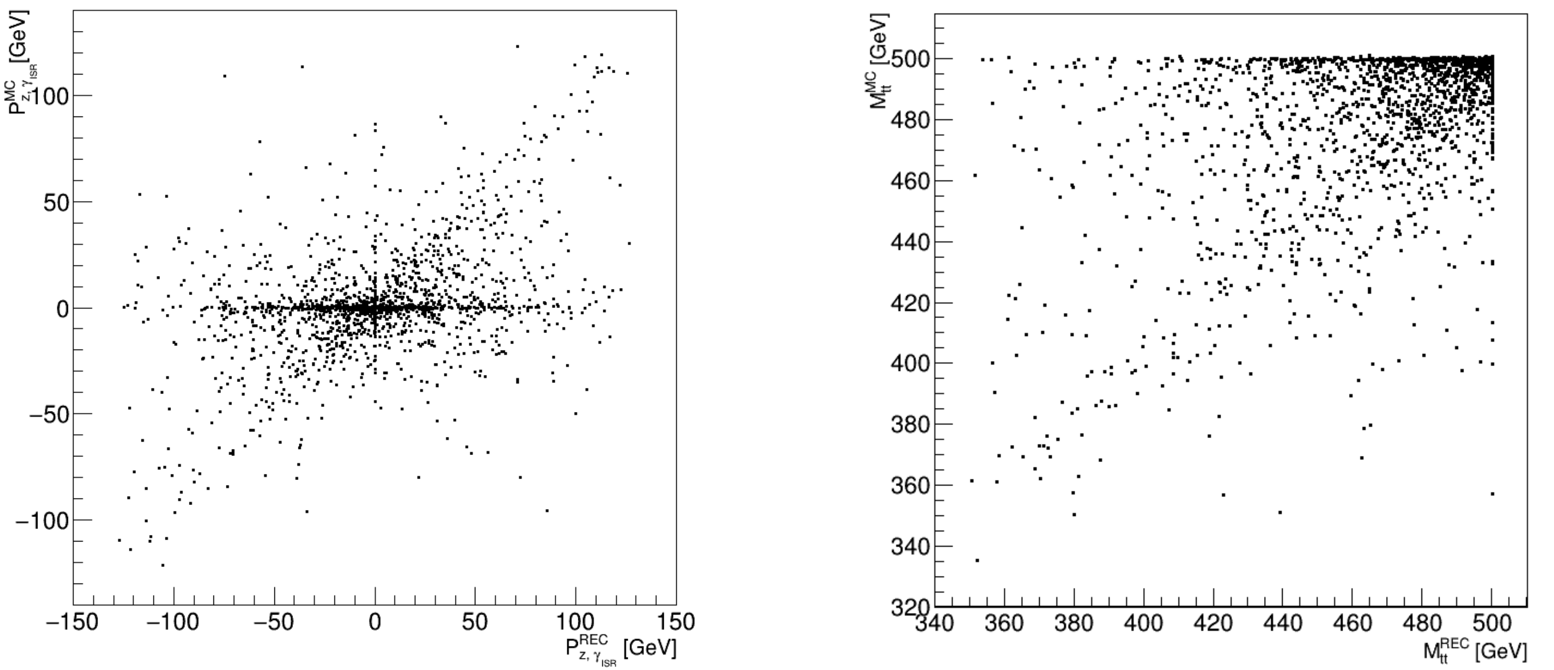




q_{min} distribution of Left polarization events (left : whole distribution, right : zoomed one)

→ *Cut on* q_{min} *is useful to reduce the background and miss combination events.*

ISR photon



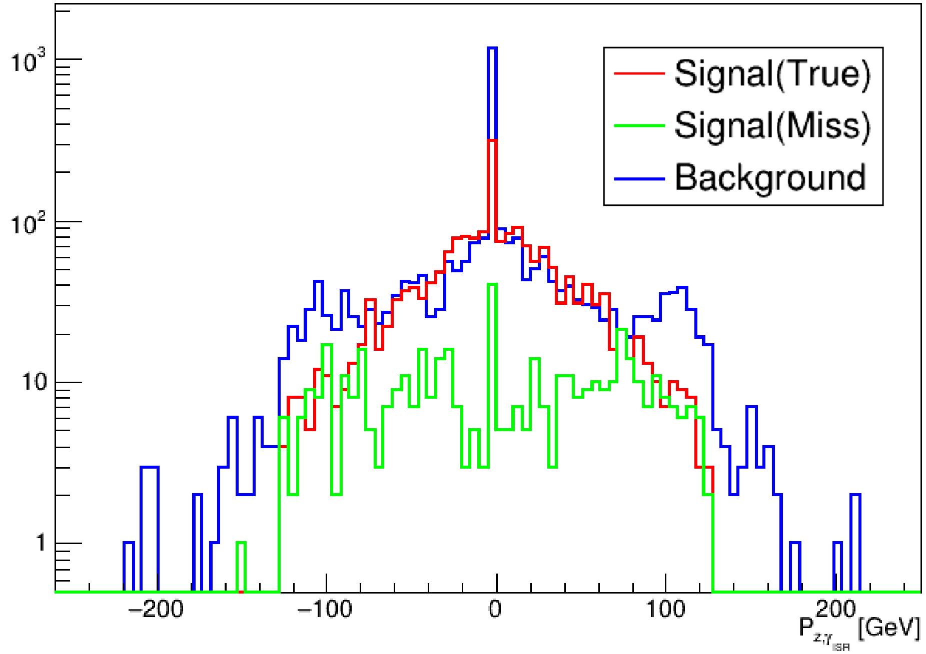
Scatter plots between MC and reconstructed of Left polarization and signal events

(left : $P_{z,\gamma_{\mathrm{ISR}}}$, right : mass of top pair)

Small correlations between MC and reconstructed are observed.

→ *Cut on $P_{z,\gamma_{ISR}}$ or M_{tt} is useful to reduce hard ISR events.*

ISR photon



$P_{z,\gamma_{\mathrm{ISR}}}$ distribution of Left polarization events

$P_{z,\gamma_{\mathrm{ISR}}}$ distribution of miss combination events are wider than true combination

→ *Cut on $P_{z,\gamma_{ISR}}$ is also useful to reduce miss combination events*

Cut table

250 fb^{-1} (-0.8,+0.3) Left	initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8	$q_{\min} < 3$ & $\lvert P_{z,\gamma}\rvert < 50$ GeV
Signal $bb\mu\mu\nu\nu$ (True)	**2961**	**2725** (e = 92.0 %)	**1921** (80.9%) (e = 80.2%)	**945** (90.7%) (e = 35.2%)
Signal $bb\mu\mu\nu\nu$ (Miss)			453 (19.1%)	97 (9.3%)
$bbll\nu\nu$ (except $bb\mu\mu\nu\nu$)	23609	387	335	71
$bbl\nu qq$	104114	40	31	3
$qqll$ (ZZ)	91478	13800	2519	21
ll (weight = 4)	212274 (→ 849096)	74961 (→ 299844)	90 (→ 360)	0
$l\nu l\nu$ (WW) (weight = 4)	377058 (→ 1508232)	1884 (→ 7536)	3 (→ 12)	0
$lll\nu l\nu$ $(llWW)$	3021	947	19	0

Matrix Element Method

We assume that full matrix squared, $|M|^2$, includes up to quadratic terms of the form factors, hence the expected number of events also includes up to quadratic terms;

$$|M|^2 = \left(1 + \sum_i \omega_i \delta F_i + \sum_{ij} \widetilde{\omega}_{ij} \delta F_i \delta F_j\right) |M|^2_{\mathrm{SM}}$$

$$N = \left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij} \delta F_i \delta F_i\right) N_{\mathrm{SM}}$$

where δF_i is deference of the form factor from SM.

Matrix Element Method

Matrix element method is based on the maximum likelihood method and a likelihood function is written by $|M|^2$ and N;

$$-2\log L(\delta F) = \chi^2(\delta F) =$$

$$-2\left(\sum_{e=1}^{N_{\mathrm{event}}} \log\left(1+\sum_i \omega_i(\Phi_e)\delta F_i + \sum_{ij}\widetilde{\omega}_{ij}(\Phi_e)\delta F_i \delta F_j\right) - N_{\mathrm{event}}\log\left(1+\sum_i \Omega_i \delta F_i + \sum_{ij}\widetilde{\Omega}_{ij}\delta F_i \delta F_j\right)\right)$$

where Φ_e is helicity angles which have sensitivity for the form factors. $\chi^2(\delta F)$ is scaled to 0 at $\delta F = 0$.

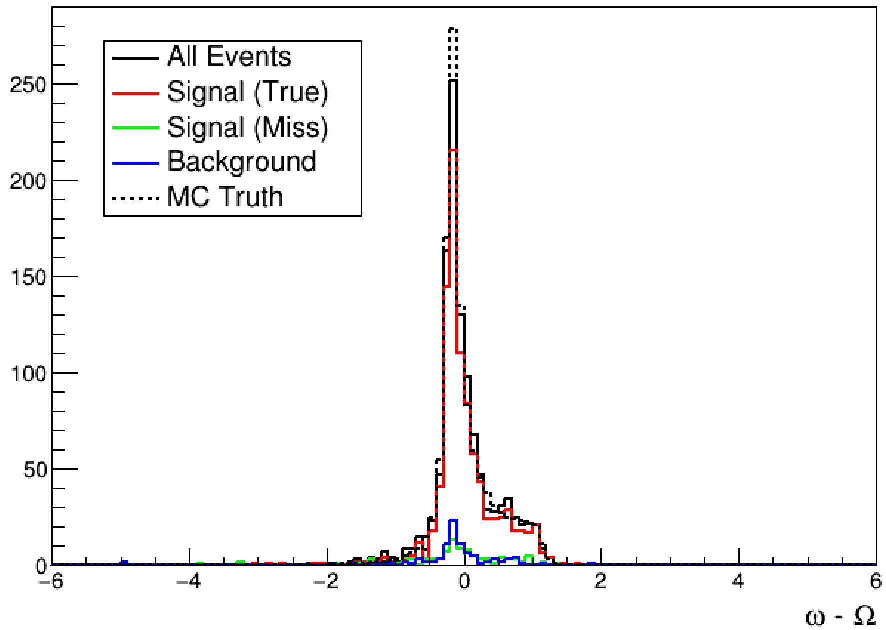
If we use the information of yields with Poisson distribution, the second term can be replaced as $N_{\mathrm{event}}\left(\sum_i \Omega_i \delta F_i^{\mathrm{SM}} + \sum_{ij}\widetilde{\Omega}_{ij}\delta F_i^{\mathrm{SM}}\delta F_j^{\mathrm{SM}}\right)$

Matrix Element Method

What we must do to fit the form factors correctly is to reconstruct ω_i correctly.

Indeed the results of fit are related with ω_i and Ω_i which are called ***optimal variables***

- $\delta F_i^{\mathrm{Fit}} \simeq \frac{<\omega_i - \Omega_i>}{<{\omega_i}^2>}$
- Covariance matrix, V_{ij} : $V_{ij}^{-1} \simeq N_{\mathrm{event}} < \omega_i \omega_j >$

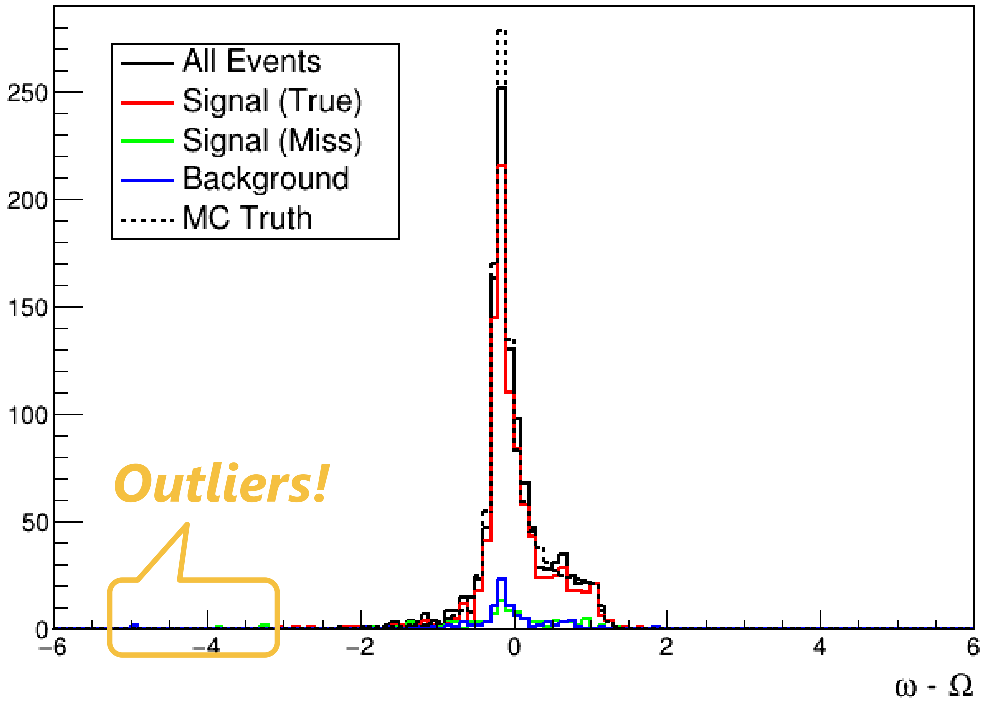


$\omega - \Omega$ distribution for δF_{1V}^{Z} of Left polarization events

Reconstructed (All Events) are similar with MC Truth

Outliers

A few events are distributed far from other events. It can be caused by detector effects and ISR effects, in other wards they are badly reconstructed events.



$\omega - \Omega$ distribution for δF_{1V}^{Z} of Left polarization events (same as last slide)

These events easily induce biases on results of fit. → ***Outliers***

We fit $\omega - \Omega$ distribution within a region not including *outliers*. Efficiency cost is only 1.6%(0.8%) for Left(Right) polarization events.

Preliminary Results without Outliers

Results of 10 parameters multi-fit

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & -0.0148 \pm 0.0129 \\ \mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0232 \pm 0.0226 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0140 \pm 0.0184 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0309 \pm 0.0286 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0736 \pm 0.0371 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.0564 \pm 0.0601 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma} & -0.0059 \pm 0.0226 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{Z} & -0.0377 \pm 0.0389 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma} & +0.0403 \pm 0.0238 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{Z} & +0.0007 \pm 0.0343 \end{bmatrix}$$

Efficiency : 35 %

CL(*) : 35 % (* It is discussed in backup slides)

This precision is comparable with semi-leptonic state analysis considering difference of statistics. ***One can fit more parameters simultaneously.***

But there are still small biases. We have room for improvement → ***Next Slide***

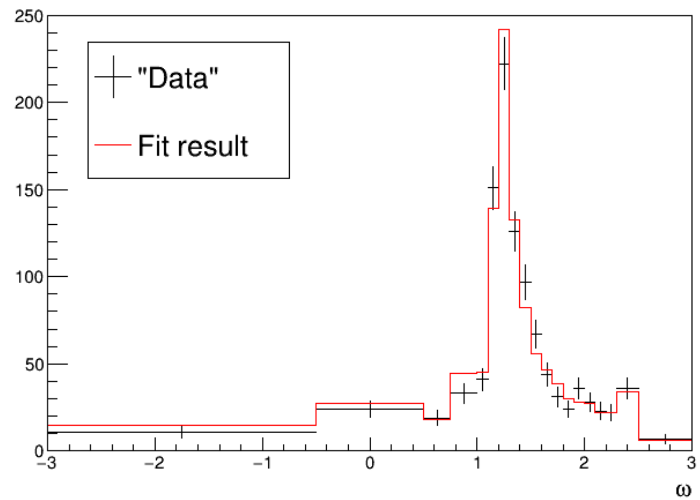
Improved method : Binned likelihood analysis

Estimate the number of events in each bin of the ω distribution described as function of δF, $N_b(\delta F)$, from the full MC simulation.

Fit $N_b(\delta F)$ to the *"data"* using the following $\chi^2(\delta F)$.

$$\chi^2(\delta F) = \sum_{b=1}^{N_{\mathrm{bin}}} \frac{\left(n_b^{\mathrm{Data}} - N_b(\delta F)\right)^2}{n_b^{\mathrm{Data}}}$$

where n_b^{Data} is the number of events in bin b of the *"data"*.



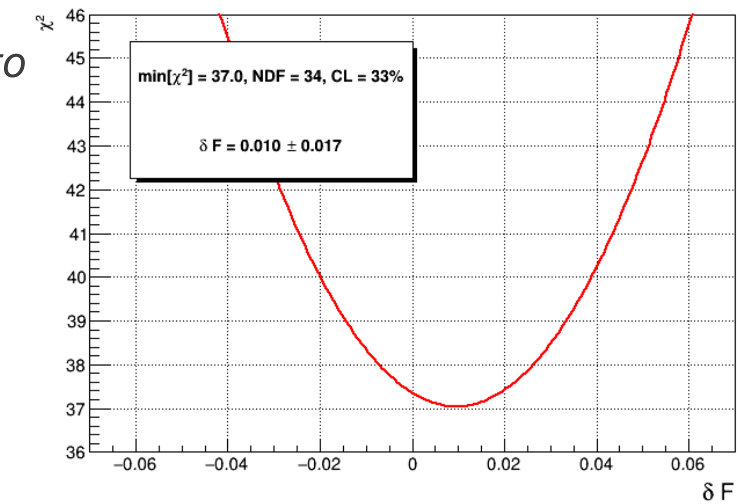
ω distribution for δF_{1V}^{Z} of Left polarization events

This method is by construction unbiased if the full MC simulation describes the "data" and one can use $\chi^2(\delta F)$ to assess the goodness of fit.

Example : Result of 1 parameter fit

$$\delta \tilde{F}_{1V}^{Z} = 0.010 \pm 0.017 \text{ (CL} = 33\%)$$

For the multi-parameter fit, more statistics of the full MC simulation is required.



$\chi^2(\delta \tilde{F}_{1V}^{Z})$ function

Summary

- Missing neutrinos and ISR/BS photon are reconstructed by kinematical reconstruction.
  - $P_{z,\gamma_{ISR}}$ cannot be reconstructed precisely, but it is useful to reduce the miss combination and hard ISR events.

- $\omega - \Omega$ distributions, which called optimal variables, can be reconstructed. One rejects outliers events and fit form factors.

→ ***Comparable results with semi-leptonic analysis. More parameters can be fitted simultaneously.***

- Small biases are still observed. (Goodness of fit is also not so great. It is discussed in backup slides)

→ ***The binned likelihood analysis can measure the parameters without biases and assess goodness of fit.***

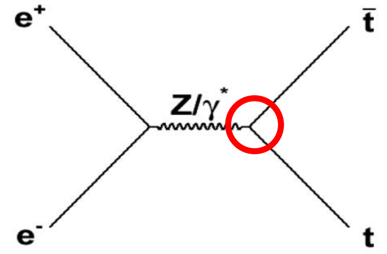
Backup

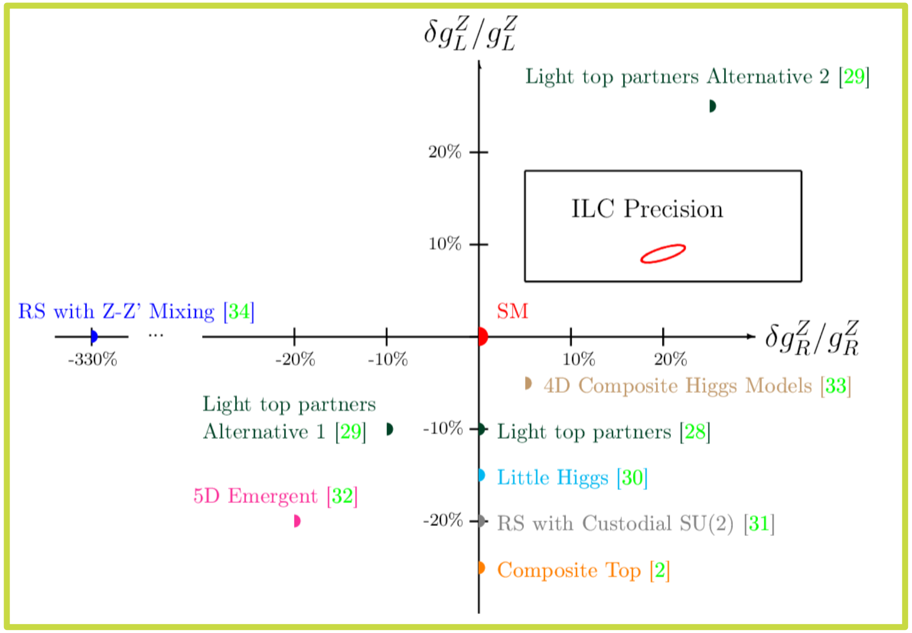
Top EW Couplings Study

- Top quark is the heaviest particle in the SM. Its large mass implies that it is strongly coupled to the mechanism of electroweak symmetry breaking (EWSB)

→ Top EW couplings are good probes for New physics behind EWSB

$$\mathcal{L}_{\rm int} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t}\gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t}\sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$



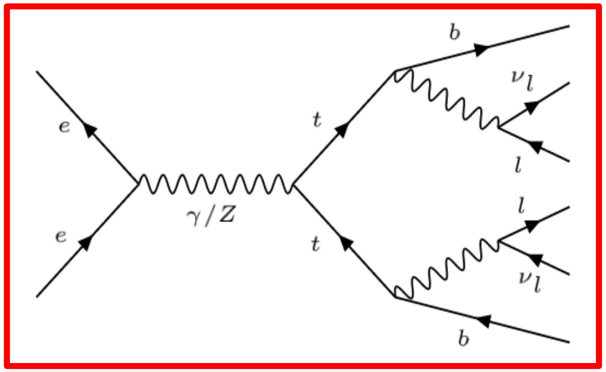


In new physics models, such as composite models, the predicted deviation of coupling constants, $g_L^Z, g_R^Z \ (= F_{1V}^Z \mp F_{1A}^Z)$ from SM is typically 10 %

Di-leptonic State of the top pair production

Top pair production has three different final states:

- Fully-hadronic state $(e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}q\bar{q}q\bar{q})$ 46.2 %
- Semi-leptonic state $(e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}q\bar{q}l\nu)$ 43.5%
- **Di-leptonic state $(e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}l\nu l\nu)$ 10.3%**



Advantage

- More observables be computed

→ Higher intrinsic sensitivity to the form factors, in principle.

Difficulty

- Two missing neutrinos
- Lower statistics : 6 times less events than the semi-leptonic state

((2/3 x 43.5 %) / (4/9 x 10.3 %) = ~ 6.3)

Pre-selection

The quality cut is necessary to reject the b-jet miss-assignment events when we don’t use the b-charge reconstruction. The cut might be also effective to reject background events.

We use only two loose constraints, called **Pre-selection**, before the kinematical reconstruction of top quark, which is useful to shorten the CPU time.

- **1 isolated μ^- and 1 isolated μ^+**
- **1 (or 2) jet has high b-tag value obtained by the LCFI Plus (b-tag1 > 0.8 or b-tag2 > 0.8)**

Other constraints that can be considered :

Thrust value, Visible energy, Mass of $\mu^-\mu^+$, …

Pre-selection : Cut table

250 fb^{-1} (-0.8, +0.3) Left	Initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8
Signal $bb\mu\mu\nu\nu$	2961	2725 (e = 92.0%)	2374 (e = 80.2%)
$bbll\nu\nu$ (except $bb\mu\mu\nu\nu$)	23609	387	335
$bbl\nu qq$	104114	40	31
$qqll$ (ZZ)	91478	13800	2519
ll (weight = 4)	212274 (→849096)	74961 (→ 299844)	90 (→ 360)
$l\nu l\nu$ (WW) (weight = 4)	377058 (→ 1508232)	1884 (→ 7536)	3 (→ 12)
$lll\nu l\nu$ $(llWW)$	3021	947	19

Pre-selection : Cut table

250 fb^{-1} (+0.8, -0.3) Right	initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8
Signal $bb\mu\mu\nu\nu$	1255	1162 (e = 92.6%)	1040 (e = 82.9%)
$bbll\nu\nu$ (except $bb\mu\mu\nu\nu$)	10181	160	138
$bbl\nu qq$	45053	18	12
$qqll$ (ZZ)	46344	6980	1237
ll (weight = 4)	161371 (→64524)	57916 (→ 231664)	61 (→ 244)

Cut table

250 fb^{-1} (-0.8,+0.3) Right	initial	$\mu^+\mu^-$	b-tag1>0.8 or b-tag2>0.8		$q_{\min} < 3$ & $\lvert P_{z,\gamma}\rvert < 50$ GeV	
Signal $bb\mu\mu\nu\nu$ (True)	**1255**	**1162** (e = 92.6 %)	**874** (84.0%)	(e = 82.9%)	**437** (94.2%)	(e = 37.0%)
Signal $bb\mu\mu\nu\nu$ (Miss)			166 (16.0%)		27 (5.8%)	
$bbll\nu\nu$ (except $bb\mu\mu\nu\nu$)	10181	160	138		30	
$bbl\nu qq$	45053	18	12		0	
$qqll$ (ZZ)	46344	6980	1237		6	
ll (weight = 4)	161371 (→ 64524)	57916 (→ 231664)	61 (→ 244)		0	

Helicity Angles

All final state particles including two neutrinos can be calculated. The 9 helicity angles which are related to the ttZ/γ vertex can be computed.

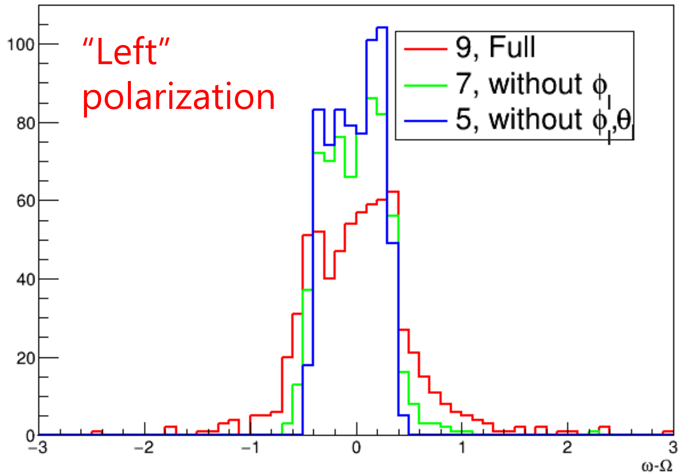
$$\theta_t, \theta_{W^+}^{t\ \mathrm{frame}}, \phi_{W^+}^{t\ \mathrm{frame}}, \theta_{\mu^+}^{W^+\ \mathrm{frame}}, \phi_{\mu^+}^{W^+\ frame}, \theta_{W^-}^{\bar{t}\ frame}, \phi_{W^-}^{\bar{t}\ frame}, \theta_{\mu^-}^{W^-frame}, \phi_{\mu^-}^{W^-frame}$$

(G. L. Kane, G. A. Ladinsky, C.-P. Yuan, Phys.Rev. D45 (1992) 124-141)

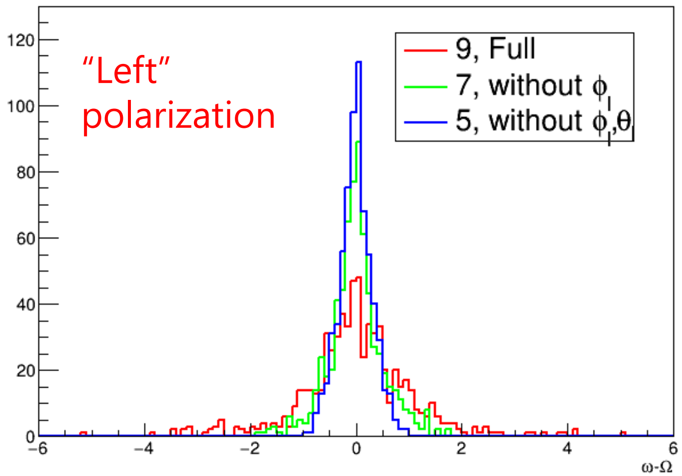
The optimal variables ω are defined at this 9-dimention phase space.

Relation of the helicity angles of $\mu^{\pm}$ and $\omega - \Omega$

$\left(\delta\tilde{F}_{2V}^{\gamma}\right)$



$\left(Re\delta\tilde{F}_{2A}^{\gamma}\right)$



When we don't use the $\phi_{\mu^{\pm}}^{W^{\pm}}$ or $(\phi_{\mu^{\pm}}^{W^{\pm}}, \theta_{\mu^{\pm}}^{W^{\pm}})$, the $\omega - \Omega$ distribution becomes sharper, hence the sensitivity becomes lower.

→ $(\phi_{\mu^{\pm}}^{W^{\pm}}, \theta_{\mu^{\pm}}^{W^{\pm}})$ has a sensitivity to the ttZ/γ.

Parameters of Likelihood function

Breit-Wigner function of mass of top and W

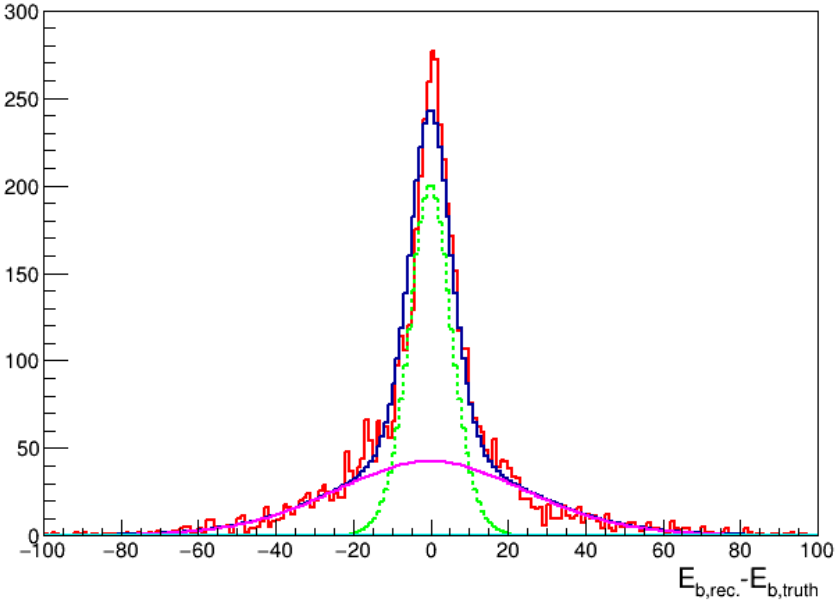
$$BW(m) \propto \frac{1}{1 + \left(\frac{m - m_0}{m_0 \Gamma_0}\right)^2}$$

$$m_{t,0} = m_{\bar{t},0} = 174, m_{W^+,0} = m_{W^-,0} = 80.4, \Gamma_0 = 5$$

Gaussian function of Beam energy spread

$$Gaus(E_{total}) \propto \exp\left[-\left(\frac{E_{total} - 500}{0.39}\right)^2\right]$$

Energy resolution of b-jets



$$G\left(\sigma_j, K; E_b^{\text{Measurement}}\right)$$

$$\propto \exp\left[-\left(\frac{E_b^{\text{Measurement}} - E_b^{\text{MC}}}{\sigma_j * \left(E_b^{\text{Measurement}}\right)^K}\right)^2\right]$$

Define the resolution function R as

$$R = c_1 G_1 + c_2 G_2 + c_3 G_3$$

Results of fit :

$$c_1 = 0.50, \sigma_{j,1} = 0.77, K_1 = 0.45$$

$$c_2 = 0.48, \sigma_{j,2} = 6.4, K_2 = 0.31$$

$$c_3 = 0.02, \sigma_{j,3} = 4.7, K_3 = 0.69$$

Goodness of Fit

The confidence level is just computed from δF^{Fit} (or $\chi^2(\delta F^{\mathrm{Fit}})$)

→ **Need to assess goodness of fit in another way**

Reminder of our assumption for the Matrix Element Method

$$|M|^2 = \left(1 + \sum_i \omega_i \delta F_i + \sum_{ij} \widetilde{\omega}_{ij} \delta F_i \delta F_j\right) |M|^2_{\mathrm{SM}}, N = \left(1 + \sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij} \delta F_i \delta F_i\right) N_{\mathrm{SM}}$$

→ One can define PDF as $f(\delta F) = \frac{\left(1+\sum_i \omega_i \delta F_i + \sum_{ij} \widetilde{\omega}_{ij} \delta F_i \delta F_j\right)}{\left(1+\sum_i \Omega_i \delta F_i + \sum_{ij} \widetilde{\Omega}_{ij} \delta F_i \delta F_i\right)} f_{\mathrm{SM}}$ where $f_{\mathrm{SM}} = \frac{|M|^2_{\mathrm{SM}}}{N_{\mathrm{SM}}}$ is PDF of SM.

Expected value of ω_i and $\widetilde{\omega}_{ij}$ for given δF can be computed from the PDF

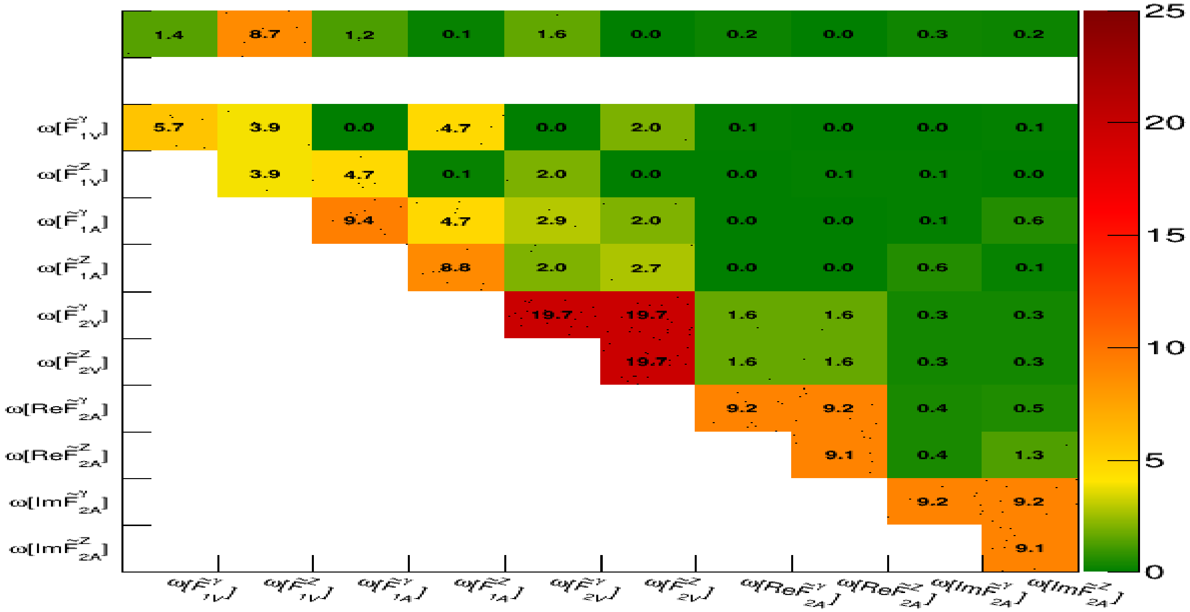
$$< \omega_i > (\delta F) = \int \omega_i f(\delta F) d\omega d\widetilde{\omega}, \qquad < \widetilde{\omega}_{ij} > (\delta F) = \int \widetilde{\omega}_{ij} f(\delta F) d\omega d\widetilde{\omega}$$

$< \omega_i > (\delta F), < \widetilde{\omega}_{ij} > (\delta F)$ should be close to $< \omega_i >_{\mathrm{data}}, < \widetilde{\omega}_{ij} >_{\mathrm{data}}$ if our assumption is correct.

Goodness of Fit

Define $\chi^2_{\mathrm{GoF},i}(\delta F)$ and $\tilde{\chi}^2_{\mathrm{GoF},ij}(\delta F)$ to assess the Goodness of Fit.

$$\chi^2_{\mathrm{GoF},i}(\delta F) = \frac{\left(<\omega_i>_{\mathrm{data}} - <\omega_i>(\delta F)\right)^2}{<\omega_i^2>_{\mathrm{data}} - <\omega_i>^2_{\mathrm{data}}}, \qquad \tilde{\chi}^2_{\mathrm{GoF},ij}(\delta F) = \frac{\left(<\tilde{\omega}_{ij}>_{\mathrm{data}} - <\tilde{\omega}_{ij}>(\delta F)\right)^2}{<\tilde{\omega}_{ij}^2>_{\mathrm{data}} - <\tilde{\omega}_{ij}>^2_{\mathrm{data}}}$$



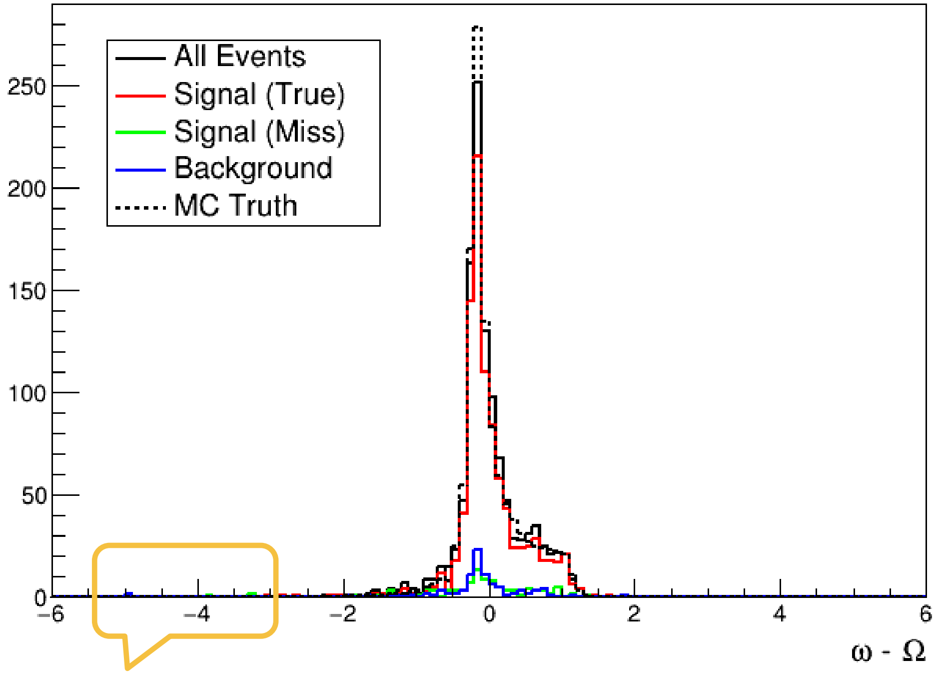
Some of $\chi^2_{GoF,i}, \tilde{\chi}^2_{GoF,ij}$ have very large values !

Table of $\chi^2_{\mathrm{GoF},i}$, $\tilde{\chi}^2_{\mathrm{GoF},ij}$ of Left polarization events

Goodness of Fit : Outliers

Large χ^2_{GoF} implies that our assumption might be wrong.

However, there is another possibility of reason → ***Outliers***



$\omega - \Omega$ distribution for δF_{1V}^{Z} of Left polarization events (same as P16.)

A few events are distributed far from other events. It can be caused by detector effects, ISR effects, etc.

→ *These events might be outliers and induce so large* χ^2_{GoF}

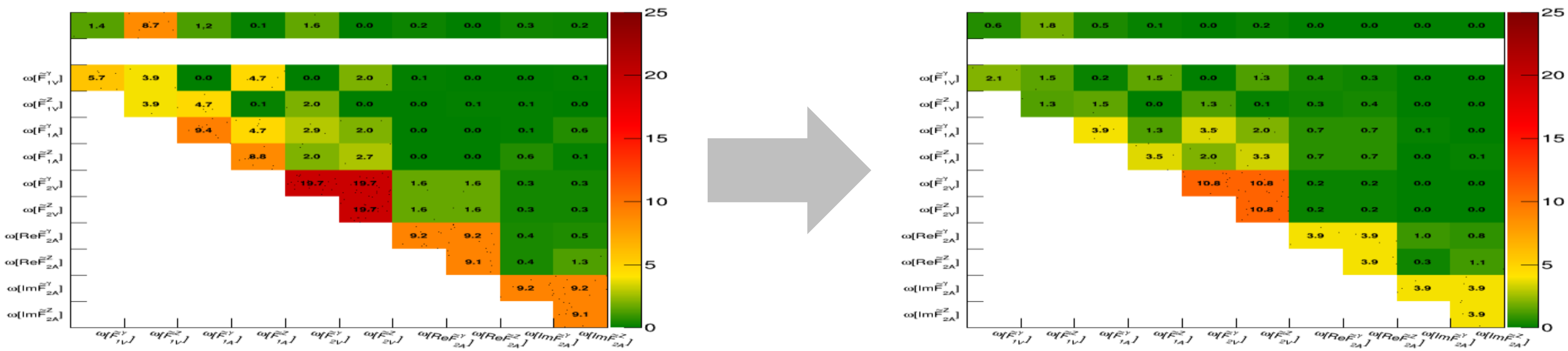
Goodness of Fit

Reject events which have too large(small) $\omega, \widetilde{\omega}$:

$$|\omega_i - \Omega_i| > 10\sigma[\omega_i^{SM}], \qquad |\widetilde{\omega}_{ij} - \widetilde{\Omega}_{ij}| > 10\sigma[\omega_{ij}^{SM}]$$

Criteria are selected *very preliminary*.

Efficiency cost is only 1.6%(0.8%) for Left(Right) polarization events.



Most of large $\chi^2_{\mathrm{GoF},i}, \widetilde{\chi}^2_{\mathrm{GoF},ij}$ become much better. It implies such large values are induced by outliers. Although some still have large values ($\sim$11), one may reduce them changing criteria for outliers.

But some of $\tilde{\chi}^2_{\mathrm{GoF},ij}$ are still 3-4. We suppose it comes from ISR effects and detector effect

Preliminary Results without Outliers

Results of 10 parameters multi-fit

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & -0.0138 \pm 0.0132 \\ \mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0284 \pm 0.0229 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0171 \pm 0.0183 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0537 \pm 0.0285 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0847 \pm 0.0401 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.1132 \pm 0.0642 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma} & -0.0160 \pm 0.0239 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{Z} & -0.0539 \pm 0.0408 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma} & +0.0428 \pm 0.0265 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{Z} & +0.0222 \pm 0.0372 \end{bmatrix} \Rightarrow \begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & -0.0148 \pm 0.0129 \\ \mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0232 \pm 0.0226 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0140 \pm 0.0184 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0309 \pm 0.0286 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0736 \pm 0.0371 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.0564 \pm 0.0601 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{\gamma} & -0.0059 \pm 0.0226 \\ \mathcal{R}e\ \delta\tilde{F}_{2A}^{Z} & -0.0377 \pm 0.0389 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{\gamma} & +0.0403 \pm 0.0238 \\ \mathcal{I}m\ \delta\tilde{F}_{2A}^{Z} & +0.0007 \pm 0.0343 \end{bmatrix}$$

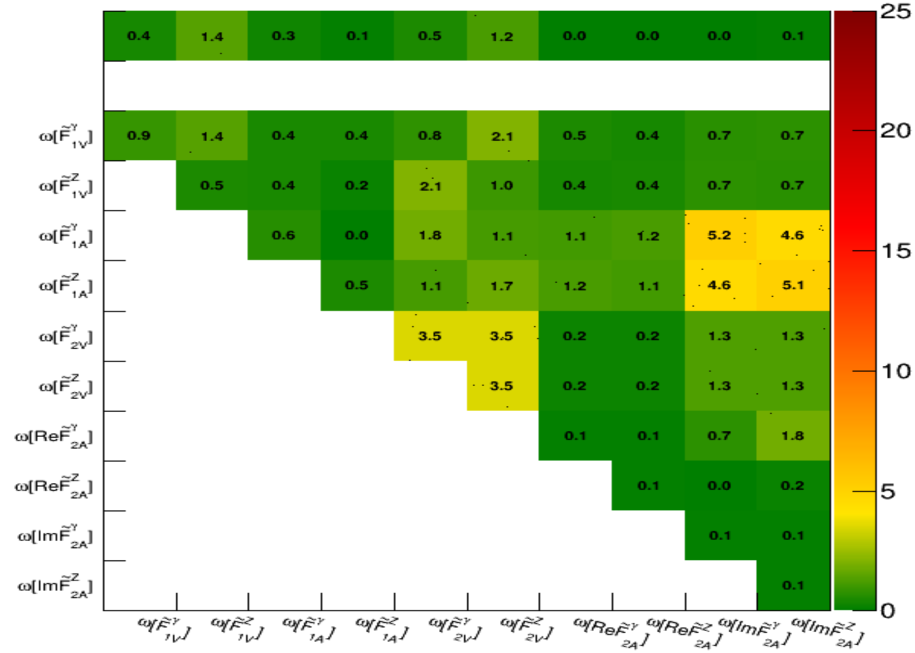
The results of fit becomes also better.

Preliminary Results without Outliers & ISR

From the MC information, one can reject events having hard ISR.

Results of 10 parameters multi-fit ($\sqrt{s} > 495$ GeV)

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1V} & -0.0177 \pm 0.0203 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1V} & -0.0186 \pm 0.0404 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{1A} & +0.0219 \pm 0.0279 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{1A} & +0.0325 \pm 0.0420 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2V} & -0.0946 \pm 0.0655 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2V} & -0.0799 \pm 0.1046 \\ \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & +0.0028 \pm 0.0308 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & +0.0645 \pm 0.0537 \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & +0.0316 \pm 0.0356 \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} & +0.0447 \pm 0.0526 \end{bmatrix}$$



Goodness of Fit

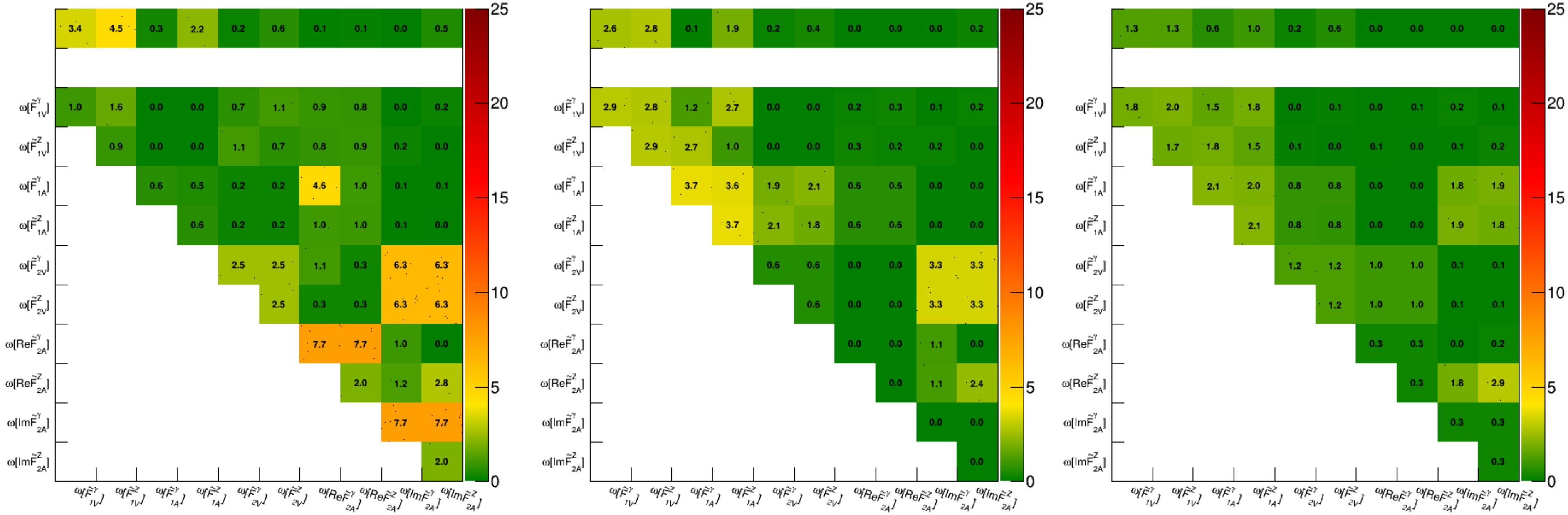


Table of $\chi^2_{\mathrm{GoF},i}$, $\tilde{\chi}^2_{\mathrm{GoF},ij}$ of Right polarization events

(left : with outliers, center : without outliers, right : without outliers & ISR)

Improved method : Binned likelihood analysis

$\chi^2(\delta F)$ is defined as following;

$$\chi^2(\delta F) = \sum_{b=1}^{N_{\text{bin}}} \frac{\left(n_b^{\text{Data}} - N_b(\delta F)\right)^2}{n_b^{\text{Data}}}$$

$N_b(\delta F)$ is obtained from the very large full MC simulation changing δF, which is called ***the template method***. However, it can be also obtained by ***the re-weighting method***

$$N_b(\delta F) = \frac{n^{\text{Data}}}{N^{\text{MC Simulation}}} \sum_{e \in b} 1 * \frac{|M|^2(\delta F)}{|M|^2_{\text{SM}}}$$

$$= \frac{n^{\text{Data}}}{N^{\text{MC Simulation}}} \sum_{e \in b} \left(1 + \sum \omega_i^{\text{Truth}} \delta F_i + \sum \tilde{\omega}_{ij}^{\text{Truth}} \delta F_i \delta F_j\right)$$

where ω_i^{Truth} and $\tilde{\omega}_{ij}^{\text{Truth}}$ are the optimal variables at MC truth level. Only one simulation is needed if one uses this method.

Since $\tilde{\omega}^{\text{Truth}}$ is a coefficient of $O(\delta F^2)$ and δF is so small, we use only ω_i^{Truth} for now.

Improved method : Binned likelihood analysis

In the definition of $\chi^2(\delta F)$, we assume the deviation is $\sqrt{n_b}$. So the n_b must be large (>10).

The following likelihood function can be used even if n_b is small.

$$-2\log L(\delta F) = -2\sum_{b=1}^{N_{bin}}\left(\begin{array}{c} n_b^{\mathrm{Data}}\ln\left(1+\sum_i o_{b,i}\delta F_i+\sum_{ij}\tilde{o}_{b,ij}\delta F_i\delta F_j\right) \\ -N_b^{\mathrm{SM}}\left(\sum_i o_{b,i}\delta F_i+\sum_{ij}\tilde{o}_{b,ij}\delta F_i\delta F_j\right)\end{array}\right)$$

where $o_{b,i} = \frac{1}{N_b^{SM}}\sum_{e\in b}\omega_i^{\mathrm{Truth}}$, $\tilde{o}_{b,ij} = \frac{1}{N_b^{SM}}\sum_{e\in b}\widetilde{\omega}_{ij}^{\mathrm{Truth}}$.

This definition is more precise because we don't use any assumptions and it is also by construction unbiased. However we cannot assess the goodness of fit from the likelihood function.