Top electroweak couplings study using di-leptonic state at $\sqrt{s} = 500$ GeV, ILC with the Matrix Element Method

AWLC2017, SLAC

Yo Sato[A]

Akimasa Ishikawa[A], Emi Kou[B], Francois Le Diberder[B], Hitoshi Yamamoto[A], Junping Tian[C], Keisuke Fujii[D],

Tohoku University[A], LAL[B], University of Tokyo[C], KEK[D]

Outline

Motivation

Kinematical reconstruction of top quark

- Strategy of kinematical reconstruction
- Fraction of wrong assignment of b-jets
- Helicity angles computation

Matrix element method analysis

- Fit of CP-Conserving form factors
- Fit of CP-Violating form factors

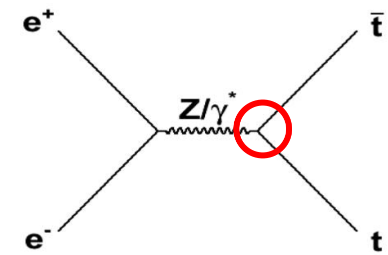
Summary

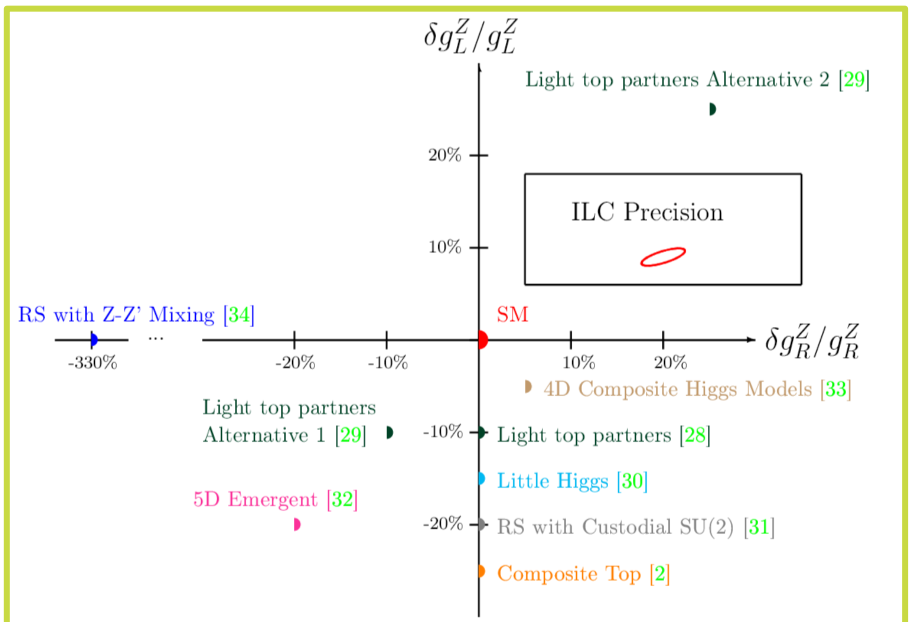
Top EW Couplings Study

- Top quark is the heaviest particle in the SM. Its large mass implies that it is strongly coupled to the mechanism of electroweak symmetry breaking (EWSB)

  → **Top EW couplings are good probes for New physics behind EWSB**

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^v \left[V_l^v \bar{t} \gamma^l (F_{1V}^v + F_{1A}^v \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^v + F_{2A}^v \gamma_5) t \right]$$



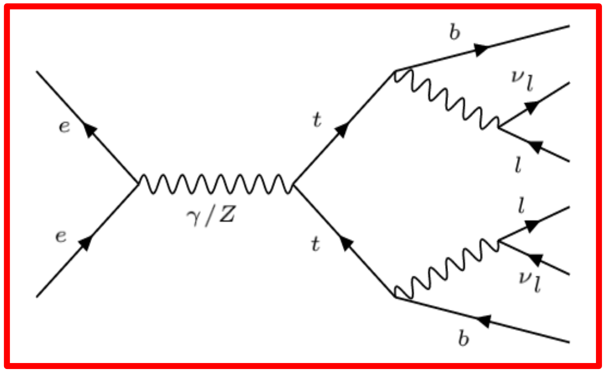


In new physics models, such as composite models, the predicted deviation of coupling constants, $g_L^Z, g_R^Z\ (= F_{1V}^Z \mp F_{1A}^Z)$ from SM is typically 10 %

Di-leptonic State of the top pair production

Top pair production has three different final states:

- Fully-hadronic state $(e^+e^- \to t\bar{t} \to b\bar{b}q\bar{q}q\bar{q})$ 46.2 %
- Semi-leptonic state $(e^+e^- \to t\bar{t} \to b\bar{b}q\bar{q}l\nu)$ 43.5%
- **Di-leptonic state $(e^+e^- \to t\bar{t} \to b\bar{b}l\nu l\nu)$ 10.3%**



Advantage

- 9 helicity angles can be computed (details will be described later)

→ Higher sensitivity to the form factors

Difficulty

- Two missing neutrinos → Difficult to reconstruct top quark.

Develop the reconstruction process in realistic situation

Set Up of Analysis

Situation	On / Off
Full simulation of ILD	On
Hadronization	On
Gluon emission from top	On
ISR/BS	On
γγ→hadrons	On
bkg. events	Off (ongoing)

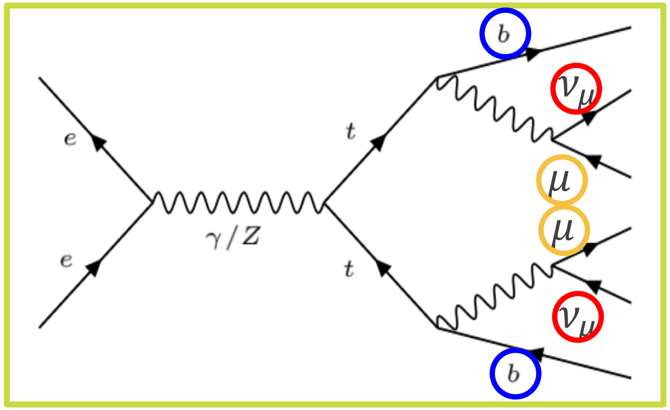
Sample (Only signal)	Di-muonic state $e^+e^- \to b\bar{b}\mu^+\nu\mu^-\bar{\nu}$
$\sqrt{s}$	500 GeV
Polarization (P_{e^-}, P_{e^+})	(-0.8, +0.3) "Left" / (+0.8, -0.3) "Right"
Integrated luminosity	500 fb^{-1} (50/50 between Left and Right)
Generator	Whizard
Detector	ILD_01_v05 (DBD ver.)

Reconstruction Process

- Isolated leptons tagging
  - Number of isolated leptons = 2 & Opposite charge each of two
- Suppression of γγ → hadrons
  - kt algorithm (cf. the Semi-leptonic analysis, R = 1.5)
- b-jet reconstruction
  - LCFI Plus (Durham algorithm)
  - The b-charge measurement is not used
- **Kinematical reconstruction of top quark**

Kinematical Reconstruction of top quark

$\boldsymbol{e^+e^- \rightarrow t\bar{t} \rightarrow b\bar{b}\mu^+\nu\mu^-\bar{\nu}}$



Measurable
- muon's : $E_{\mu^+}, \theta_{\mu^+}, \phi_{\mu^+}, E_{\mu^-}, \theta_{\mu^-}, \phi_{\mu^-}$
- b-jet's : $E_{b1}, \theta_{b1}, \phi_{b1}, E_{b2}, \theta_{b2}, \phi_{b2}$

Missing
- neutrino's : $E_\nu, \theta_\nu, \phi_\nu, E_{\bar{\nu}}, \theta_{\bar{\nu}}, \phi_{\bar{\nu}}$
- **=> 6 unknowns**

To recover them, impose the kinematical constraints;

- Initial state constraints : $(\sqrt{s}, \vec{P}_{\text{init.}}) = (500, \vec{0})$
- Mass constraints : $m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$

=> 8 constraints (2 in excess)

We don't need E_{b1} and E_{b2} which are relatively difficult to reconstruct.

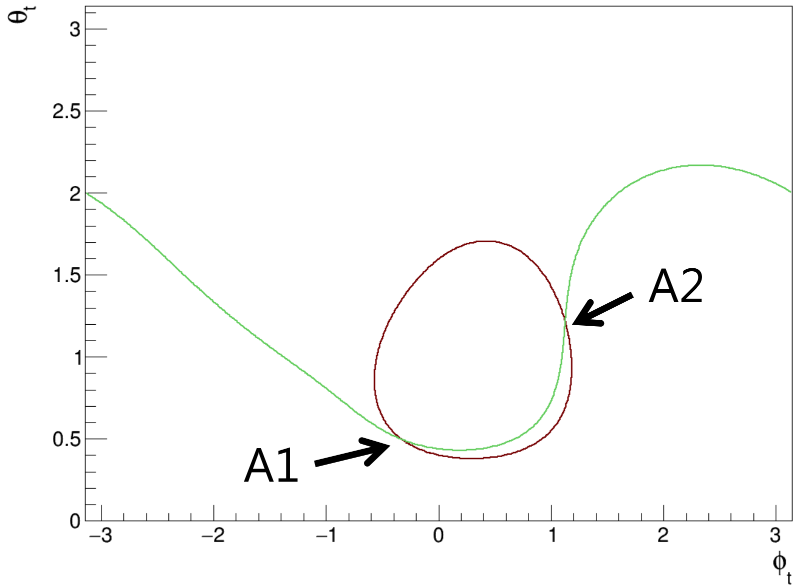
→ Just use to decide the assignment of b-jets

Kinematical Reconstruction of top quark

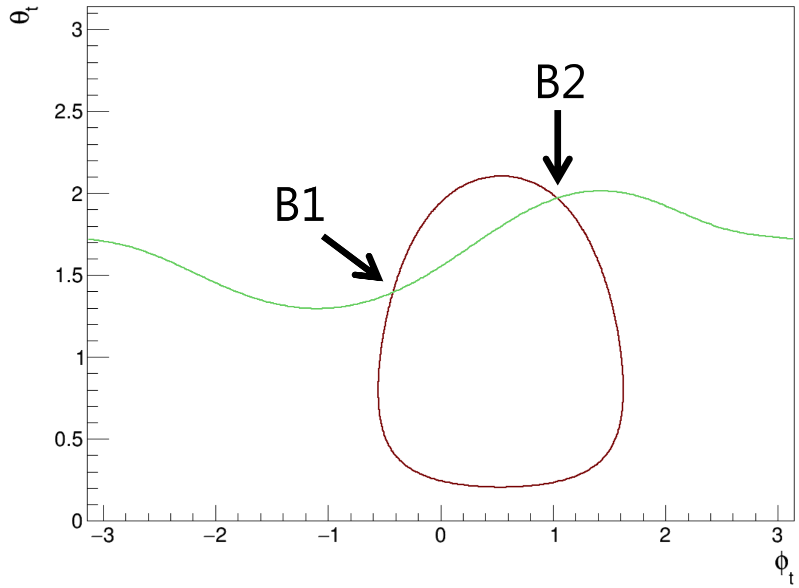
To detect the solution, we solve the following equations.

$$E_{\mu^\pm}^{W^\pm \text{ rest frame}}(\theta_t, \phi_t) = m_{W^\pm}/2 \text{ (Red : } \mu^+\text{, Green : } \mu^-\text{)}$$

assignment A (correct), $b1 = b,\ b2 = \bar{b}$



assignment B (wrong), $b1 = \bar{b},\ b2 = b$



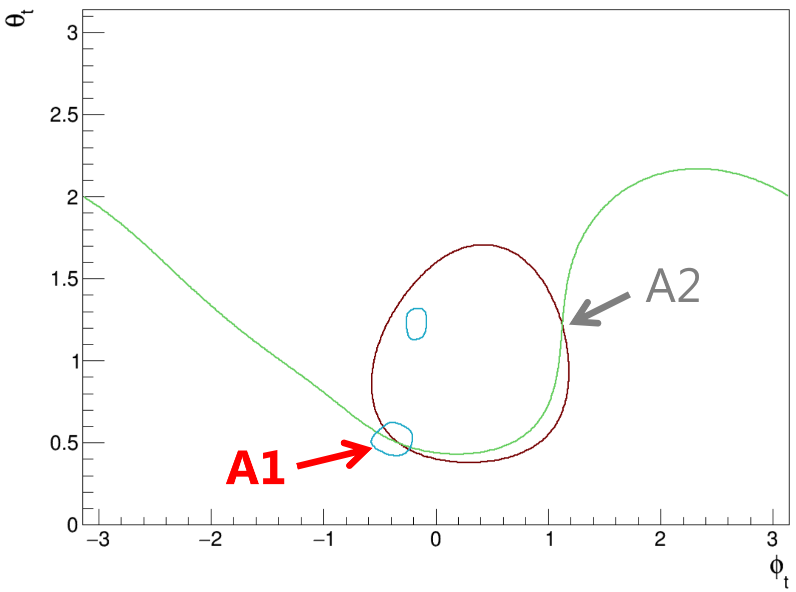
Typically, 4 candidates exist for each event.

We need to select the optimal solution from these candidates.

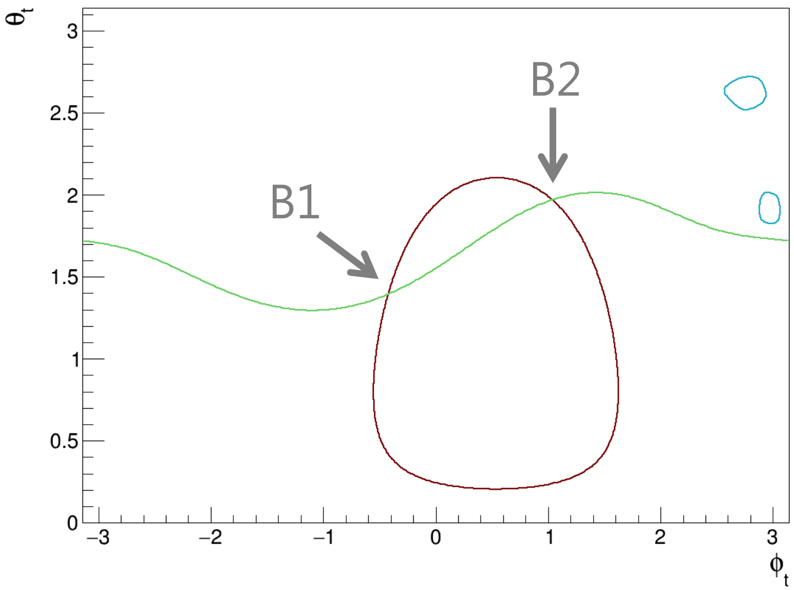
Kinematical Reconstruction of top quark

$$\chi_b^2(\theta_t,\phi_t) \equiv \left(\frac{E_b(\theta_t,\phi_t)-E_b^{\text{meas.}}}{\sigma\left[E_b^{\text{meas.}}\right]}\right)^2 + \left(\frac{E_{\bar{b}}(\theta_t,\phi_t)-E_{\bar{b}}^{\text{meas.}}}{\sigma\left[E_{\bar{b}}^{\text{meas.}}\right]}\right)^2 = 2 \text{ (Blue)}$$

assignment A (correct), $b1 = b,\ b2 = \bar{b}$



assignment B (wrong), $b1 = \bar{b},\ b2 = b$



The candidate A1 has the minimum χ_b^2

→ The assignment A is selected and the solution is $(\theta_t,\phi_t) \simeq (0.5, -0.35)$

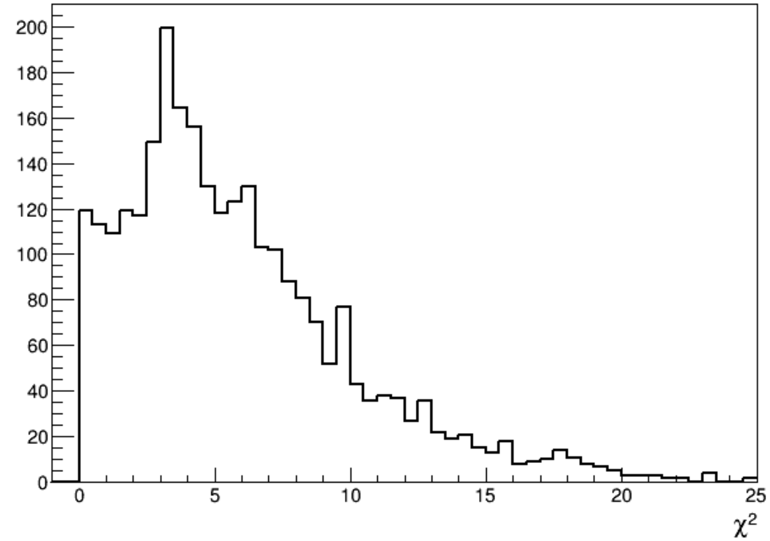
Kinematical Reconstruction of top quark

Technically, to obtain the solution, we minimize χ^2_{tot} ;

$$\chi^2_{tot}(\theta_t, \phi_t) = \chi^2_\mu(\theta_t, \phi_t) + \chi^2_b(\theta_t, \phi_t)$$

where $\chi^2_\mu(\theta_t, \phi_t) \equiv \left(\frac{E_{\mu^+}^{(W^+ \text{ rest frame})}(\theta_t, \phi_t) - m_{W^+}/2}{\sigma\left[E_{\mu^+}^{(W^+ \text{ rest frame})}\right]} \right)^2 + \left(\frac{E_{\mu^-}^{(W^- \text{ rest frame})}(\theta_t, \phi_t) - m_{W^-}/2}{\sigma\left[E_{\mu^-}^{(W^- \text{ rest frame})}\right]} \right)^2$

χ^2_μ is dominant to determine (θ_t, ϕ_t) because $\sigma\left[E_\mu^{(W \text{ rest frame})}\right] \ll \sigma[E_b]$



χ^2_{tot} distribution

F_{wrong} : Fraction of the Wrong Assignment of b-jets

F_{wrong} (the fraction of the wrong assignment of b-jets) **= 22 %**

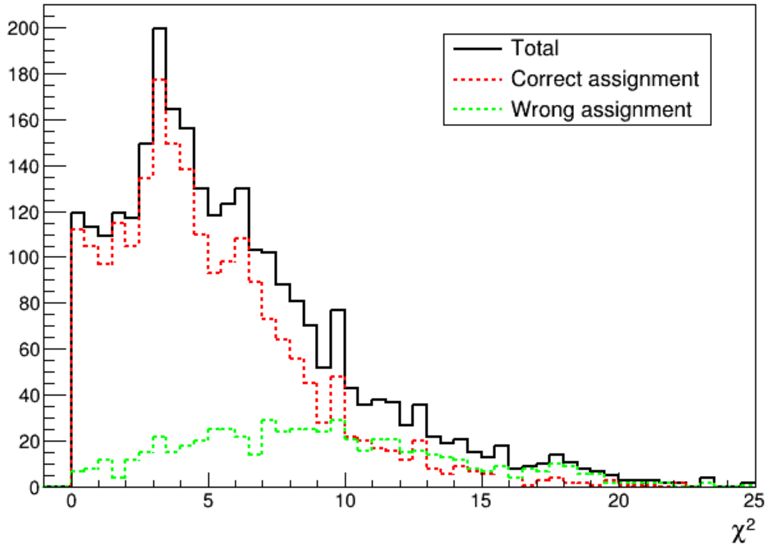
When we use samples not including ISR, F_{wrong} = 8 %

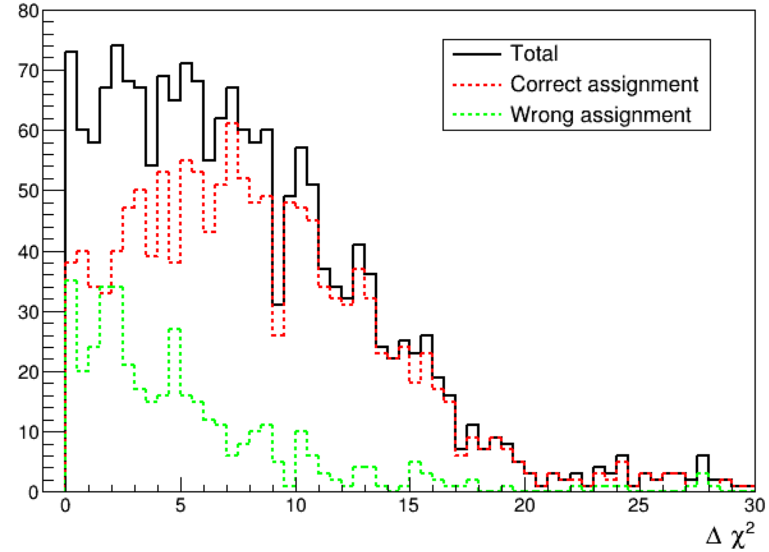
→ ISR significantly affects the assignment problem.

We use two quantities to reduce F_{wrong}

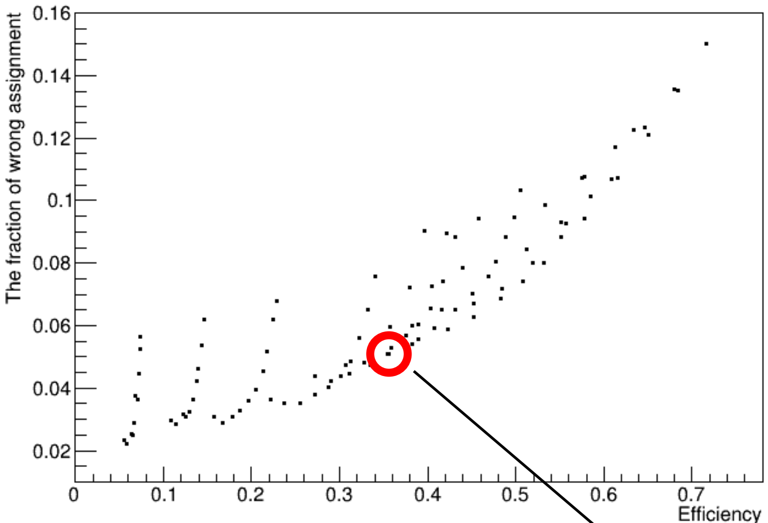
χ^2_{tot} (as mentioned)

$\Delta\chi^2_{tot} = \left|\chi^2_{tot,\text{assignment A}} - \chi^2_{tot,\text{assignment B}}\right|$





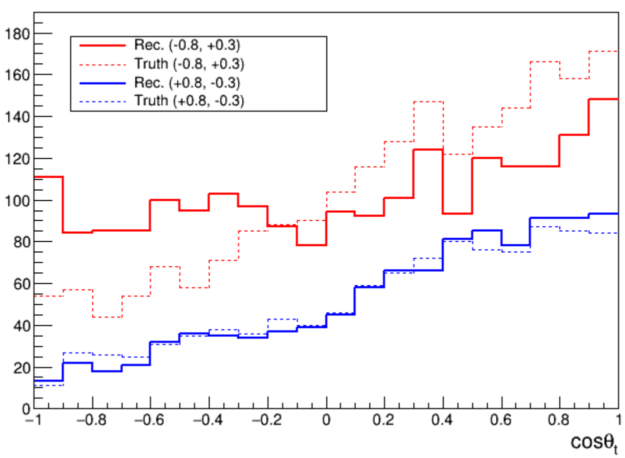
F_{wrong} : Fraction of the Wrong Assignment of b-jets



Efficiency vs. F_{wrong}

We investigate F_{wrong} and the efficiency varying the set of criteria for $(\chi^2_{tot}, \Delta\chi^2_{tot})$

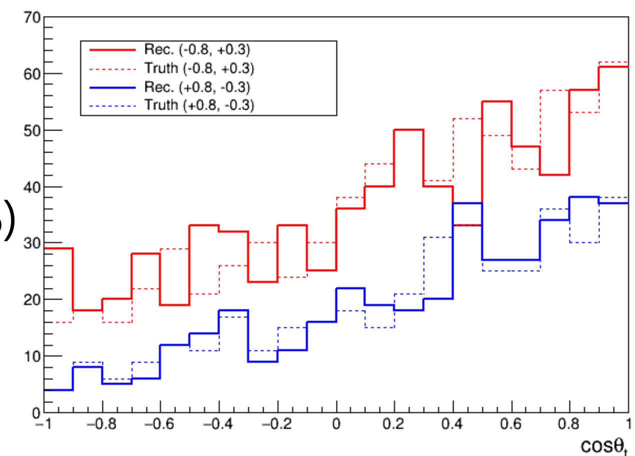
The polar angle distribution of top is improved by the quality cut.



$\chi^2_{tot} < 5$, $\Delta\chi^2_{tot} > 6$

(F_{wrong} = 5.0 %

total efficiency = 28 %)



Helicity Angles Computation

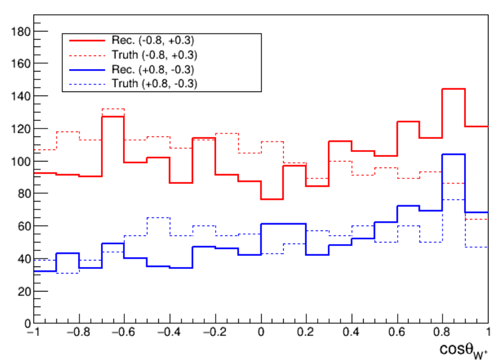
All final state particles including two neutrinos can be calculated. The 9 helicity angles which are related to the ttZ/γ vertex are computed.

$$\theta_t, \theta_{W^+}^{t\ \mathrm{frame}}, \phi_{W^+}^{t\ \mathrm{frame}}, \theta_{\mu^+}^{W^+\ \mathrm{frame}}, \phi_{\mu^+}^{W^+\ frame}, \theta_{W^-}^{\bar{t}\ frame}, \phi_{W^-}^{\bar{t}\ frame}, \theta_{\mu^-}^{W^-frame}, \phi_{\mu^-}^{W^-frame}$$

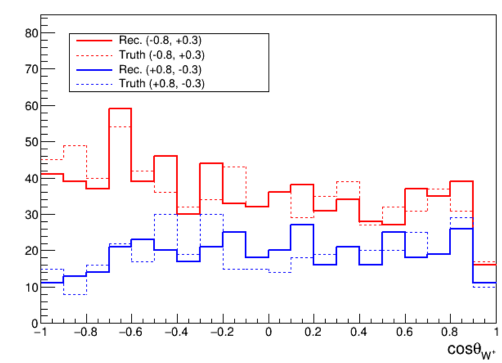
(G. L. Kane, G. A. Ladinsky, C.-P. Yuan, Phys.Rev. D45 (1992) 124-141)

eg)

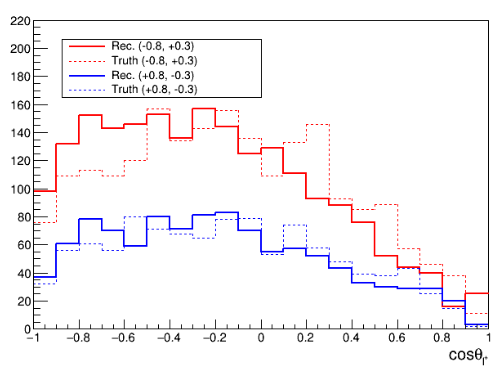
$\cos\theta_{W^+}^{t\ \mathrm{frame}}$

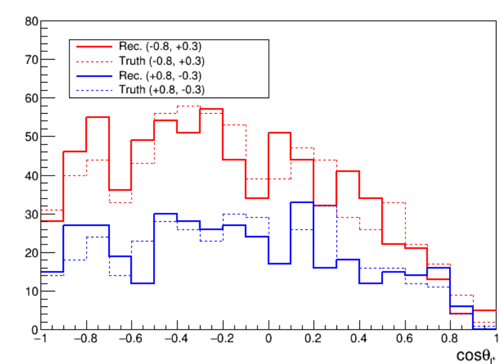


$\chi^2_{tot} < 5$, $\Delta\chi^2_{tot} > 6$



$\cos\theta_{\mu^+}^{W^+\ \mathrm{frame}}$





Matrix Element Method Analysis

Matrix element method is based on the maximum likelihood method.

$$-2\log L(F)\left(=\chi^2(F)\right) = -2\left(\sum_{e=1}^{N_{\rm event}} \log|M|^2(\Phi_e, F) - N(F)\right)$$

$|M|^2$: the full matrix element, Φ_e : the 9 helicity angles, F : the form factors,
$N(F)$: the expected number of events.

The minimization of $\chi^2(F)$ automatically introduces the derivatives;

$$\omega_i(\Phi_e) = \frac{1}{|M|^2(\Phi_e)}\frac{\partial|M|^2(\Phi_e)}{\partial F_i}\bigg|_{F\ {\rm at\ SM}}, \qquad \Omega_i = \frac{1}{N}\frac{\partial N}{\partial F_i}\bigg|_{F\ {\rm at\ SM}}$$

The results of fit are related with $\omega_i(\Phi_e)$ and Ω_i;

- $\delta F_i\ (= F_{\rm fit} - F_{\rm SM}) \simeq \frac{<\omega_i - \Omega_i>}{<(\omega_i - \Omega_i)^2>}$
- covariance matrix, V_{ij} ;

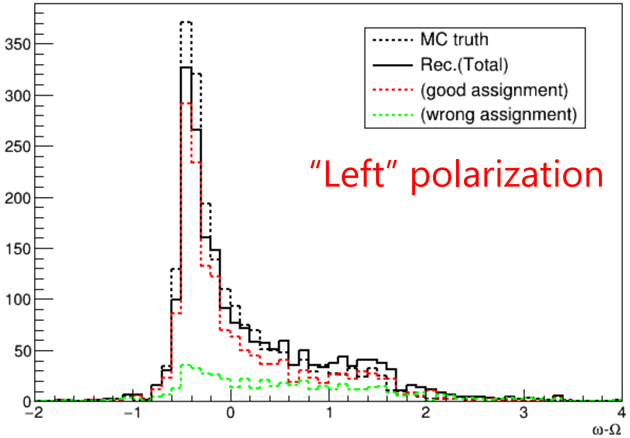
$$V_{ij}^{-1} = N_{\rm event} < (\omega_i - \Omega_i)(\omega_j - \Omega_j) >$$

Fit of the CP-Conserving form factors

Result of $\delta\tilde{F}_{1V}^{\gamma}$ fit (the others are fixed at SM)

Before the quality cut (total efficiency 77%)

$$\delta\tilde{F}_{1V}^{\gamma} = 0.0223 \pm 0.0066,\ \ \chi^2_{\text{test}} = 11.4 \Leftrightarrow 0.07\%\ \text{CL}$$



The histogram of $\omega - \Omega$ for $\delta\tilde{F}_{1V}^{\gamma}$ (before quality cut)

The $\omega - \Omega$ distribution of the wrong assignment (Green) is

- shifted to positive → bias
- blunter → over estimates the precision

* $\chi^2_{\text{test}} = \sum \delta F_i V_{ij}^{-1} \delta F_j$: the chi-square test

Fit of the CP-Conserving form factors

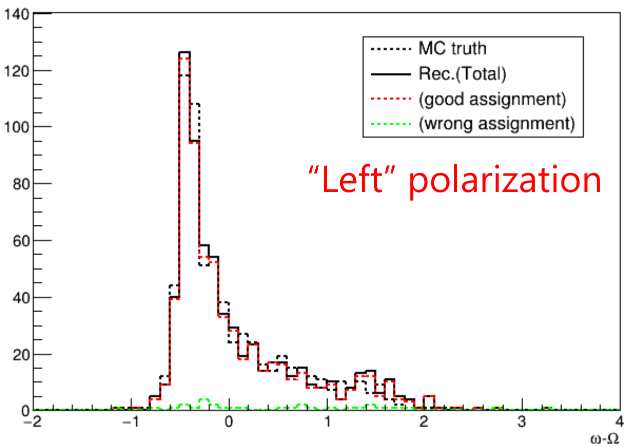
Result of $\delta\tilde{F}_{1V}^{\gamma}$ fit (the others are fixed at SM)

Before the quality cut (total efficiency 77%)

$$\delta\tilde{F}_{1V}^{\gamma} = 0.0223 \pm 0.0066, \;\; \chi^2_{\text{test}} = 11.4 \Leftrightarrow 0.07\%\ \text{CL}$$

After the quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, total efficiency 28%)

$$\delta\tilde{F}_{1V}^{\gamma} = 0.0075 \pm 0.0115, \;\; \chi^2_{\text{test}} = 0.43 \Leftrightarrow 51\%\ \text{CL}$$



The histogram of $\omega - \Omega$ for $\delta\tilde{F}_{1V}^{\gamma}$ (after quality cut)

Good agreement between MC truth and Rec.

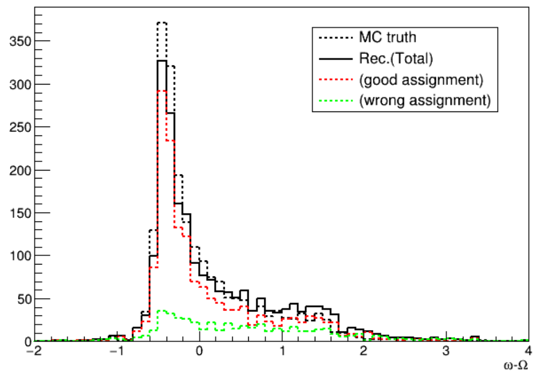
→ The bias disappears.

→ The error becomes larger ($\sim\sqrt{N}$)

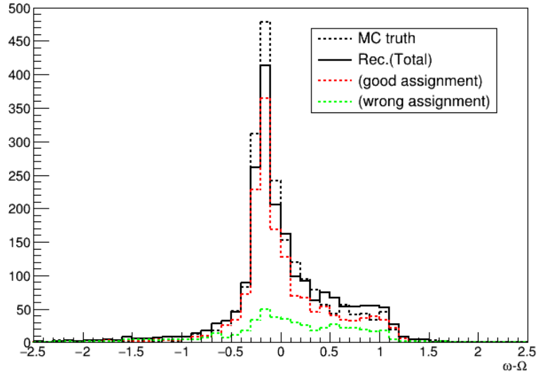
The distributions of $\omega - \Omega$ (bef. the quality cut)

"Left" polarization

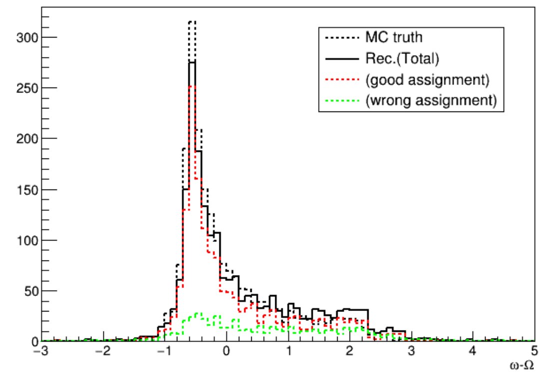
$\left(\delta\tilde{F}_{1V}^{\gamma}\right)$



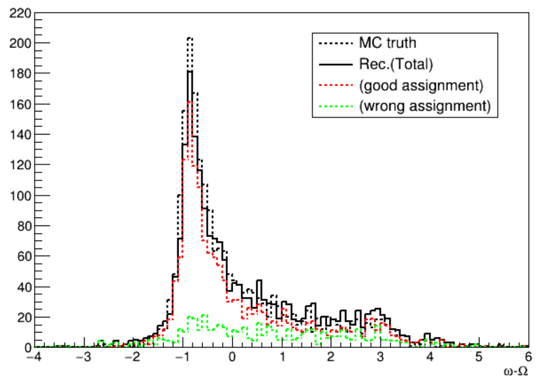
$\left(\delta\tilde{F}_{1V}^{Z}\right)$



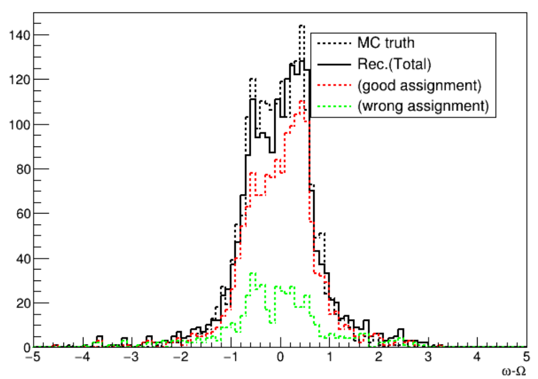
$\left(\delta\tilde{F}_{1A}^{\gamma}\right)$



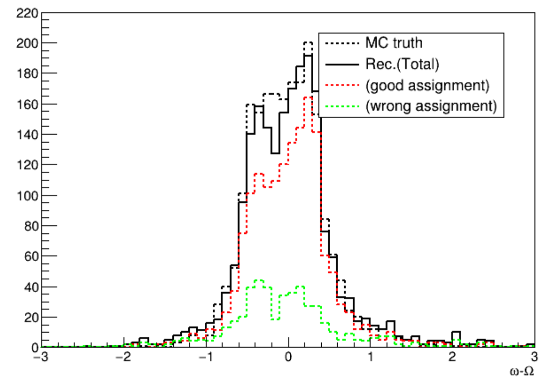
$\left(\delta\tilde{F}_{1A}^{Z}\right)$



$\left(\delta\tilde{F}_{2V}^{\gamma}\right)$



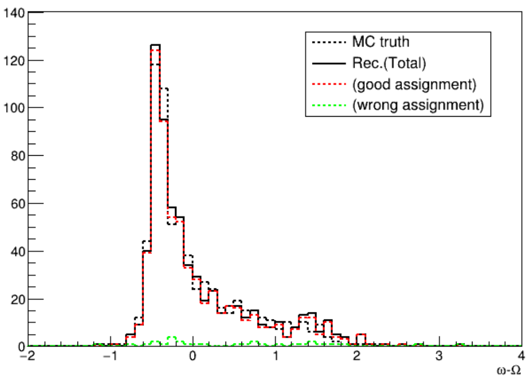
$\left(\delta\tilde{F}_{2V}^{Z}\right)$



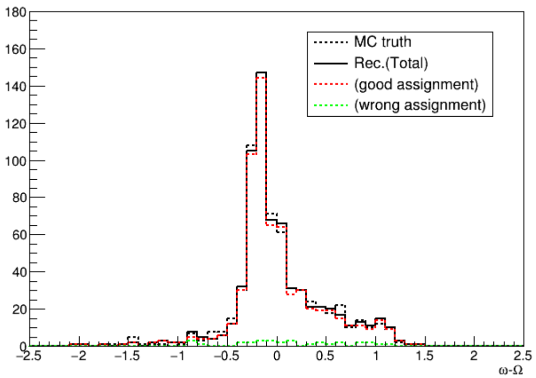
The distributions of $\omega - \Omega$ (aft. the quality cut)

"Left" polarization

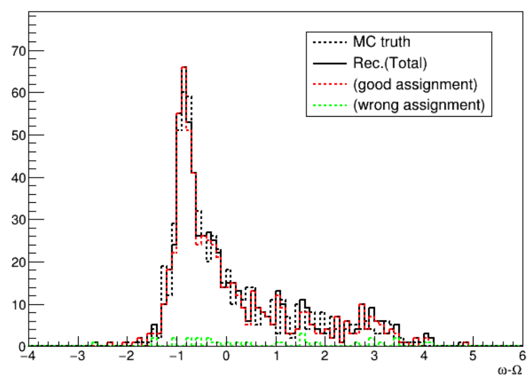
$(\delta\tilde{F}_{1V}^{\gamma})$



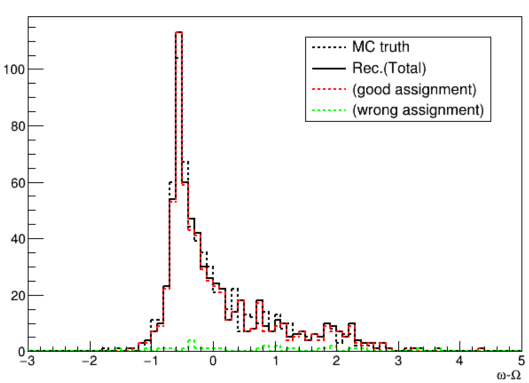
$(\delta\tilde{F}_{1V}^{Z})$



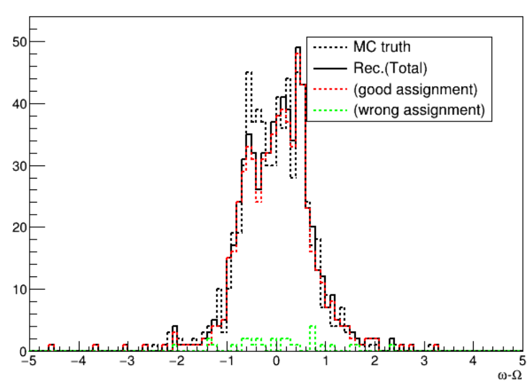
$(\delta\tilde{F}_{1A}^{\gamma})$



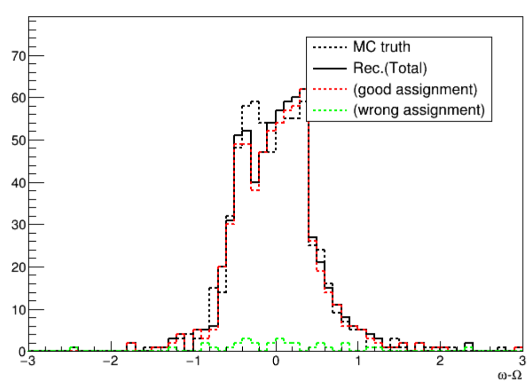
$(\delta\tilde{F}_{1A}^{Z})$



$(\delta\tilde{F}_{2V}^{\gamma})$



$(\delta\tilde{F}_{2V}^{Z})$



Fit of the CP-Conserving form factors

Results of 6 CPC form factors fit

Before quality cut (total efficiency 77%)

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & +0.0188 \pm 0.0089 \\ \mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0293 \pm 0.0161 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0280 \pm 0.0133 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.2250 \pm 0.0202 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & -0.0246 \pm 0.0260 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.1448 \pm 0.0435 \end{bmatrix}$$

$$\chi^2_{\text{test}} = 166 \Leftrightarrow \sim 0\%\ \text{CL}$$

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, total efficiency 28%)

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}_{1V}^{\gamma} & +0.0088 \pm 0.0154 \\ \mathcal{R}e\ \delta\tilde{F}_{1V}^{Z} & +0.0339 \pm 0.0270 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{\gamma} & +0.0233 \pm 0.0221 \\ \mathcal{R}e\ \delta\tilde{F}_{1A}^{Z} & +0.0704 \pm 0.0340 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{\gamma} & +0.0788 \pm 0.0461 \\ \mathcal{R}e\ \delta\tilde{F}_{2V}^{Z} & +0.1244 \pm 0.0762 \end{bmatrix}$$

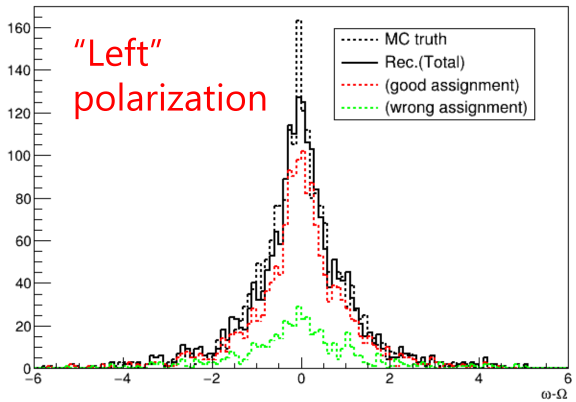
$$\chi^2_{\text{test}} = 10.0 \Leftrightarrow 12.5\%\ \text{CL}$$

Fit of the CP-Violating form factors

Result of $Re\delta\tilde{F}_{2A}^{\gamma}$ fit (the others are fixed at SM)

Before the quality cut (total efficiency 77%)

$$Re\delta\tilde{F}_{2A}^{\gamma} = -0.0172 \pm 0.0185,\ \ \chi^2_{\text{test}} = 0.87 \Leftrightarrow 35\%\ \text{CL}$$



The histogram of $\omega - \Omega$ for $Re\delta\tilde{F}_{2A}^{\gamma}$ (before quality cut)

The $\omega - \Omega$ distribution of the wrong assignment (Green) is

- centered at 0
  → no apparent effect on the bias
  → χ^2_{test} is misleading
  → if we use a CP-Violating sample, the wrong assignment will dilute the effect of CPV
- blunter → over estimates the precision

* $\chi^2_{\text{test}} = \sum \delta F_i V_{ij}^{-1} \delta F_j$: the chi-square test

Fit of the CP-Violating form factors

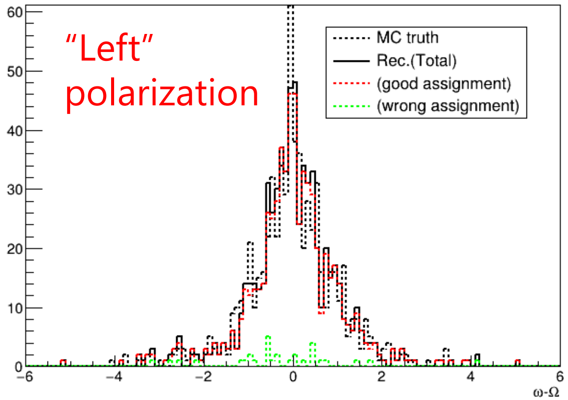
Result of $Re\delta\tilde{F}_{2A}^{\gamma}$ fit (the others are fixed at SM)

Before the quality cut (total efficiency 77%)

$$Re\delta\tilde{F}_{2A}^{\gamma} = -0.0172 \pm 0.0185,\ \chi^2_{\text{test}} = 0.87 \Leftrightarrow 35\%\ \text{CL}$$

After the quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, total efficiency 28%)

$$Re\delta\tilde{F}_{2A}^{\gamma} = -0.0052 \pm 0.0287,\ \chi^2_{\text{test}} = 0.034 \Leftrightarrow 85\%\ \text{CL}$$



The histogram of $\omega - \Omega$ for $Re\delta\tilde{F}_{2A}^{\gamma}$
(after quality cut)

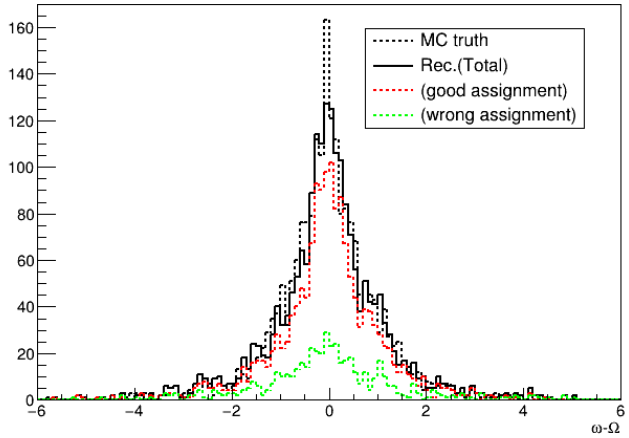
Good agreement between MC truth and Rec.

→ The error is estimated correctly.

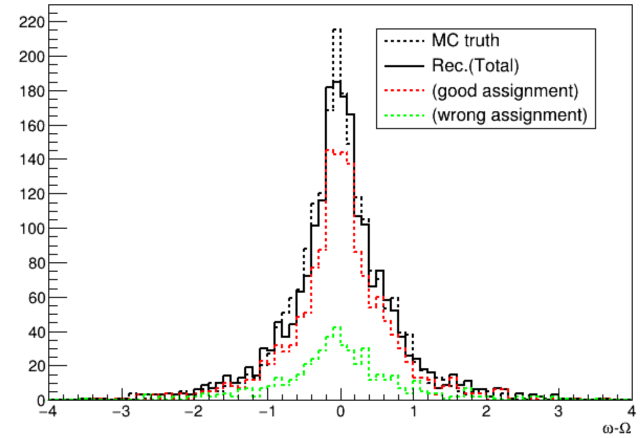
The distributions of $\omega - \Omega$ (bef. the quality cut)

"Left" polarization

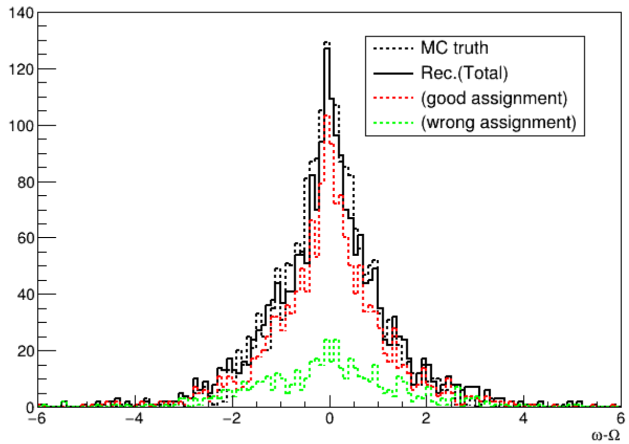
$\left(Re\delta\tilde{F}_{2A}^{\gamma}\right)$



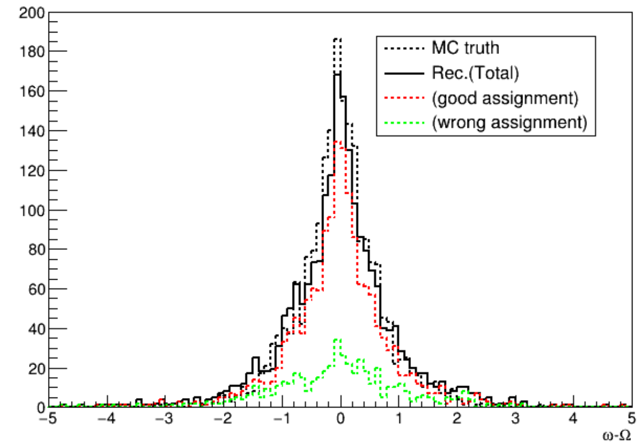
$\left(Re\delta\tilde{F}_{2A}^{Z}\right)$



$\left(Im\delta\tilde{F}_{2A}^{\gamma}\right)$



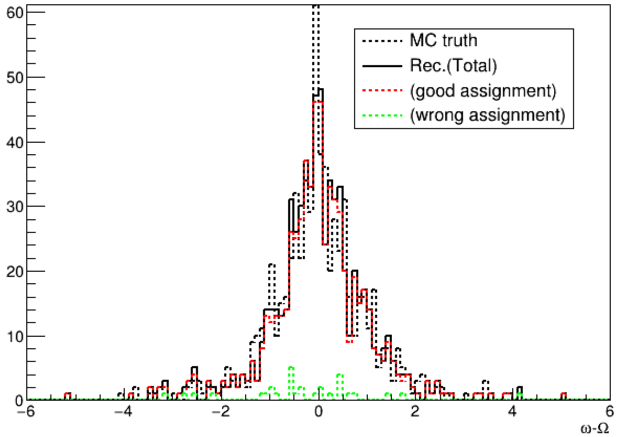
$\left(Im\delta\tilde{F}_{2A}^{Z}\right)$



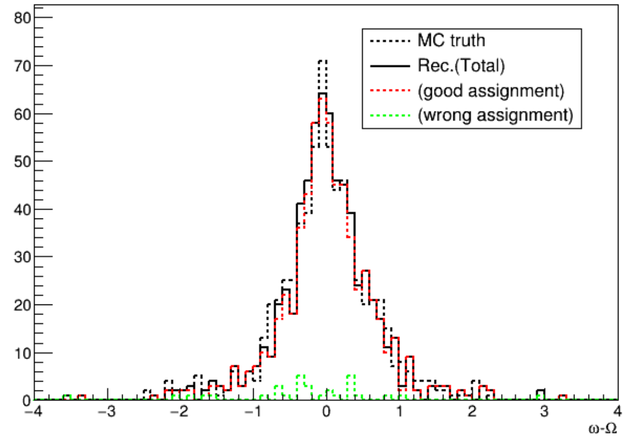
The distributions of $\omega - \Omega$ (aft. the quality cut)

"Left" polarization

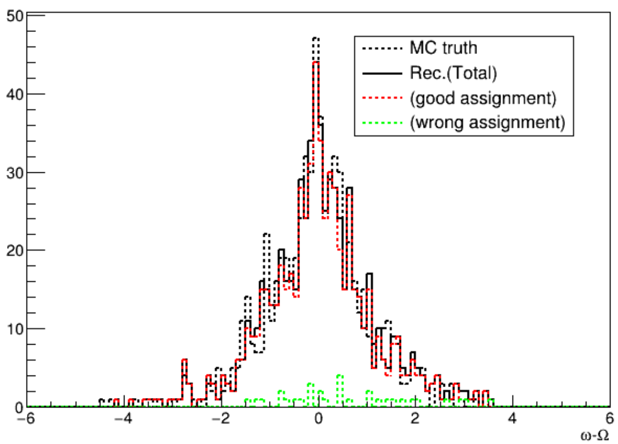
$(Re\delta\tilde{F}_{2A}^{\gamma})$



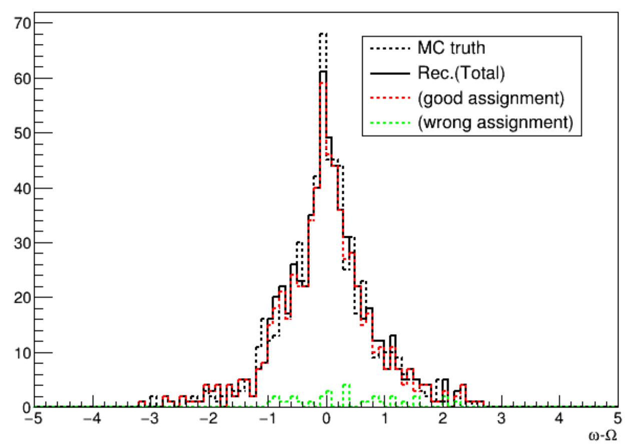
$(Re\delta\tilde{F}_{2A}^{Z})$



$(Im\delta\tilde{F}_{2A}^{\gamma})$



$(Im\delta\tilde{F}_{2A}^{Z})$



Fit of the CP-Violating form factors

Results of 4 CPV form factors fit

Before quality cut (total efficiency 77%)

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & -0.0196 \pm 0.0185 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & +0.0307 \pm 0.0357 \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & -0.0324 \pm 0.0177 \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} & +0.0111 \pm 0.0239 \end{bmatrix}$$

$$\chi^2_{\text{test}} = 5.0 \Leftrightarrow 29\%\ \text{CL}$$

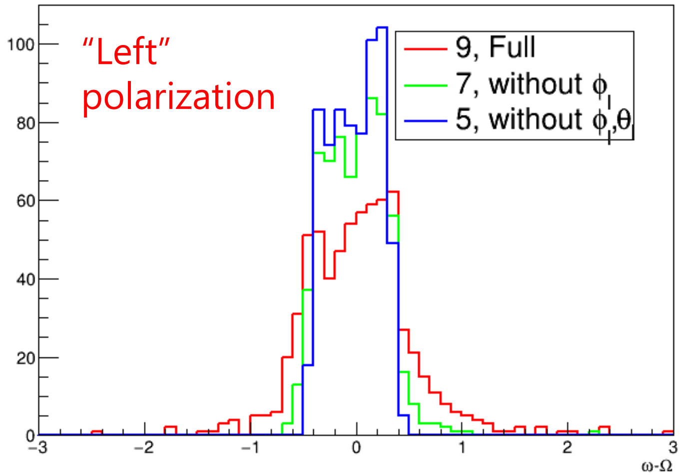
After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, total efficiency 28%)

$$\begin{bmatrix} \mathcal{R}e\ \delta\tilde{F}^{\gamma}_{2A} & -0.0022 \pm 0.0287 \\ \mathcal{R}e\ \delta\tilde{F}^{Z}_{2A} & +0.0423 \pm 0.0567 \\ \mathcal{I}m\ \delta\tilde{F}^{\gamma}_{2A} & -0.0026 \pm 0.0300 \\ \mathcal{I}m\ \delta\tilde{F}^{Z}_{2A} & +0.0148 \pm 0.0419 \end{bmatrix}$$

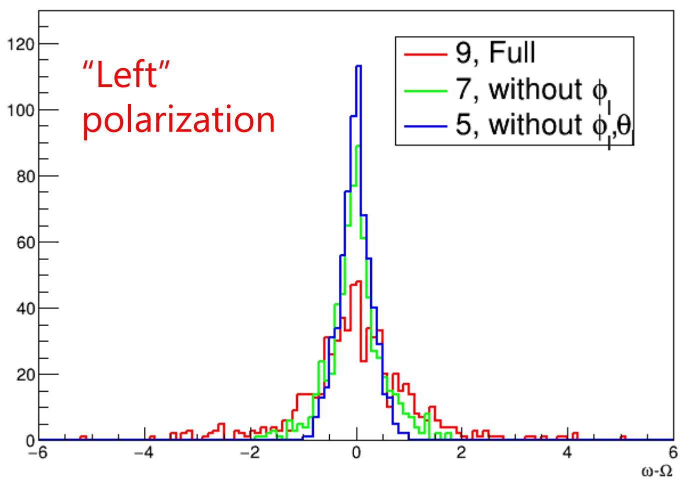
$$\chi^2_{\text{test}} = 0.64 \Leftrightarrow 96\%\ \text{CL}$$

Relation of the helicity angles of $\mu^{\pm}$ and $\omega - \Omega$

$(\delta\tilde{F}_{2V}^{\gamma})$



$(Re\delta\tilde{F}_{2A}^{\gamma})$



When we don't use the $\phi_{\mu^{\pm}}^{W^{\pm}}$ or $(\phi_{\mu^{\pm}}^{W^{\pm}}, \theta_{\mu^{\pm}}^{W^{\pm}})$, the $\omega - \Omega$ distribution becomes sharper, hence the sensitivity becomes lower.

→ $(\phi_{\mu^{\pm}}^{W^{\pm}}, \theta_{\mu^{\pm}}^{W^{\pm}})$ has a sensitivity to the ttZ/γ.

Summary

- **Di-leptonic state analysis produces the 9 helicity angles which are sensitive to the form factors.**
- **Reconstruct top quark imposing the kinematical constraints**
  - ISR significantly affects the assignment problem of b-jets
  - The quality cut improves the fraction of wrong assignment of b-jets, hence the angular distributions.
- **Fit the form factors with the Matrix element method**
  - CPC : After quality cut, results are consistent with SM.
  - CPV : The wrong fraction has no effects on the bias, but it will dilute the CPV effects if we use a CPV sample.

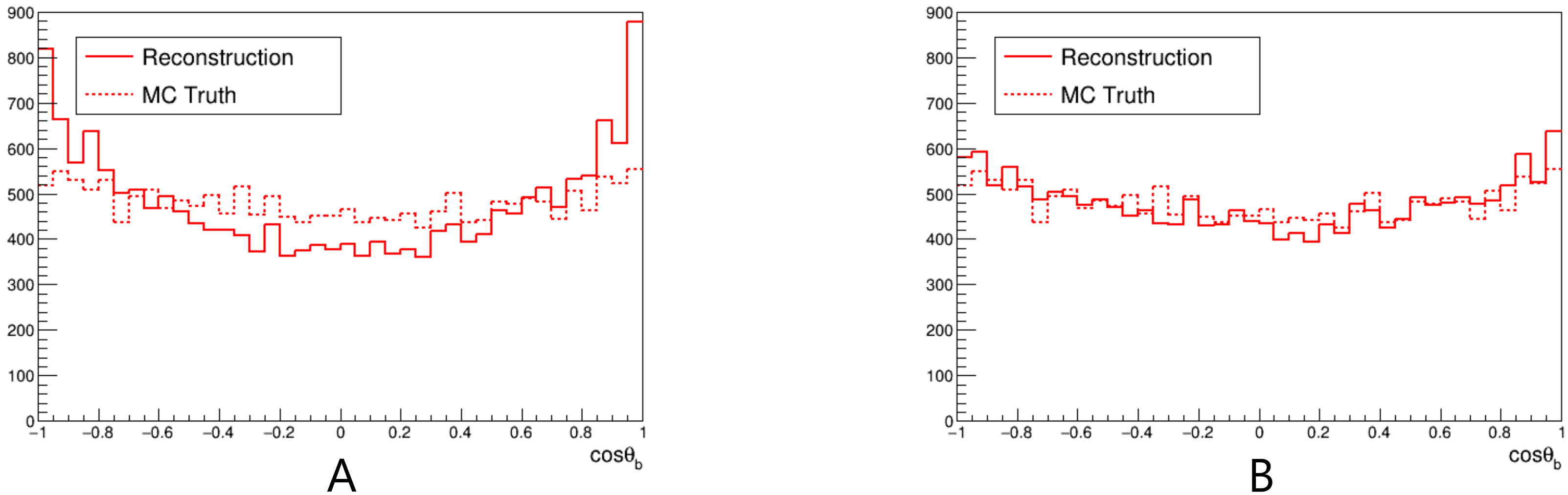
Back up

Suppression of γγ → hadrons & b-jet reconstruction

Particles from γγ → hadrons are mostly emitted along the beam direction. The direction of the b-jet is affected by these particles.

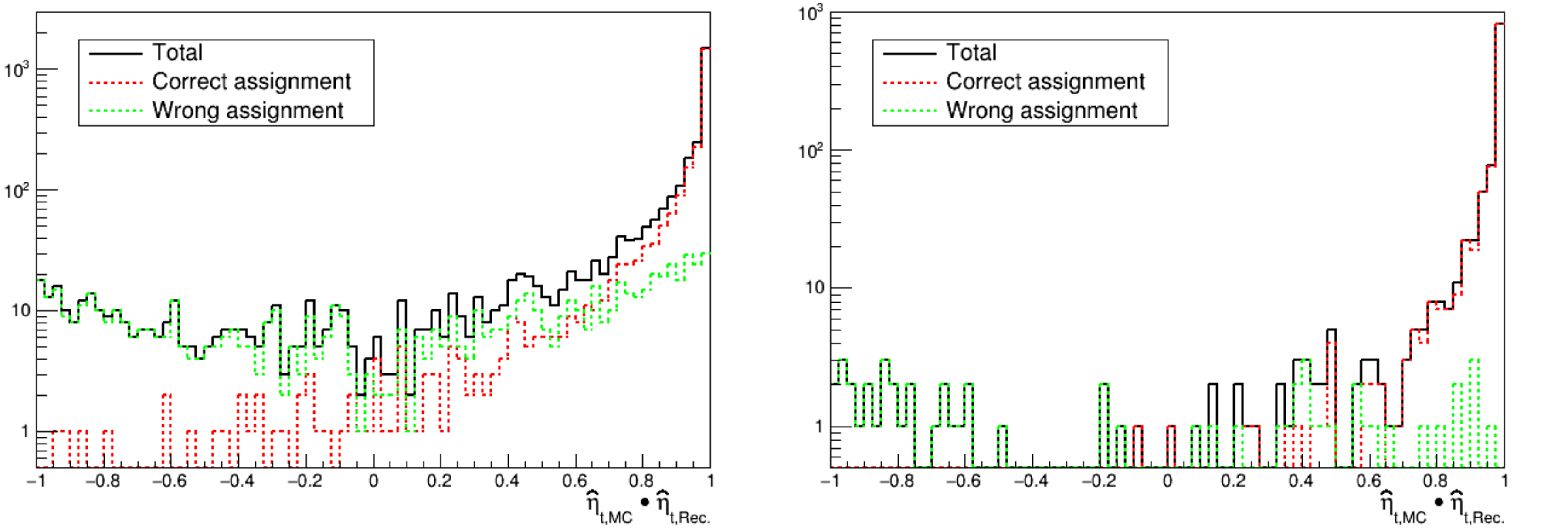
Suppress these particles using the kt algorithm (R=1.5).

→ The direction of the b-jet is improved.



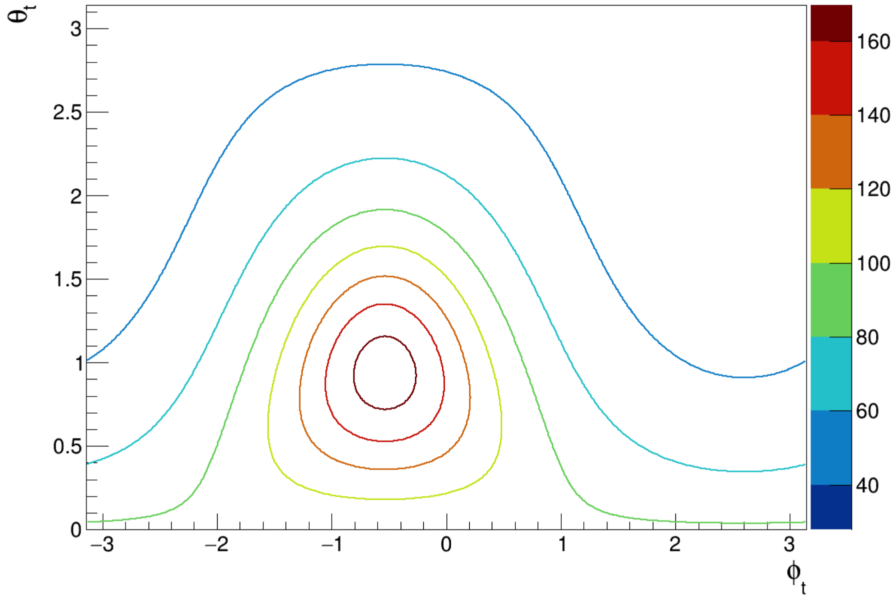
The polar angle distribution b-jets. A: without the suppression of γγ →hadrons, B: with the suppression of γγ → hadrons

Scalar product, $\hat{\eta}_{t,\mathrm{MC}} \cdot \hat{\eta}_{t,\mathrm{Rec.}}$



Kinematical reconstruction of top

To select the optimal solution, we compare E_b and $E_{\bar{b}}$ between calculated by (θ_t, ϕ_t) and measured by the b-jet reconstruction.



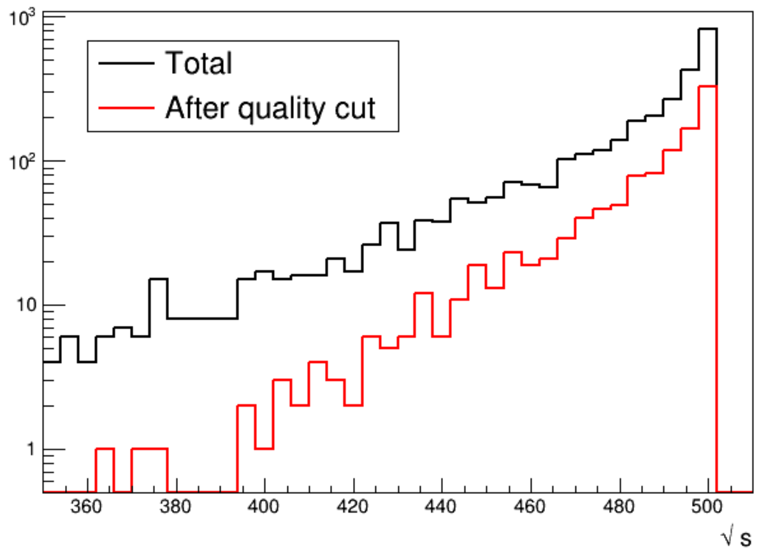
$E_b(\theta_t, \phi_t)$ in the case of assignment A

$$\chi_b^2(\theta_t, \phi_t) = \left(\frac{E_b(\theta_t, \phi_t) - E_b^{\text{meas.}}}{\sigma[E_b^{\text{meas.}}]}\right)^2 + \left(\frac{E_{\bar{b}}(\theta_t, \phi_t) - E_{\bar{b}}^{\text{meas.}}}{\sigma[E_{\bar{b}}^{\text{meas.}}]}\right)^2$$

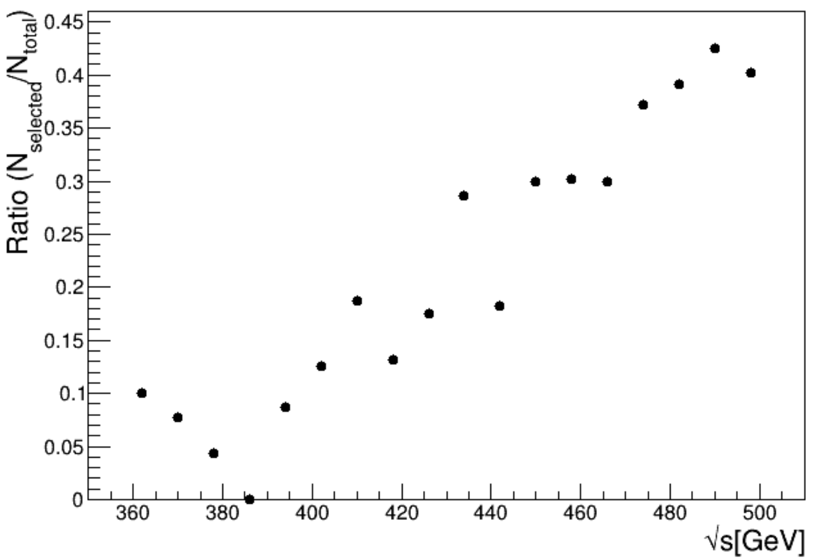
Compute χ_b^2 for each candidate → **Pick one which has the smallest χ_b^2**

Luminosity spectrum

Because we impose the initial state constraints, the events which have low $\sqrt{s}$ are badly reconstructed.



Luminosity spectrum
Black : Total events, Red : After quality cut



Ratio of luminosity spectrum (Red/Black)

The quality cut reduces low $\sqrt{s}$ events, but there are still a tail.

Luminosity spectrum

Tried to fit the energy of ISR photon along beam direction;

$$e^+e^- \to b\bar{b}\mu^+\nu\mu^-\bar{\nu} + \gamma_{\mathrm{ISR}}$$

→ Another parameter, K

- $|K| = E_\gamma/250$, hence $\sqrt{s} = 500 * \sqrt{1-|K|}$
- If γ is emitted in the $e^-(e^+)$ direction, K is positive (negative).

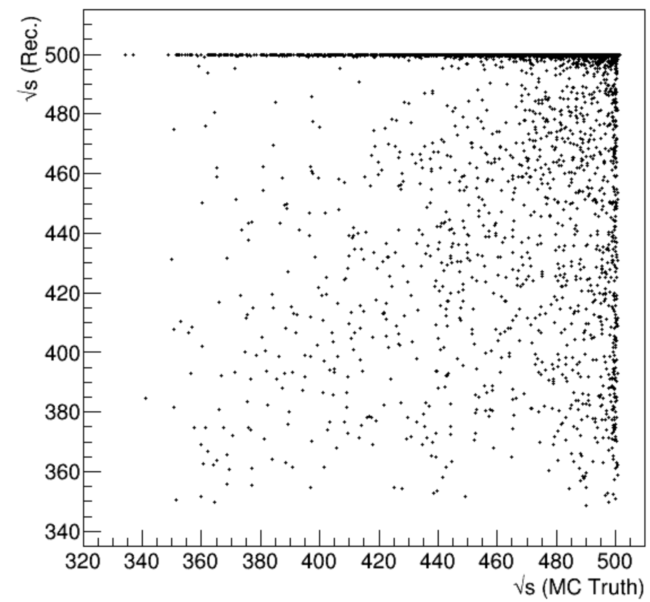
Then one minimizes $\chi^2_{tot}{}'(\theta_t, \phi_t, K)$;

$\chi^2_{tot}{}'(\theta_t, \phi_t, K) = \chi^2_{tot}(\theta_t, \phi_t, K) - 2\log \mathrm{PDF}_K(K)$

→ Reconstructed $\sqrt{s}$ don't correlate MC truth.

→ The constraints are not enough.

Now we fix $K = 0$ (i.e. use $\chi^2_{tot}(\theta_t, \phi_t)$)



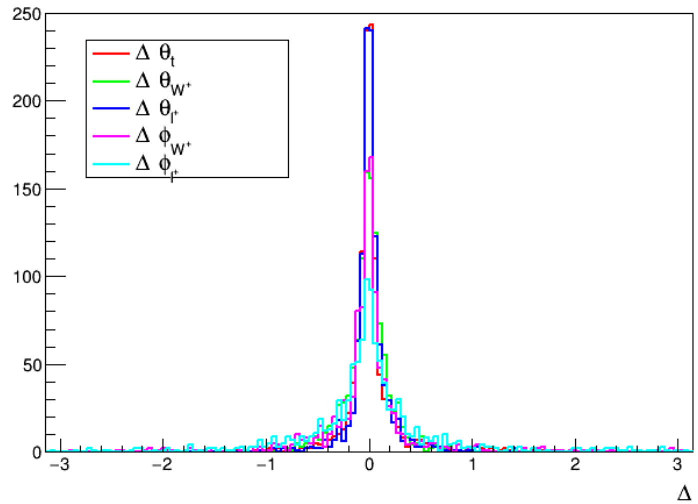
$\sqrt{s}$ (MC Truth vs. Rec.)

$\widetilde{F}^{Z}_{2V}$ fit (The simplest case)

Other ways to reduce the bias

- Convolve the $|M|^2$ with the resolution function of the helicity angles

$$|M|^2 \quad * \quad \text{[histogram]} \quad = \quad |M|^2_{\text{cov.}}$$



The deviation of each helicity angles

- Use other quantities for the quality cut.

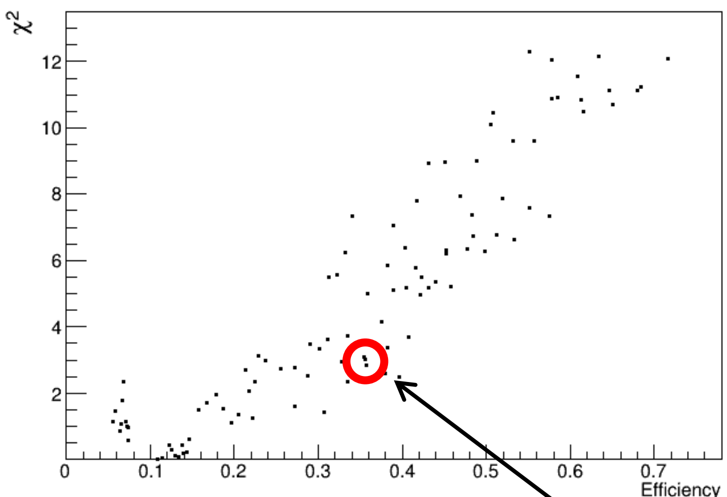
eg) $\left|\chi^2_{tot,\text{caseA1(B1)}} - \chi^2_{tot,\text{caseA2(B2)}}\right|$

$\widetilde{F}_{2V}^{Z}$ Fit (The simplest case)

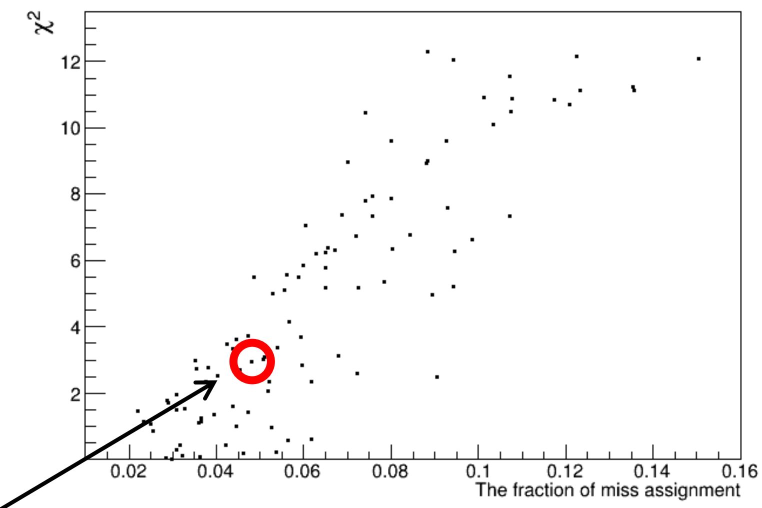
(Fix the other form factors at the SM)

Before quality cut

$\delta\tilde{F}_{2V}^{Z} = 0.117 \pm 0.033,\ \chi^2_{\text{test}} = 12.6$ (confidence level = 0.03%)



χ^2 vs Efficiency



χ^2 vs F_{wrong}

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 36%)

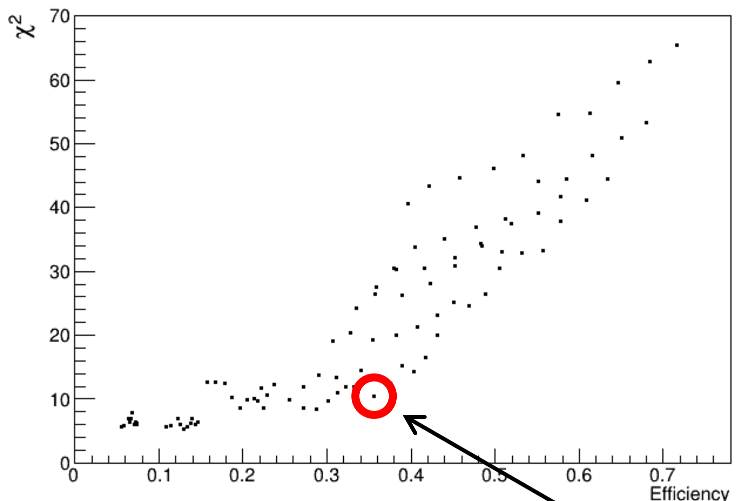
$\delta\tilde{F}_{2V}^{Z} = 0.096 \pm 0.055,\ \chi^2_{\text{test}} = 3.0$ (confidence level = 8.3%)

6 CPC form factors fit

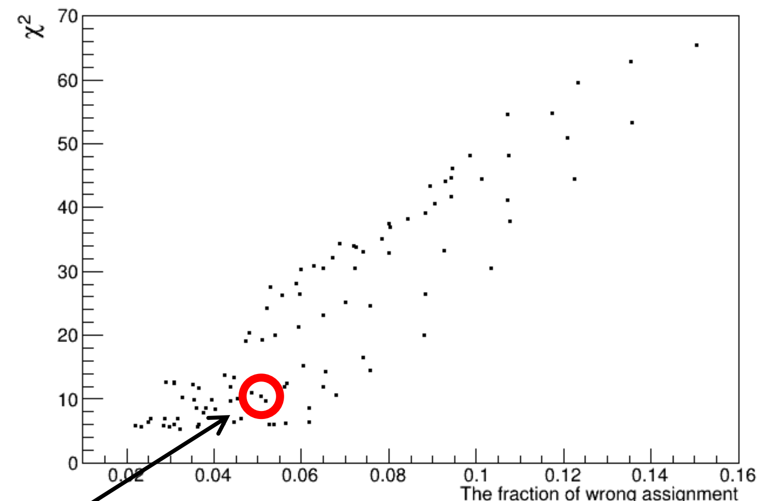
Fit 6 form factors $(\tilde{F}_{1V}^{\gamma}, \tilde{F}_{1V}^{Z}, \tilde{F}_{1A}^{\gamma}, \tilde{F}_{1A}^{Z}, \tilde{F}_{2V}^{\gamma}, \tilde{F}_{2V}^{Z})$

Before quality cut

$< \sigma_F > =$ 0.021, $\chi^2 = 141$ (confidence level ~ 0 %)



χ^2 vs Efficiency



χ^2 vs F_{wrong}

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 36%)

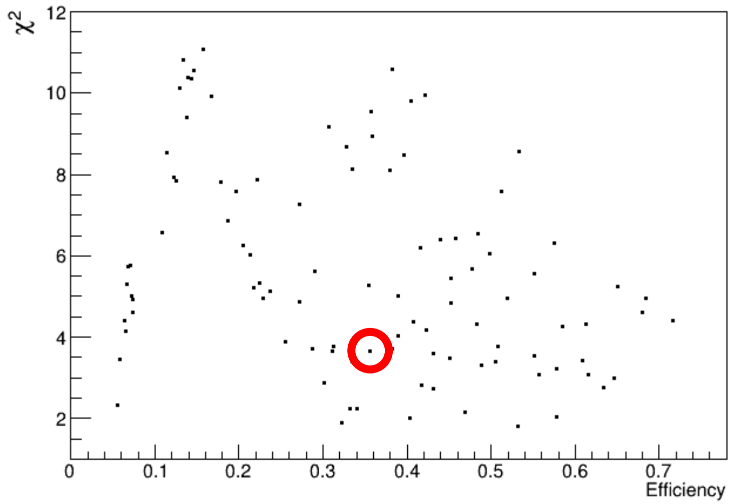
$< \sigma_F > =$ 0.035, $\chi^2 = 10.5$ (confidence level = 11 %)

4 CP Violating Form Factors Fit

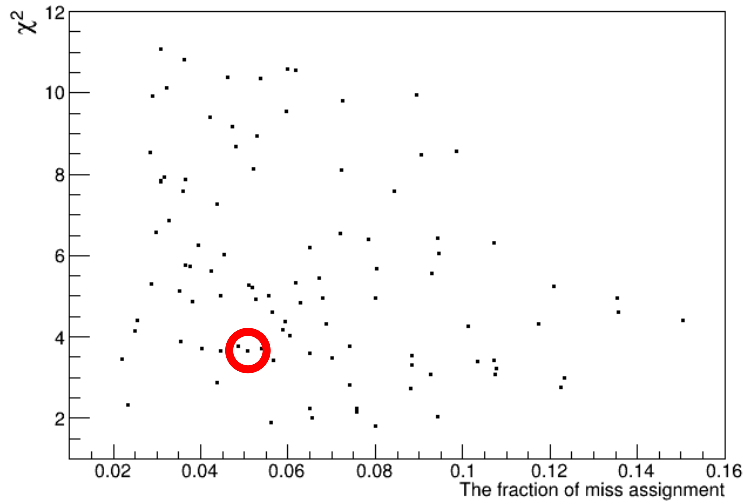
Fit 4 form factors $(Re\tilde{F}_{2A}^{\gamma}, Re\tilde{F}_{2A}^{Z}, Im\tilde{F}_{2A}^{\gamma}, Im\tilde{F}_{2A}^{Z})$

Before quality cut

$< \sigma_F >= 0.026,\ \chi^2 = 8.6$ (confidence level = 7.2 %)



χ^2 vs Efficiency



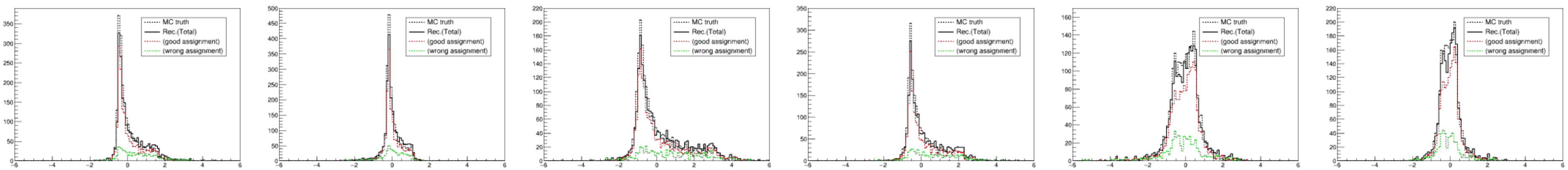
χ^2 vs F_{wrong}

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 35%)

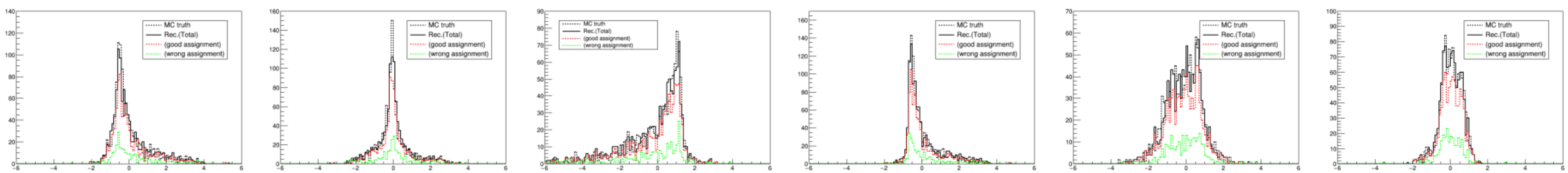
$< \sigma_F >= 0.038, \chi^2 = 3.7$ (confidence level = 45 %)

The distributions of $\omega - \Omega$ (bef. the quality cut)

"Left" polarization

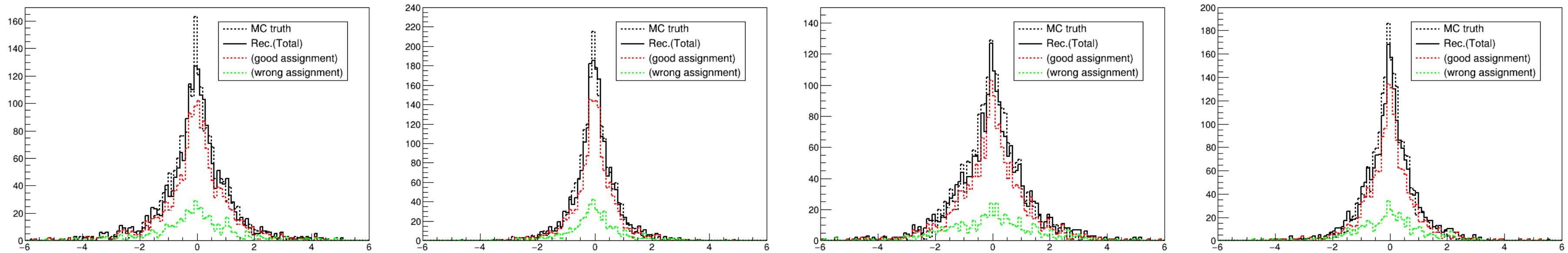


"Right" polarization



The distributions of $\omega - \Omega$ (bef. the quality cut)

“Left” polarization



“Right” polarization