Top electroweak couplings study using di-leptonic state at $\sqrt{s} = 500$ GeV, ILC with the Matrix Element Method

Workshop on top physics at the LC 2017

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Top EW couplings

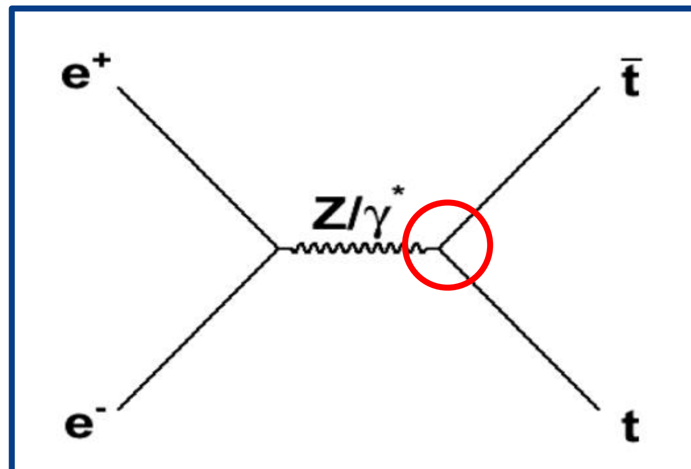
- Top quark is the heaviest particle in the SM. Its large mass implies that it is strongly coupled to the mechanism of electroweak symmetry breaking (EWSB)

  → **Top EW couplings are good probes for New physics behind EWSB**

$$\mathcal{L}_{\text{int}} = \sum_{v=\gamma,Z} g^{v} \left[V_l^{v} \bar{t} \gamma^{l} (F_{1V}^{v} + F_{1A}^{v} \gamma_5) t + \frac{i}{2m_t} \partial_\nu V_l \bar{t} \sigma^{l\nu} (F_{2V}^{v} + F_{2A}^{v} \gamma_5) t \right]$$

eg.)

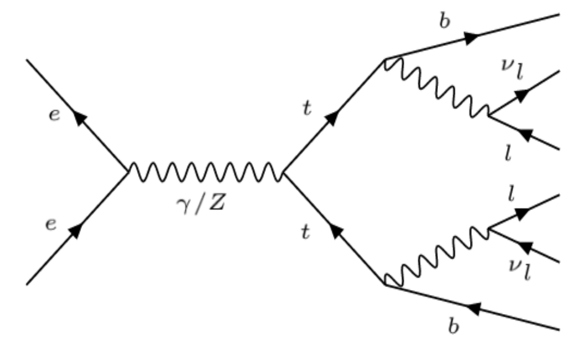
- Composite models yield typically 10% deviation of ${g_{L,R}}^{Z} (= F_{1V}^{Z} \pm F_{1A}^{Z})$ from SM
- In the 2HDM, $F_{2A}^{\gamma/Z}$ which is a CP-violating parameter can be non-zero



Di-leptonic state of the top pair production

Top pair production has three different final states:

- Fully-hadronic state $\left(e^+e^- \to t\bar{t} \to b\bar{b}q\bar{q}q\bar{q}\right)$ 46.2 %
- Semi-leptonic state $\left(e^+e^- \to t\bar{t} \to b\bar{b}q\bar{q}lv\right)$ 43.5%
- **Di-leptonic state $\left(e^+e^- \to t\bar{t} \to b\bar{b}lvlv\right)$ 10.3%**



Advantage

- 9 helicity angles can be computed (details will be described later)

→ Higher sensitivity to the form factors

Difficulty

- Two missing neutrinos → Difficult to reconstruct top quark.

→ **Develop the reconstruction process in realistic situation**

Set up of analysis

Situation	LCWS16, Morioka	Top@LC 17, CERN
Full simulation of ILD	✓	✓
Hadronization	✓	✓
Gluon emission from top	Off	✓
ISR/BS	Off	✓
γγ→hadrons	Off	✓
bkg. events	Off	Off (ongoing)

Sample (Only signal)	Di-muonic state (SM-LO) $e^+e^- \to b\bar{b}\mu^+\nu\mu^-\bar{\nu}$
$\sqrt{s}$	500 GeV
Polarization (P_{e^-}, P_{e^+})	(-0.8, +0.3) "Left" / (+0.8, -0.3) "Right"
Integrated luminosity	500 fb^{-1} (50/50 between Left and Right)
Generator	Whizard
Detector	ILD_01_v05 (DBD ver.)

Reconstruction process

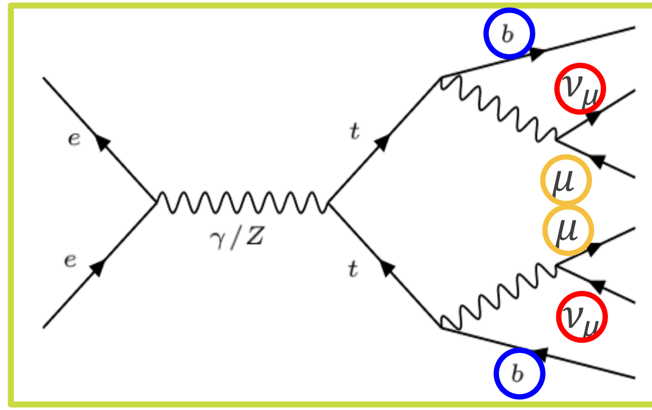
- Isolated leptons tagging
  - Number of isolated leptons = 2 & Opposite charge each of two
- Suppression of γγ → hadrons
  - kt algorithm (cf. the Semi-leptonic analysis, R = 1.5)
- b-jet reconstruction
  - LCFI Plus (Durham algorithm)
  - The b-charge measurement is not used (It will be useful)
- **Kinematical reconstruction of top quark**

Kinematical reconstruction of top

$e^+e^- \to t\bar{t} \to b\bar{b}\mu^+\nu\mu^-\bar{\nu}$

Measurable
- muon's : $E_{\mu^+}, \theta_{\mu^+}, \phi_{\mu^+}, E_{\mu^-}, \theta_{\mu^-}, \phi_{\mu^-}$
- b-jet's : $E_{b1}, \theta_{b1}, \phi_{b1}, E_{b2}, \theta_{b2}, \phi_{b2}$

Missing
- neutrino's : $E_{\nu}, \theta_{\nu}, \phi_{\nu}, E_{\bar{\nu}}, \theta_{\bar{\nu}}, \phi_{\bar{\nu}}$
- **=> 6 unknowns**



To recover them, impose the kinematical constraints ;

- Initial state constraints : $(\sqrt{s}, \vec{P}_{\text{init.}}) = (500, \vec{0})$
- Mass constraints : $m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$

=> 8 constraints (2 in excess)

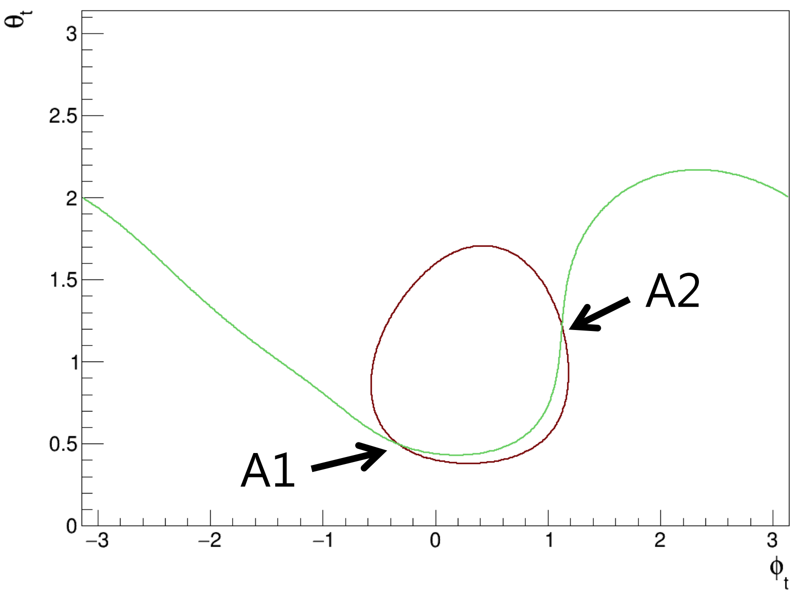
We don't use E_{b1} and E_{b2} which are relatively difficult to reconstruct.

Kinematical reconstruction of top

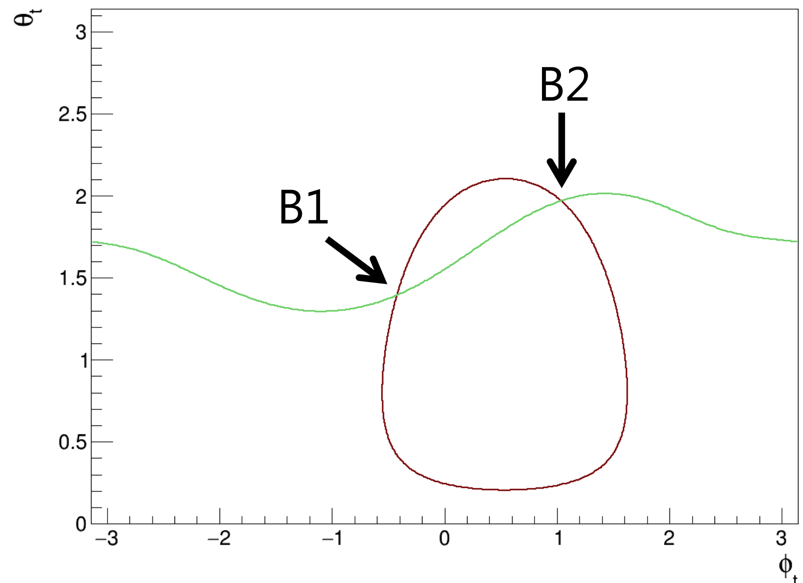
To detect the solution, we solve the following equations.

$$E_{\mu^\pm}^{W^\pm \text{ rest frame}}(\theta_t, \phi_t) = m_{W^\pm}/2 \text{ (Red : } \mu^+\text{, Green : } \mu^-\text{)}$$

assignment A (correct), $b1 = b,\ b2 = \bar{b}$



assignment B (wrong), $b1 = \bar{b},\ b2 = b$

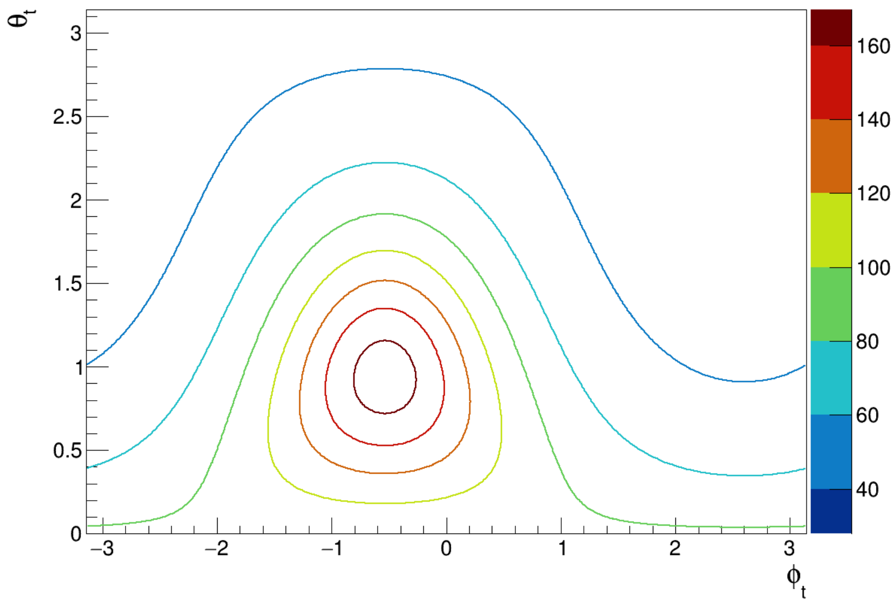


Typically, 4 candidates exist for each event.

We need to select the optimal solution from these candidates.

Kinematical reconstruction of top

To select the optimal solution, we compare E_b and $E_{\bar{b}}$ between calculated by (θ_t, ϕ_t) and measured by the b-jet reconstruction.



$E_b(\theta_t, \phi_t)$ in the case of assignment A

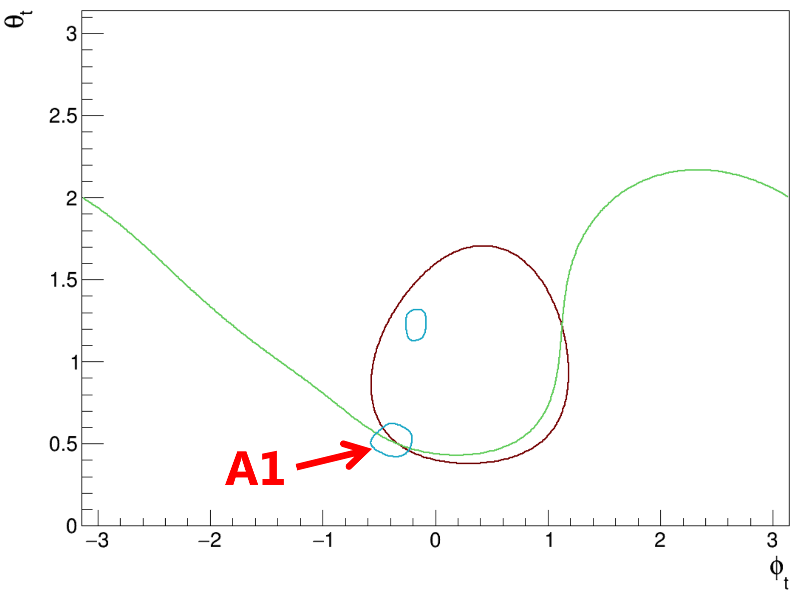
$$\chi_b^2(\theta_t, \phi_t) = \left(\frac{E_b(\theta_t, \phi_t) - E_b^{\text{meas.}}}{\sigma[E_b^{\text{meas.}}]}\right)^2 + \left(\frac{E_{\bar{b}}(\theta_t, \phi_t) - E_{\bar{b}}^{\text{meas.}}}{\sigma[E_{\bar{b}}^{\text{meas.}}]}\right)^2$$

Compute χ_b^2 for each candidate → **Pick one which has the smallest χ_b^2**

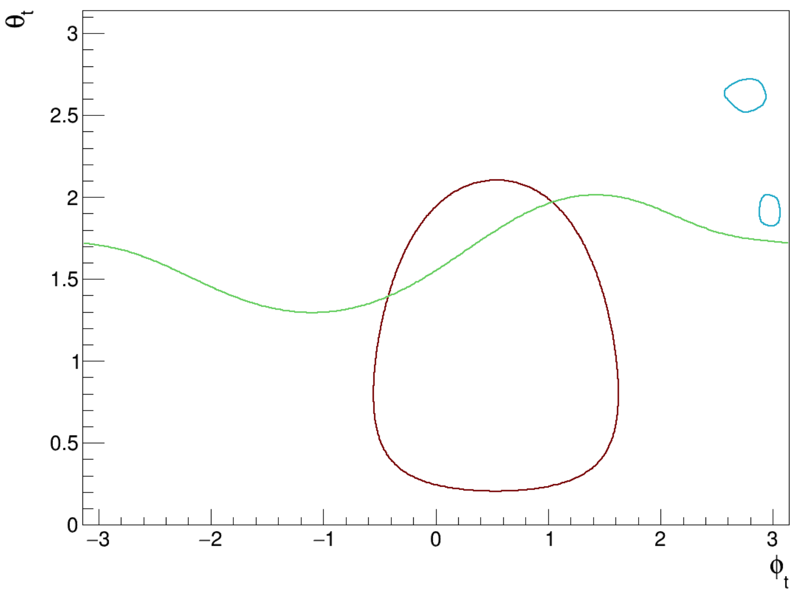
Kinematical reconstruction of top

$\chi_b^2\,(\theta_t, \phi_t) = 2$ (Blue)

assignment A (correct), $b1 = b,\ b2 = \bar{b}$



assignment B (wrong), $b1 = \bar{b},\ b2 = b$



The candidate A1 has the minimum χ_b^2.

→ The assignment A is selected and the solution is $(\theta_t, \phi_t) \simeq (0.5, -0.35)$

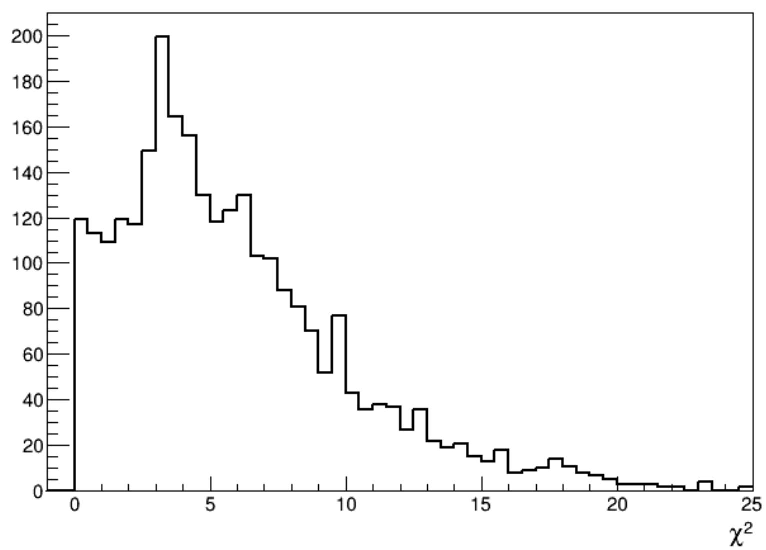
Kinematical reconstruction of top

Technically, to obtain the solution, we minimize χ^2_{tot} ;

$$\chi^2_{tot}(\theta_t,\phi_t) = \chi^2_\mu(\theta_t,\phi_t) + \chi^2_b(\theta_t,\phi_t)$$

where $\chi^2_\mu(\theta_t,\phi_t) \equiv \left(\frac{E_{\mu^+}^{(W^+\text{ rest frame})}(\theta_t,\phi_t) - m_{W^+}/2}{\sigma\left[E_{\mu^+}^{(W^+\text{ rest frame})}\right]}\right)^2 + \left(\frac{E_{\mu^-}^{(W^-\text{ rest frame})}(\theta_t,\phi_t) - m_{W^-}/2}{\sigma\left[E_{\mu^-}^{(W^-\text{ rest frame})}\right]}\right)^2$

χ^2_μ is dominant to determine (θ_t,ϕ_t) because $\sigma\left[E_\mu^{(W\text{ rest frame})}\right] \ll \sigma[E_b]$



χ^2_{tot} distribution

F_{wrong} : the fraction of the wrong assignment of b-jets

F_{wrong} (the fraction of the wrong assignment of b-jets) **= 22 %**

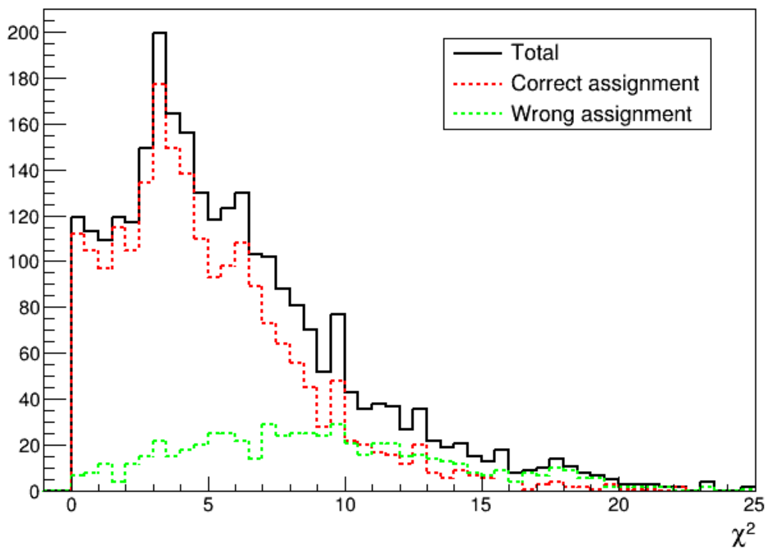
When we use samples not including the ISR, F_{wrong} = 8 %

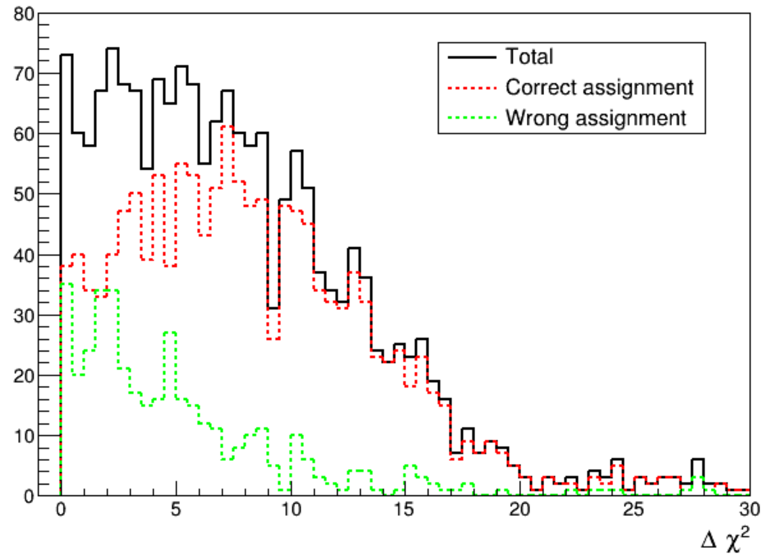
→ ISR significantly affects the assignment problem.

We use two quantities to reduce F_{wrong}

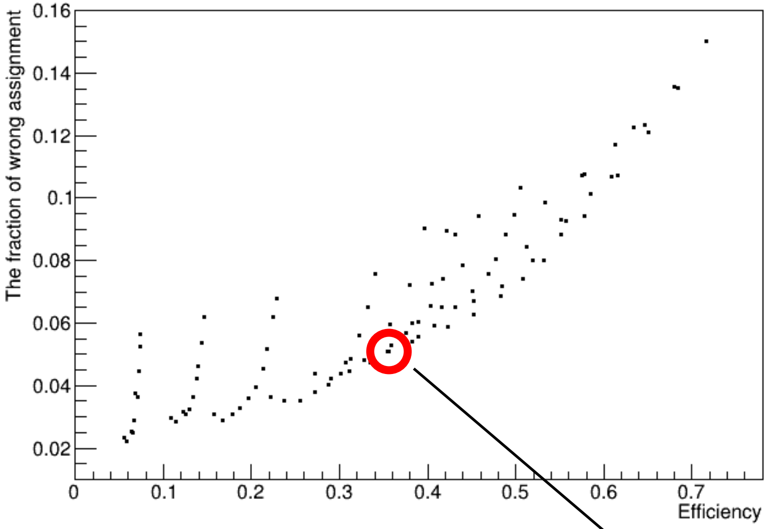
χ^2_{tot} (as mentioned)

$\Delta\chi^2_{tot} = \left|\chi^2_{tot,\text{assignment A}} - \chi^2_{tot,\text{assignment B}}\right|$





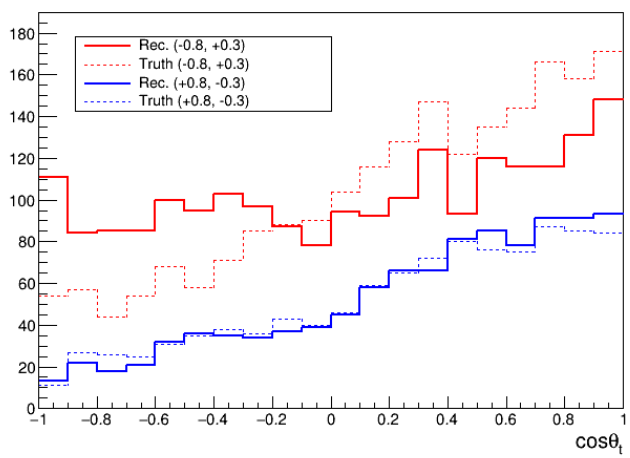
F_{wrong} : the fraction of the wrong assignment of b-jets



Efficiency vs. F_{wrong}

We investigate F_{wrong} and the efficiency varying the set of criteria for $(\chi^2_{tot}, \Delta\chi^2_{tot})$

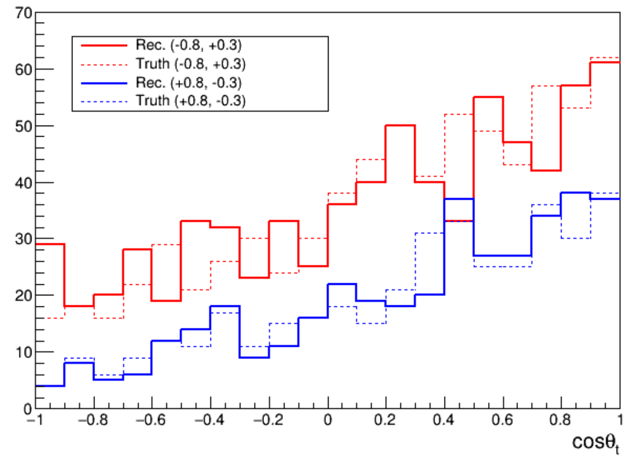
The polar angle distribution of top is improved by the quality cut.



χ^2_{tot}<5, $\Delta\chi^2_{tot}$>6

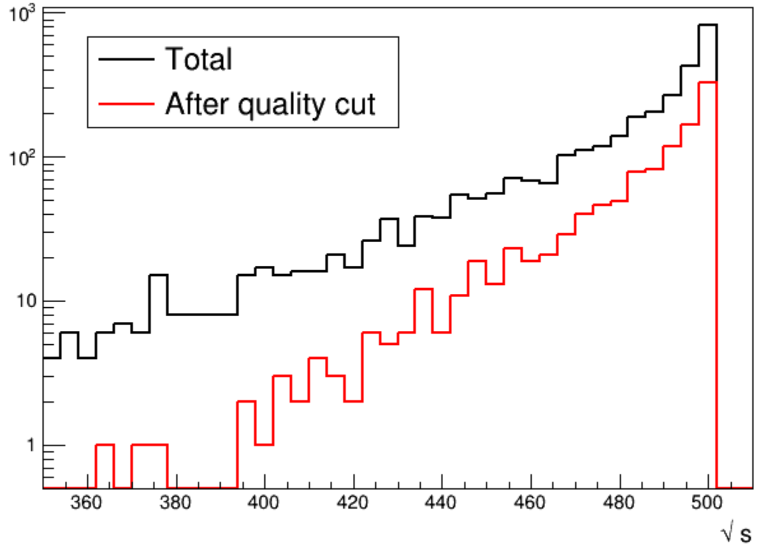
(F_{wrong} = 5.0 %

Efficiency = 36 %)

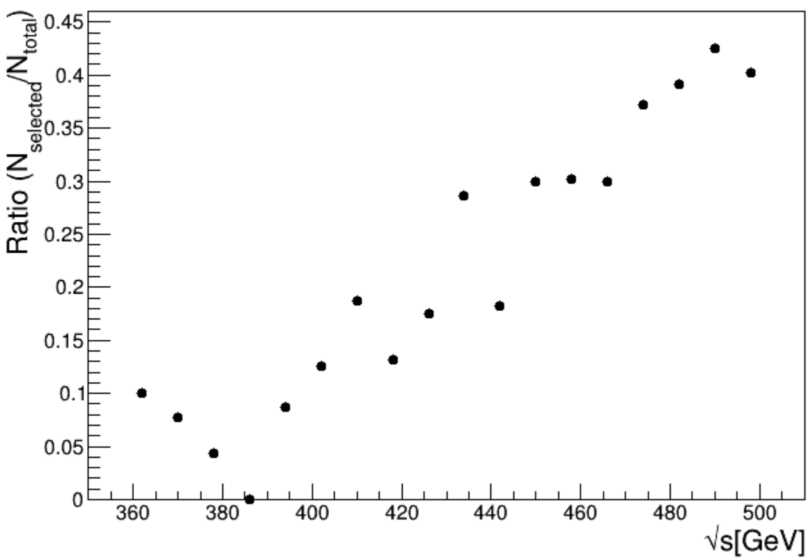


Luminosity spectrum

Because we impose the initial state constraints, the events which have low $\sqrt{s}$ are badly reconstructed.



Luminosity spectrum
Black : Total events, Red : After quality cut



Ratio of luminosity spectrum (Red/Black)

The quality cut reduces low $\sqrt{s}$ events, but there are still a tail.

Luminosity spectrum

Tried to fit the energy of ISR photon along beam direction;

$$e^+e^- \to b\bar{b}\mu^+\nu\mu^-\bar{\nu} + \gamma_{\text{ISR}}$$

→ Another parameter, K

- $|K| = E_\gamma/250$, hence $\sqrt{s} = 500 * \sqrt{1-|K|}$
- If γ is emitted in the $e^-(e^+)$ direction, K is positive (negative).

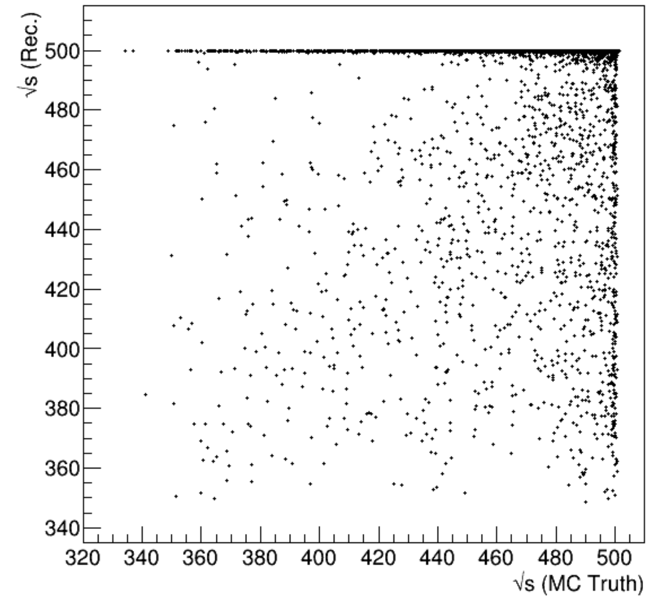
Then one minimizes $\chi^{2}_{tot}{}'(\theta_t, \phi_t, K)$;

$$\chi^{2}_{tot}{}'(\theta_t, \phi_t, K) = \chi^{2}_{tot}(\theta_t, \phi_t, K) - 2\log \mathrm{PDF}_K(K)$$

→ Reconstructed $\sqrt{s}$ don't correlate MC truth.

→ The constraints are not enough.

Now we fix $K = 0$ (i.e. use $\chi^{2}_{tot}(\theta_t, \phi_t)$)



$\sqrt{s}$ (MC Truth vs. Rec.)

9 helicity angles computation

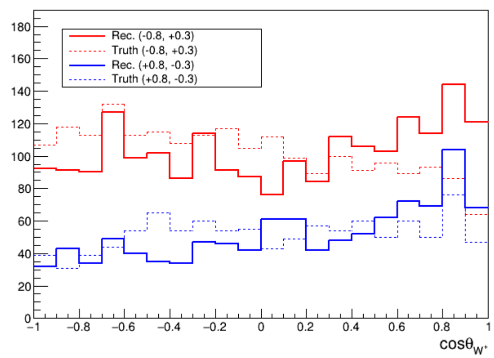
All final state particles including two neutrinos can be calculated. The 9 helicity angles which are related to the ttZ/γ vertex are computed.

$$\theta_t, \theta_{W^+}^{t\ \mathrm{frame}}, \phi_{W^+}^{t\ \mathrm{frame}}, \theta_{\mu^+}^{W^+\ \mathrm{frame}}, \phi_{\mu^+}^{W^+\ frame}, \theta_{W^-}^{\bar{t}\ frame}, \phi_{W^-}^{\bar{t}\ frame}, \theta_{\mu^-}^{W^-frame}, \phi_{\mu^-}^{W^-frame}$$

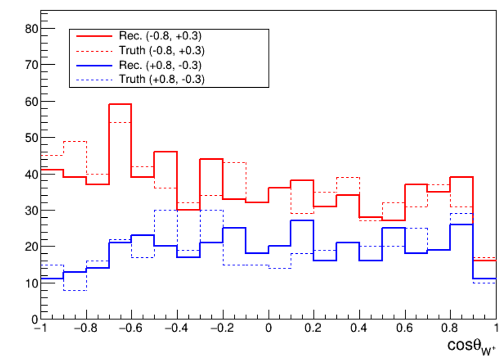
(G. L. Kane, G. A. Ladinsky, C.-P. Yuan, Phys.Rev. D45 (1992) 124-141)

eg)

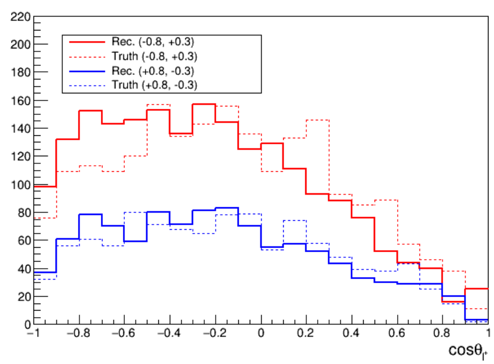
$\cos\theta_{W^+}^{t\ \mathrm{frame}}$

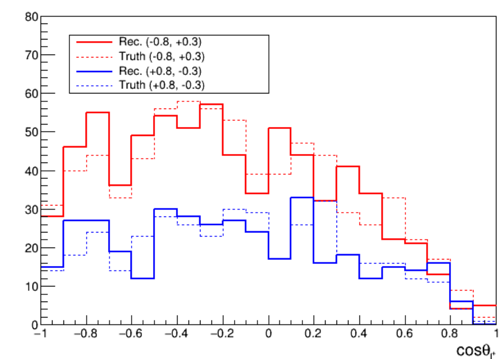


χ^2_{tot}<5, $\Delta\chi^2_{tot}$>6



$\cos\theta_{\mu^+}^{W^+\ \mathrm{frame}}$





Matrix element method analysis

Matrix element method is based on the maximum likelihood method.

The $|M|^2$ (SM-LO) is used as the probability density function.

We use the 9-dim distribution and the cross section simultaneously

→ Fit any the form factors.

1. Only $\tilde{F}_{2V}^{Z}$ (The simplest case)
2. 6 CPC form factors $\left(\tilde{F}_{1V}^{\gamma}, \tilde{F}_{1V}^{Z}, \tilde{F}_{1A}^{\gamma}, \tilde{F}_{1A}^{Z}, \tilde{F}_{2V}^{\gamma}, \tilde{F}_{2V}^{Z}\right)$
3. 4 CPV form factors $\left(Re\tilde{F}_{2A}^{\gamma}, Re\tilde{F}_{2A}^{Z}, Im\tilde{F}_{2A}^{\gamma}, Im\tilde{F}_{2A}^{Z}\right)$

Matrix element method analysis

To estimate the goodness of fit, we use chi-squared test ;

$$\chi^2 = \sum \delta F_i V_{ij}^{-1} \delta F_j$$

where

δF_i : the deviation of the form factor from SM value

V_{ij} : the covariance matrix of the form factor

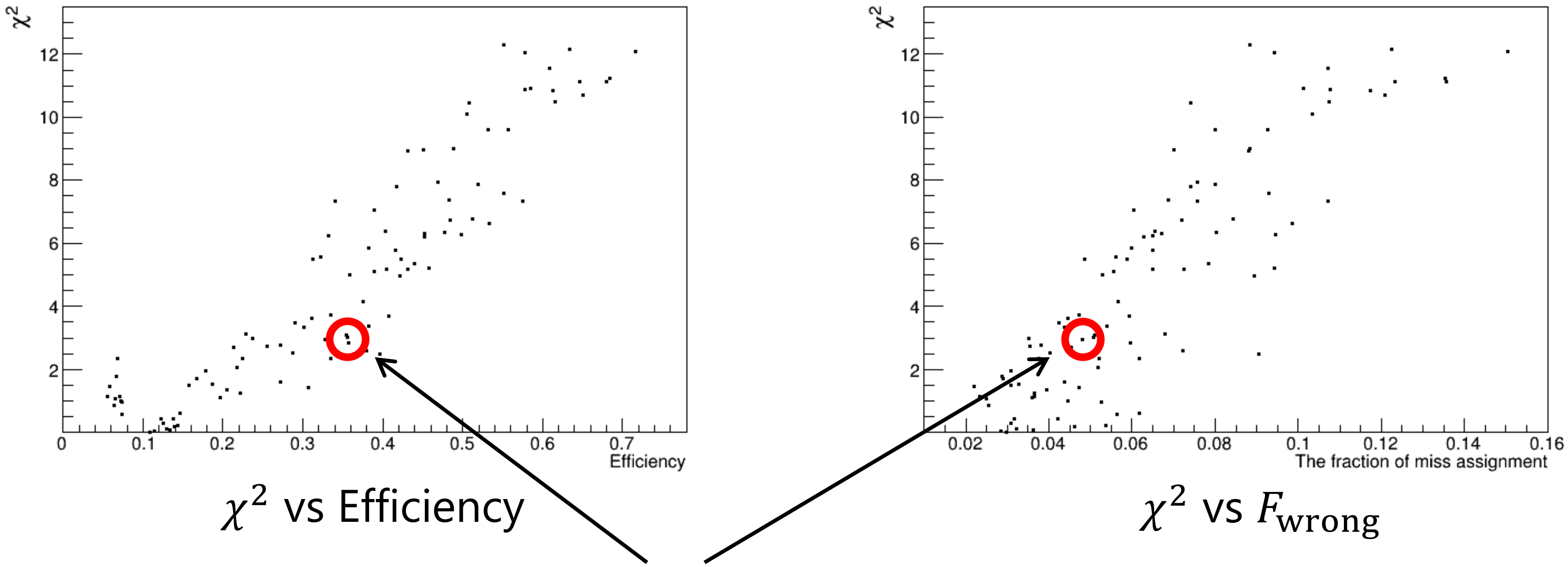
From χ^2 and the degree of freedom, the confidence level is computed.

$\tilde{F}_{2V}^{Z}$ fit (The simplest case)

(Fix the other form factors at the SM, $\delta F = 0$)

Before quality cut

$\delta\tilde{F}_{2V}^{Z} = 0.117 \pm 0.033,\ \chi^2 = 12.6$ (confidence level = 0.03%)



χ^2 vs Efficiency

χ^2 vs F_{wrong}

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 36%)

$\delta\tilde{F}_{2V}^{Z} = 0.096 \pm 0.055,\ \chi^2 = 3.0$ (confidence level = 8.3%)

$\tilde{F}_{2V}^{Z}$ fit (The simplest case)

Use only $\cos\theta_t$ and the cross section

9 helicity angles → $\cos\theta_t$

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 36%)

$\delta\tilde{F}_{2V}^{Z} = -0.074 \pm 0.087$, $\chi^2 = 0.71$ (confidence level = 40 %)

- The error becomes 1.6 factor larger from the 9 helicity angles case
- The bias disappears

Investigate the error and bias changing the number of angles (ongoing)

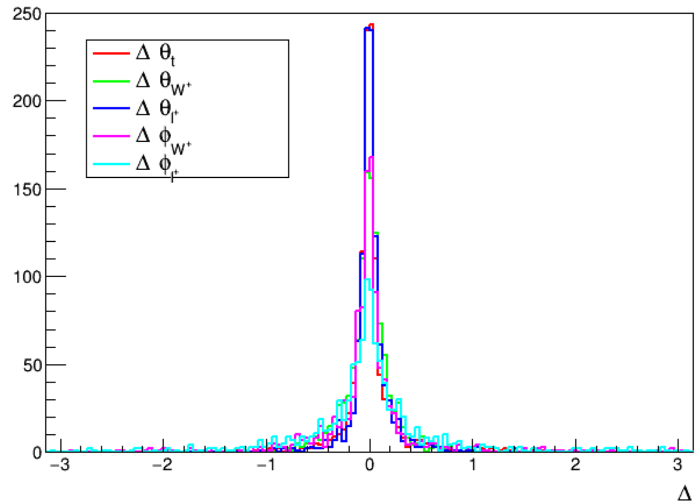
9 helicity angles → 7 helicity angles → … → $\cos\theta_t$

$\widetilde{F}_{2V}^{Z}$ fit (The simplest case)

Other ways to reduce the bias

- Convolve the $|M|^2$ with the resolution function of the helicity angles

$$|M|^2 \quad * \quad \text{(resolution function)} \quad = |M|^2_{\text{cov.}}$$



The deviation of each helicity angles

- Use other quantities for the quality cut.

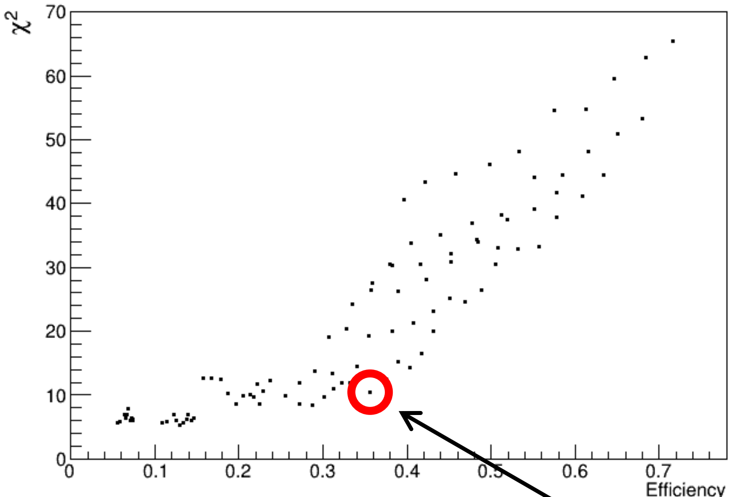
eg) $\left|\chi^2_{tot,\text{caseA1(B1)}} - \chi^2_{tot,\text{caseA2(B2)}}\right|$

6 CPC form factors fit

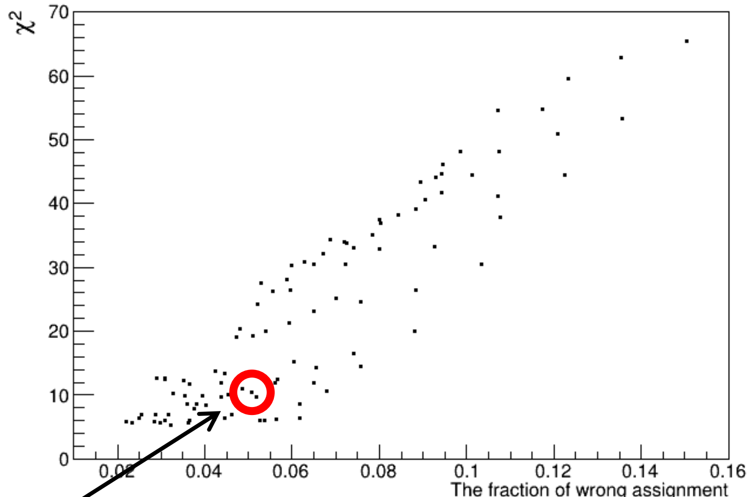
Fit 6 form factors $(\tilde{F}_{1V}^{\gamma}, \tilde{F}_{1V}^{Z}, \tilde{F}_{1A}^{\gamma}, \tilde{F}_{1A}^{Z}, \tilde{F}_{2V}^{\gamma}, \tilde{F}_{2V}^{Z})$

Before quality cut

$< \sigma_F > =$ 0.021, $\chi^2 = 141$ (confidence level ~ 0 %)



χ^2 vs Efficiency



χ^2 vs F_{wrong}

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 36%)

$< \sigma_F > =$ 0.035, $\chi^2 = 10.5$ (confidence level = 11 %)

4 CPV form factors fit

Fit 4 form factors $\left(Re\tilde{F}_{2A}^{\gamma}, Re\tilde{F}_{2A}^{Z}, Im\tilde{F}_{2A}^{\gamma}, Im\tilde{F}_{2A}^{Z}\right)$

Before quality cut

$< \sigma_F > =$ 0.026, χ^2 = 8.6 (confidence level = 7.2 %)

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 36%)

$< \sigma_F > =$ 0.038, χ^2 = 3.7 (confidence level = 45 %)

Even though the fraction of the wrong assignment of b-jets (F_{wrong}) is large, the results are almost consistent with SM.

→ Need to use samples which have the deviation of these form factors to investigate the effects of F_{wrong}

Summary

- **Di-leptonic state analysis produces the 9 helicity angles which are sensitive to the form factors.**
- **Reconstruct top quark imposing the kinematical constraints**
  - ISR significantly affects the assignment problem of b-jets
  - The quality cut improves the fraction of wrong assignment of b-jets, hence the angular distributions.
- **Fit the form factors with the Matrix element method**
  - CPC : After quality cut, results are consistent with SM. The small bias will be reduced by convoluting the resolution functions etc.
  - CPV : Need to investigate the effects on CPV form factors using samples which have deviation of these form factors.

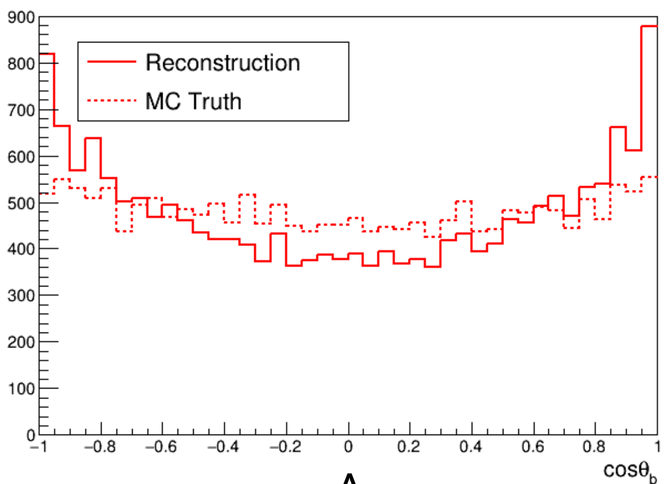
Back up

Suppression of γγ → hadrons & b-jet reconstruction

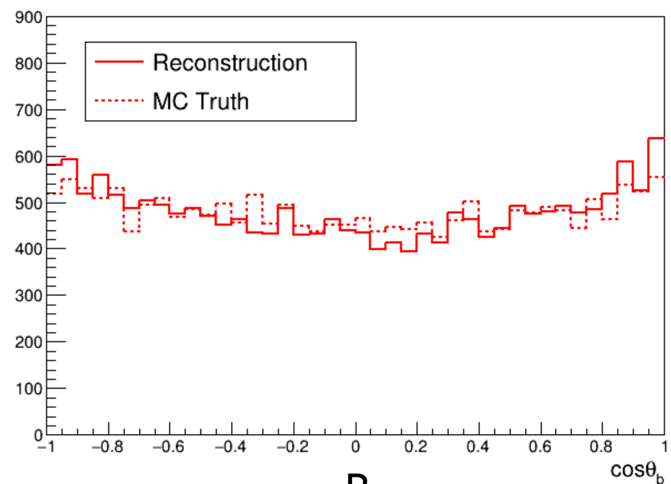
Particles from γγ → hadrons are mostly emitted along the beam direction. The direction of the b-jet is affected by these particles.

Suppress these particles using the kt algorithm (R=1.5).

→ The direction of the b-jet is improved.



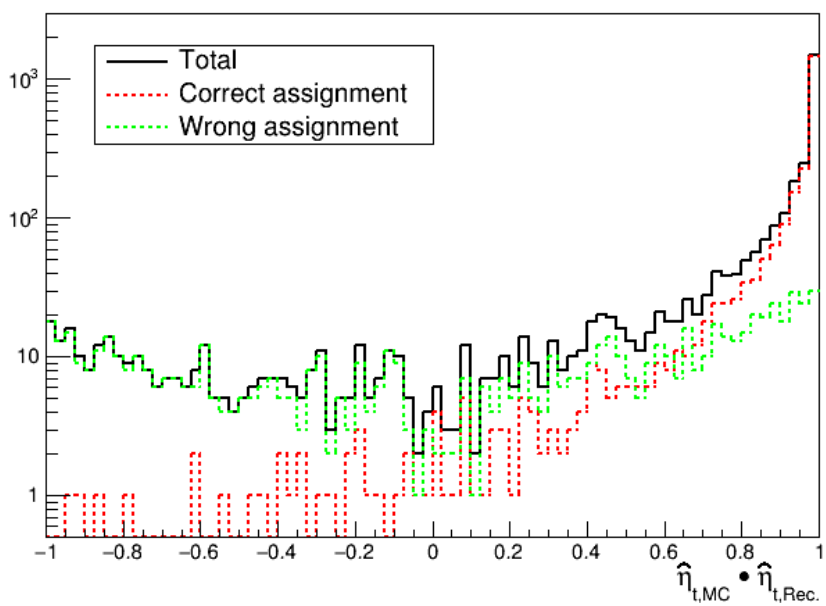
A

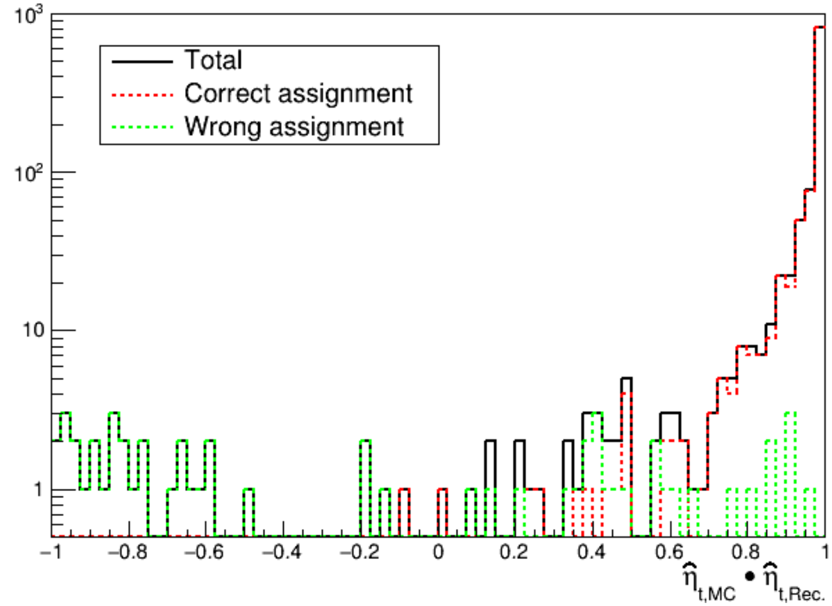


B

The polar angle distribution b-jets. A: without the suppression of γγ →hadrons, B: with the suppression of γγ → hadrons

Scalar product, $\hat{\eta}_{t,\mathrm{MC}} \cdot \hat{\eta}_{t,\mathrm{Rec.}}$



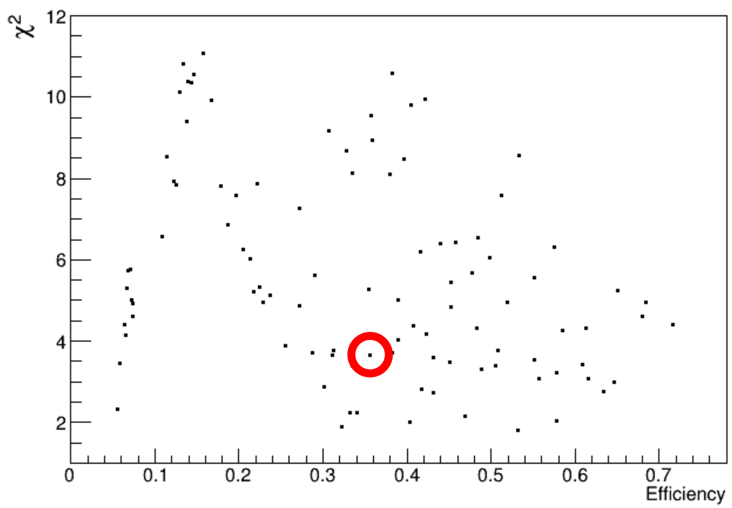


CPV form factors fit

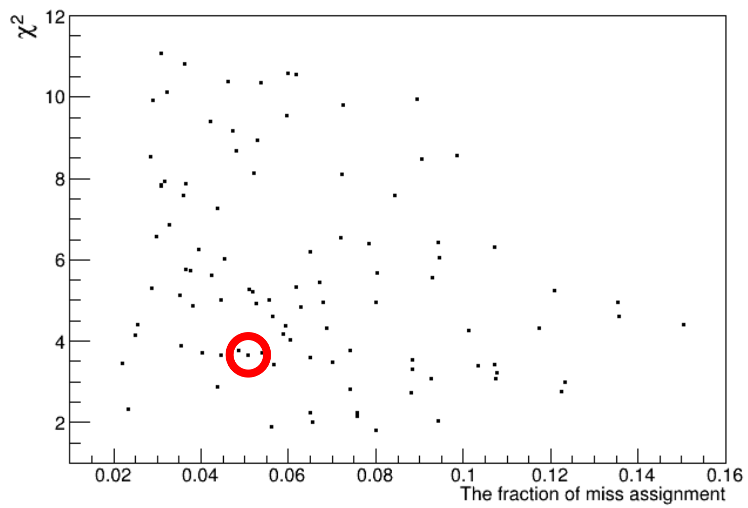
Fit 4 form factors $\left(Re\tilde{F}_{2A}^{\gamma}, Re\tilde{F}_{2A}^{Z}, Im\tilde{F}_{2A}^{\gamma}, Im\tilde{F}_{2A}^{Z}\right)$

Before quality cut

$< \sigma_F >= 0.026,\ \chi^2 = 8.6$ (confidence level = 7.2 %)



χ^2 vs Efficiency



χ^2 vs F_{wrong}

After quality cut ($\chi^2_{tot} < 5$ & $\Delta\chi^2_{tot} > 6$, efficiency 35%)

$< \sigma_F >= 0.038,\ \chi^2 = 3.7$ (confidence level = 45 %)