

A simulation study on measurement of
the polarization asymmetry A_{LR}
using the initial state radiation at the ILC
with center-of-mass energy of 250 GeV

TOHOKU UNIVERSITY
Experimental Particle Physics Labo.



M2 Takayuki Ueno



eeZ couplings study

Lagrangian for Z-fermion interference

$$\mathcal{L} = \frac{g}{\cos \theta_W} Z_\mu \bar{\Psi} \gamma^\mu (s_L P_L + s_R P_R) \Psi$$

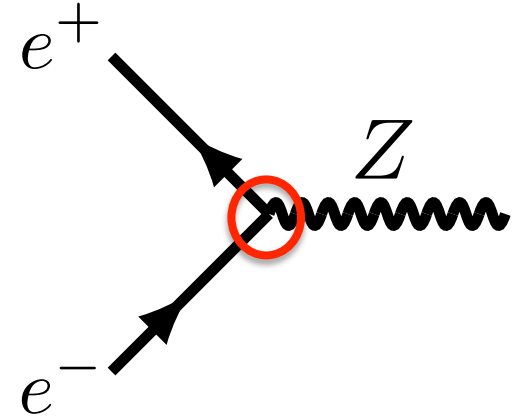
$s_{L/R}$: Eigenvalue of $T_3 - Q \sin^2 \theta_W$

Q : Operator for Electric Charge

T_3 : Operator for Weak Isospin

θ_W : Weak Mixing Angle

$P_{L/R}$: Projection Operator $(P_L \equiv \frac{1 - \gamma_5}{2} \quad P_R \equiv \frac{1 + \gamma_5}{2})$



For electron,

$$s_L = -\frac{1}{2} + \sin \theta_W \quad s_R = \sin \theta_W$$



This causes left-light polarization asymmetry

A_{LR}

$$A_{LR} \equiv \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

$\sigma_{L/R}$: the cross-section with $(P_{e-}, P_{e+}) = (\mp 1.0, \pm 1.0)$

Because of $\sigma_{L/R} \propto s_{L/R}^2$,

$$A_{LR} = \frac{s_L^2 - s_R^2}{s_L^2 + s_R^2}$$

$$= \frac{(-\frac{1}{2} + \sin \theta_W)^2 - (\sin \theta_W)^2}{(-\frac{1}{2} + \sin \theta_W)^2 + (\sin \theta_W)^2}$$

$$= \frac{2(1 - 4 \sin^2 \theta_W)}{1 + (1 - 4 \sin^2 \theta_W)^2}$$



For electron

The A_{LR} is very sensitive to the weak mixing angle θ_W

A_{LR}

Experimentally,

$$A_{LR} = \frac{\sigma_L^{meas} - \sigma_R^{meas}}{\sigma_L^{meas} + \sigma_R^{meas}} \frac{1 + \langle P_{e^-} \rangle \langle P_{e^+} \rangle}{\langle P_{e^-} \rangle + \langle P_{e^+} \rangle}$$

$\sigma_{L/R}^{meas}$: the cross-section with $(P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$

$\langle P_{e^-/e^+} \rangle$: the magnitude of electron/positron polarization

$$P_{e^-/e^+} \equiv \frac{N_R - N_L}{N_R + N_L}$$

$N_{R/L}$: Number of right/left-handed electron (positron) in a bunch

The ILC is suitable for research of the eeZ coupling because of beam polarization

Effective Field Theory for Higgs precision measurement

General SU(2)×U(1) gauge invariant Lagrangian with dimension-6 operators in addition to the SM

$$\begin{aligned}
 \Delta\mathcal{L} = & \frac{c_H}{2v^2} \partial^\mu(\Phi^\dagger\Phi)\partial_\mu(\Phi^\dagger\Phi) + \frac{c_T}{2v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\Phi^\dagger \overleftrightarrow{D}_\mu \Phi) - \frac{c_6\lambda}{v^2} (\Phi^\dagger\Phi)^3 \\
 & + \frac{g^2 c_{WW}}{m_W^2} \Phi^\dagger\Phi W_{\mu\nu}^a W_{\mu\nu}^a + \frac{4gg' c_{WB}}{m_W^2} \Phi^\dagger t^a \Phi W_{\mu\nu}^a B^{\mu\nu} \\
 & + \frac{g'^2 c_{BB}}{m_W^2} \Phi^\dagger\Phi B_{\mu\nu} B^{\mu\nu} + \frac{g^3 c_{3W}}{m_W^2} \epsilon_{abc} W_{\mu\nu}^a W_\rho^{b\nu} W^{c\rho\mu} \\
 & + i \frac{c_{HL}}{v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu L) + 4i \frac{c'_{HL}}{v^2} (\Phi^\dagger t^a \overleftrightarrow{D}^\mu \Phi)(\bar{L}\gamma_\mu t^a L) \\
 & + i \frac{c_{HE}}{v^2} (\Phi^\dagger \overleftrightarrow{D}^\mu \Phi)(\bar{e}\gamma_\mu e) + c_{\tau\Phi} \frac{y_\tau}{v^2} (\Phi^\dagger\Phi) \bar{L}_3 \cdot \Phi_{\tau R} + h.c.
 \end{aligned}$$

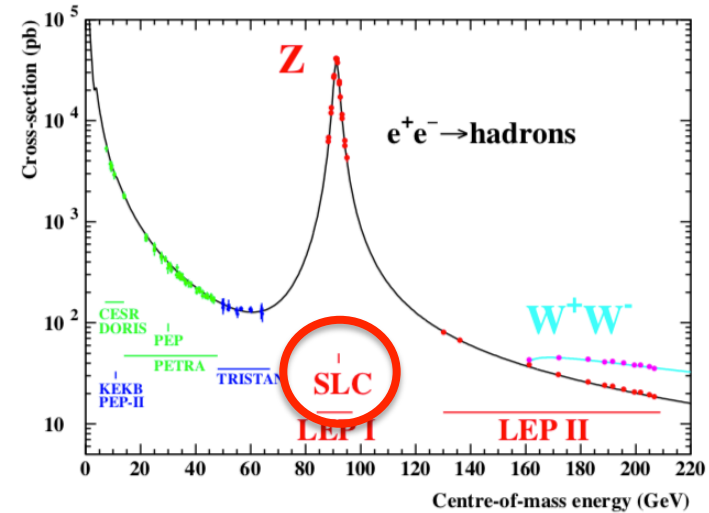
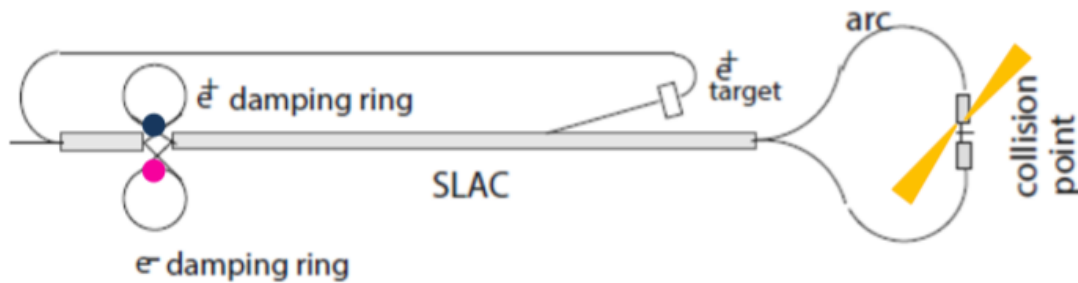
Using this framework, Higgs coupling is fitted

$$\sin^{*2} \theta_W = \sin^2 \theta_W + \frac{\sin^2 \theta_W}{\cos^2 \theta_W - \sin^2 \theta_W} (c'_{HL} + 8c_{WB} - c_0^2 c_T) - \frac{1}{2} c_{HE} - \sin^2 \theta_W (c_{HL} - c_{HE})$$

$$\sin \theta_W \text{ is defined by } 4 \sin \theta_W \cos \theta_W = \frac{4\pi\alpha}{\sqrt{2}G_F m_Z^2}$$

$\sin^* \theta_W$: effective $\sin \theta_W$ on Z pole

Previous Study



Stanford Linear Collider (SLC)

arXiv:hep-ex/0509008

- The world's first electron-positron linear collider
- The center-of-mass energy of the e^+e^- collisions $\sim m_Z$ (91 GeV)
- Longitudinal polarization of electron beam was established
 - > reached $\sim 80\%$ in the end of its operation.
- 600 thousand Z decays collected by the SLD detector



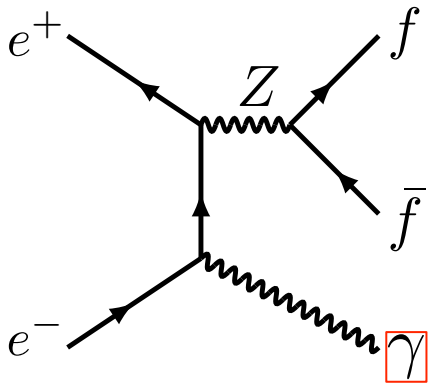
- Previous Value -

$$\frac{\Delta A_{LR}}{A_{LR}} \approx 1.5\%$$

Goal of this study

Goal of this study

to estimate how the statistical error of the A_{LR} can be reduced
at the ILC with center-of-mass energy of 250 GeV



COM Energy after ISR : $\sqrt{s'} = 91.2 \text{ GeV} (= M_Z)$



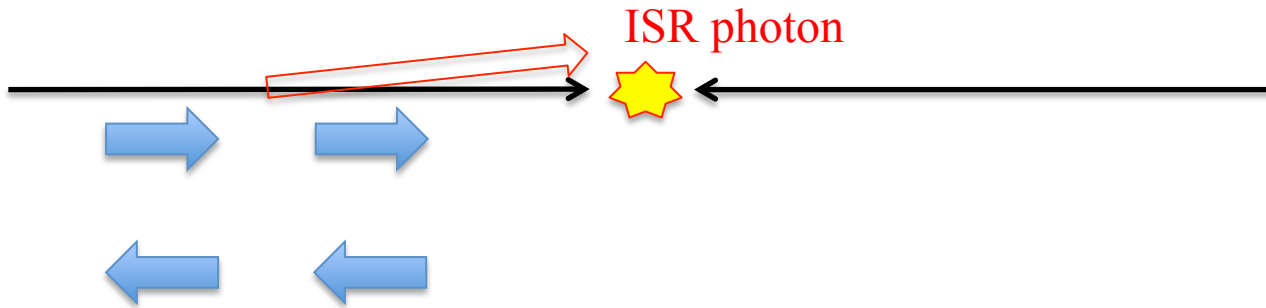
Energy of ISR : $E_{ISR} \approx 108 \text{ GeV}$

$$\sqrt{s'} = \sqrt{4E_{beam}(E_{beam} - E_{ISR})}$$

Helicity

if $E_{kin} \gg E_0 \rightarrow m_e \approx 0$

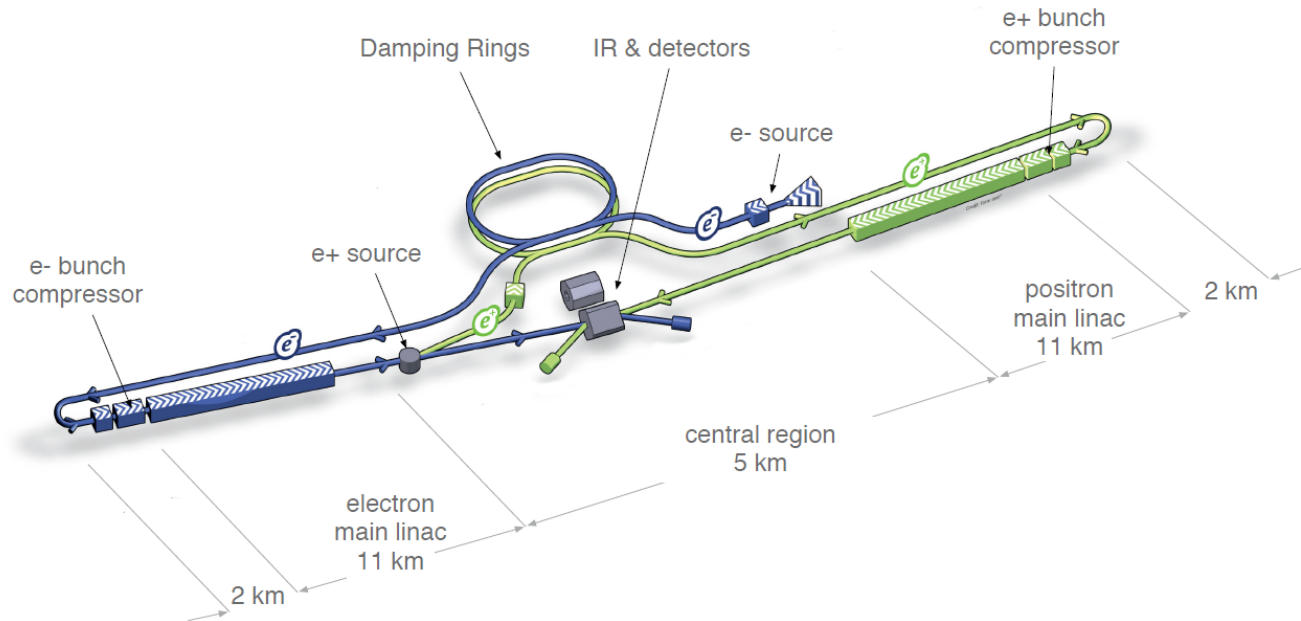
 : the direction of the spin of beam



The sign of the beam helicity is conserved

A_{LR} will be possible to be measured at the ILC with COM energy of 250 GeV

International Linear Collider



- **Electron-positron collider** with a center-of-mass energy of 250 GeV
- Polarized electron/positron beam $(P_{e^-}, P_{e^+}) = (\mp 0.8, \pm 0.3)$
- Candidate detector : **SiD detector** and ILD detector

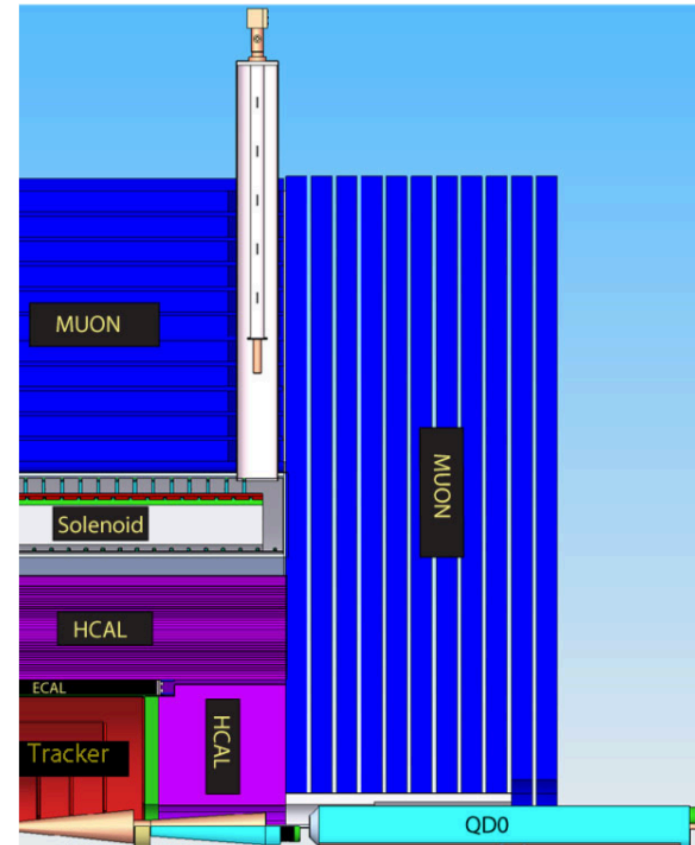
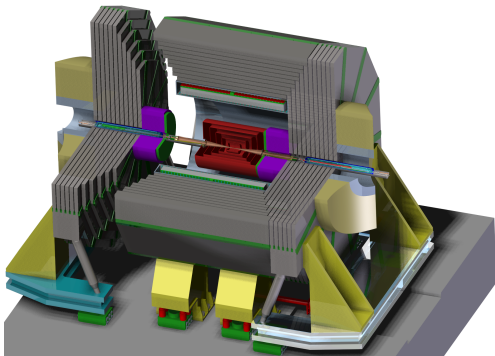
SiD Detector

SiD

provides excellent momentum and energy resolution over the broad range of particles energies expected at the ILC

due to

- 5T solenoidal magnetic field,
- a vertex detector with silicon pixels
- a main tracker with silicon strips et al.



Parameter setup

Event Generation : WHIZARD 1.95

- Accelerator parameter: based on the Technical Design Report (TDR)

Parton shower & hadronization	Pythia 6.4
Center-of-mass energy	250 GeV
Beam Polarization	(-0.8,+0.3) / (+0.8,-0.3)
Integrated luminosity	250 fb ⁻¹ / 250 fb ⁻¹
ISR option	ON
Beamstrahlung	CIRCE2
$\sin^2\theta_W$	0.22225
A_{LR}^{lepton}	0.21930

Detector Simulation : a fast simulation (Delphes)

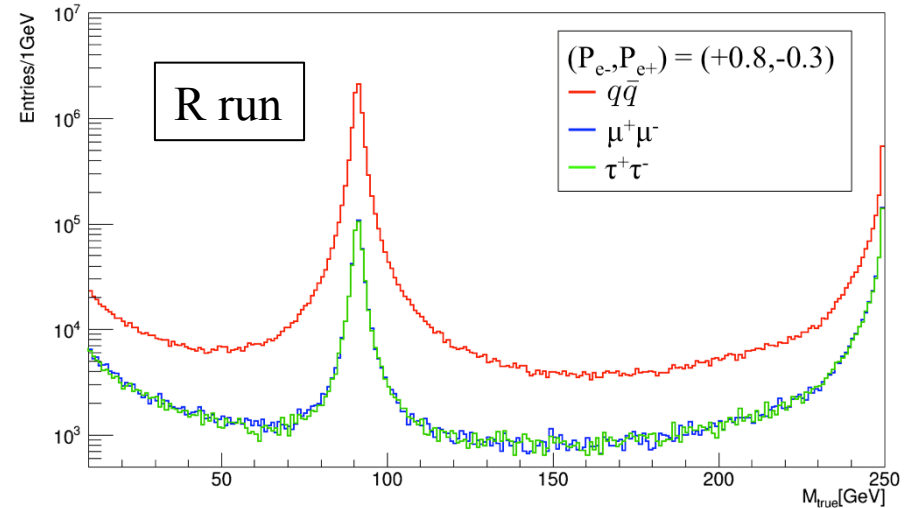
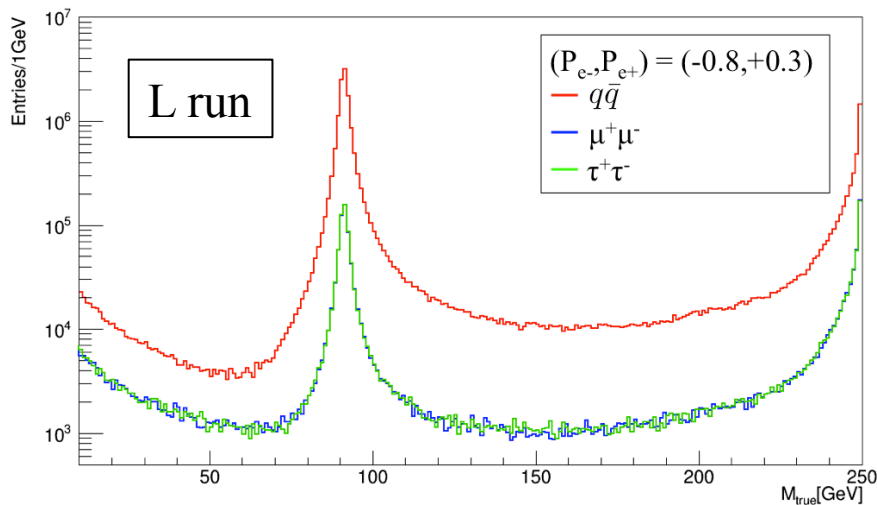
- based on the TDR

Signal Event

L run : sample with $(P_{e^-}, P_{e^+}) = (-0.8, +0.3)$

R run : sample with $(P_{e^-}, P_{e^+}) = (+0.8, -0.3)$

$M_{f\bar{f}}(true)$ distribution in the $e^+e^- \rightarrow f\bar{f}$ process



The A_{LR} is irrespective of the final state of $e^+e^- \rightarrow f\bar{f}$

Signal Process for this analysis

$e^+e^- \rightarrow q\bar{q}$ via Z boson

Background

The list of background events

$$e^+ e^- \rightarrow ZZ \rightarrow 4 \text{ fermions}$$

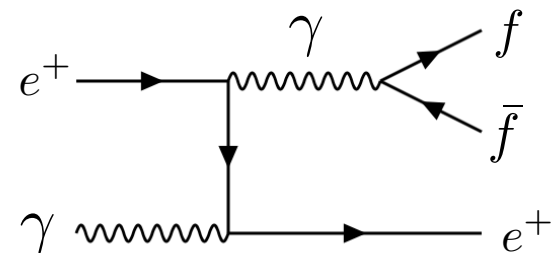
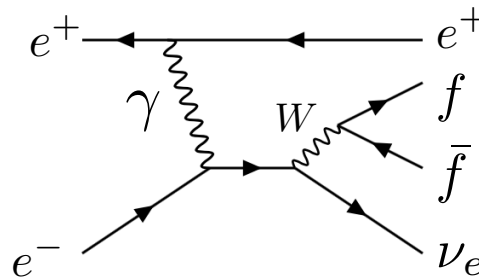
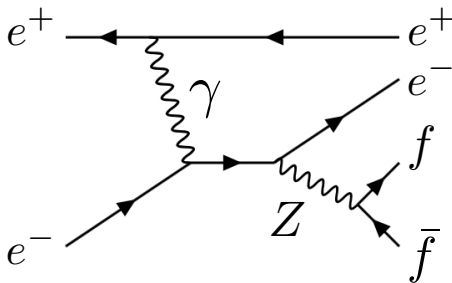
$$e^+ e^- \rightarrow W^+ W^- \rightarrow 4 \text{ fermions}$$

$$e^+ e^- \rightarrow \text{single } Z \rightarrow 4 \text{ fermions}$$

$$e^+ e^- \rightarrow \text{single } W \rightarrow 4 \text{ fermions}$$

$$e\gamma, \gamma\gamma \rightarrow X$$

Examples of the Feynman diagrams



Jet Clustering

Jet clustering is based on **the anti- k_T algorithm**

$$d_{ij} \equiv \min(k_{Ti}^{-2}, k_{Tj}^{-2}) \frac{\Delta_{ij}^2}{R^2}$$
$$d_{ii} \equiv k_{Ti}^{-2}$$
$$d_{min} \equiv \min(d_{ij}, d_{ii})$$

i, j : cluster

k_T : transverse momentum

Δ : distance between clusters

R : a parameter to be adjusted

if $d_{min} = d_{ij}$, combine i and j with each weight corresponding to their energy.

if $d_{min} = d_{ii}$, regard i as a jet and remove it from the list of the clusters.

This procedure is repeated until no clusters are left.

In this study, only 2jet events are selected.

Background Rejection Cut

Event Selection

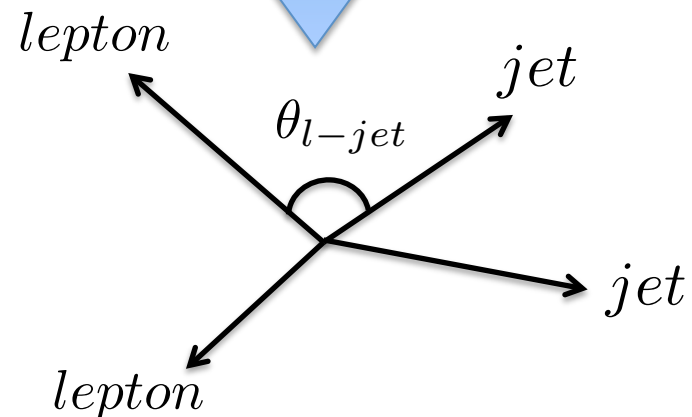
$$5 \leq N_{\text{chargedtrack}} \leq 25$$

– to reduce $e\gamma, \gamma\gamma \rightarrow X$ events

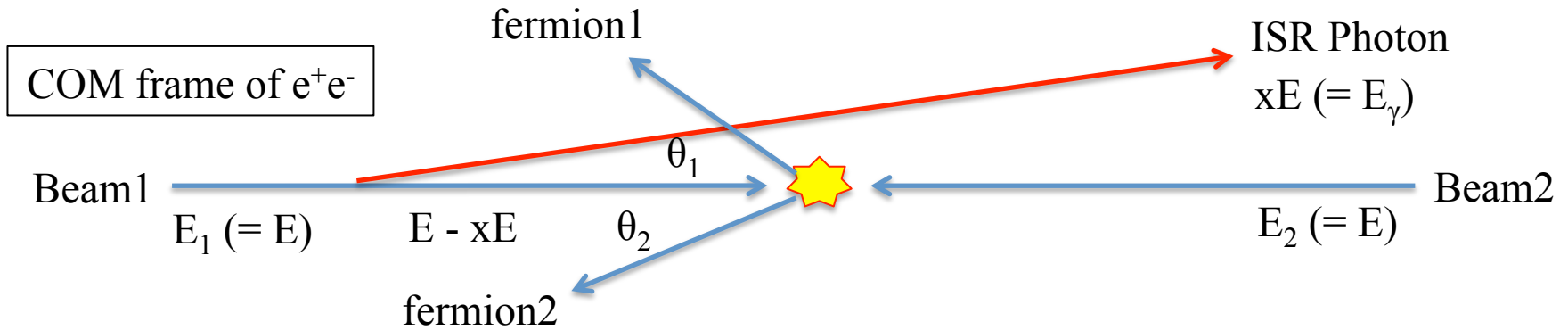
$$\rho = \sqrt{2E_l(1 - \cos\theta_{l-jet})} < 1.6 \quad (\text{for all combination})$$

– to reduce $e^+e^- \rightarrow 4$ fermions events

$$50 \text{ GeV} < M_{2jet}(\text{reco}) < 107 \text{ GeV}$$



Reconstruction of $x \equiv E_\gamma/E_{\text{beam}}$



Assumption

ISR photon travels collinearly with the beam pipe / Only one ISR photon is emitted

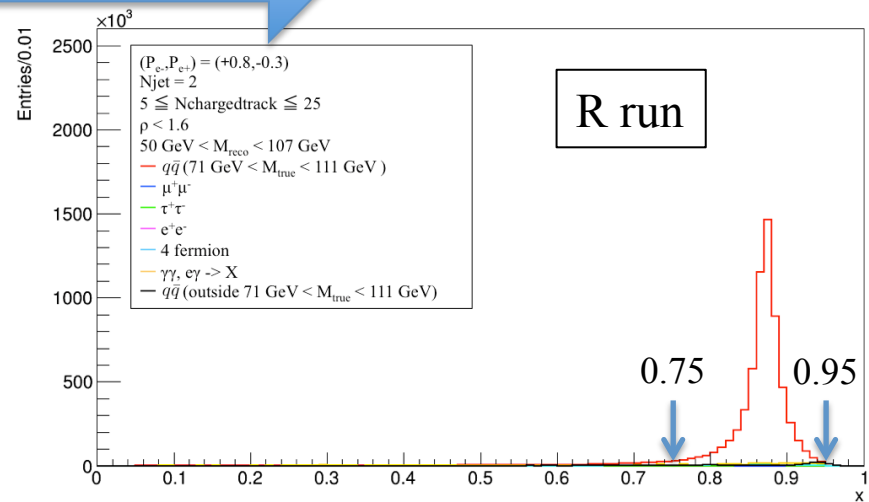
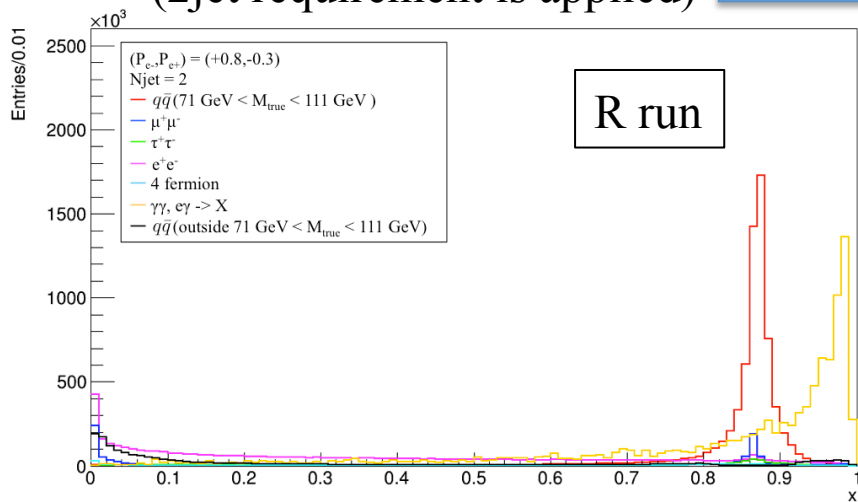
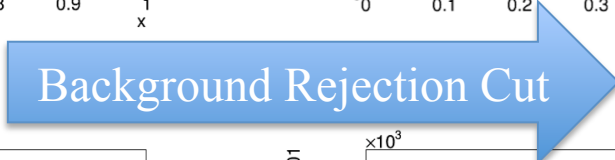
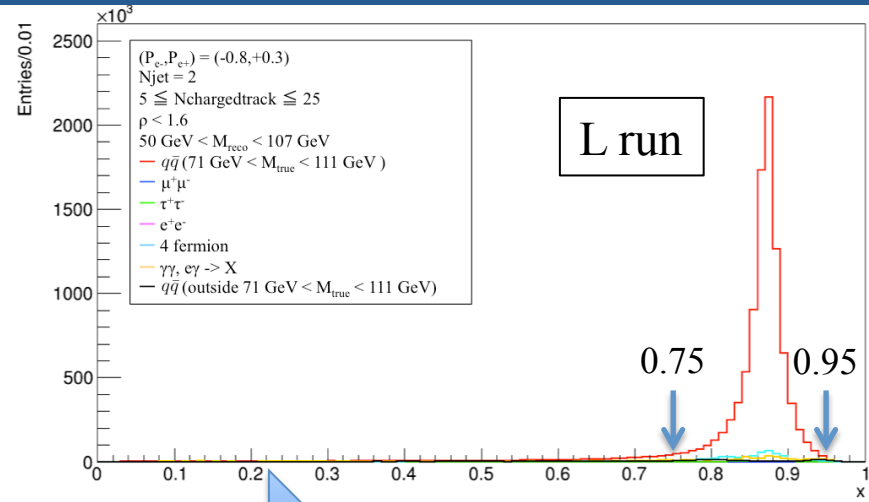
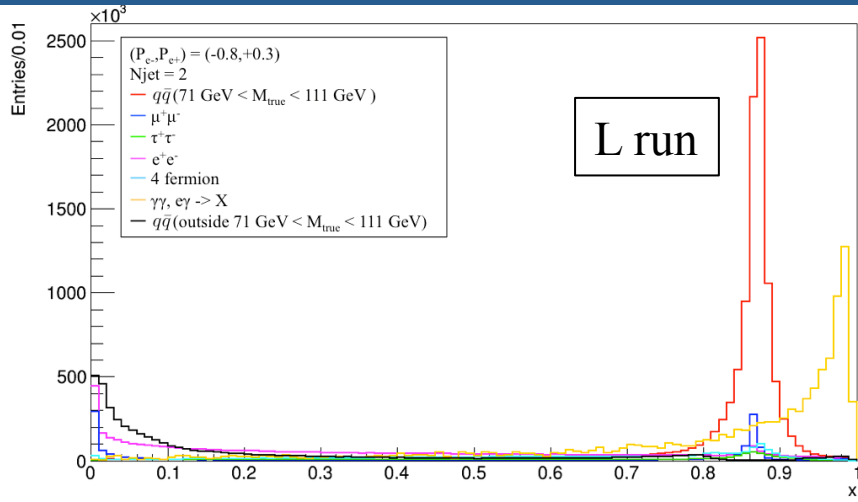
$$x = \frac{2|\beta|}{1 + |\beta|} \dots (1) \quad \frac{|\sin(\theta_1 + \theta_2)|}{\sin\theta_1 + \sin\theta_2} = \frac{\eta}{\gamma} = |\beta| \dots (2)$$

β : the velocity of the recoil system

The x can be reconstructed from θ_1 and θ_2
 → No need to detect ISR photon

$$\sqrt{s} = 250 \text{ GeV}, \quad \sqrt{s'} = M_Z \iff x = 0.8670$$

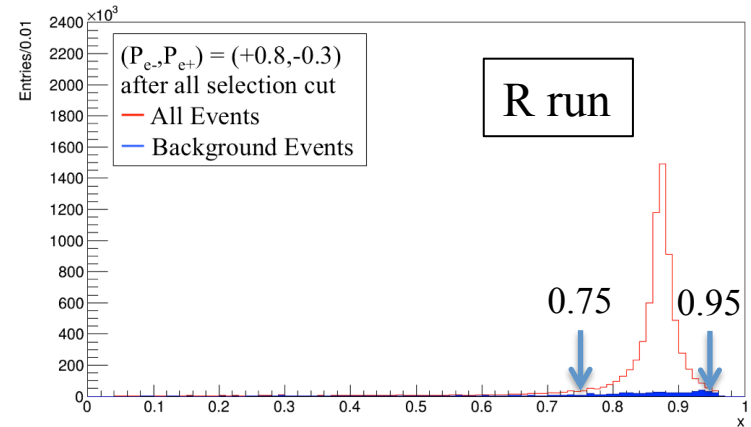
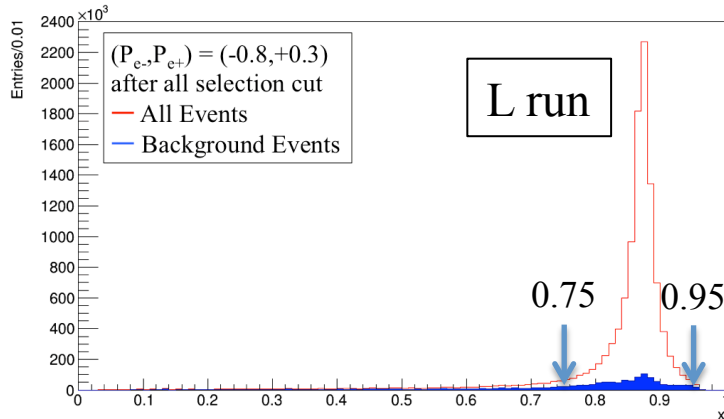
x value distribution



Reconstruct only the events satisfying $0.75 < x < 0.95$

Derivation of A_{LR}

The x distribution



Assume the absolute value of background is estimated correctly by the MC data.

	N_L (250 fb ⁻¹)	N_R (250 fb ⁻¹)
All Events (measured)	10434326	6794226
Background Events (MC)	1051278	469413

$$N_L^{signal} = N_L^{total}(meas) - N_L^{bkg}(MC) = 9383048$$

$$N_R^{signal} = N_R^{total}(meas) - N_R^{bkg}(MC) = 6324813$$

$$A_{LR} = \frac{N_L^{signal} - N_R^{signal}}{N_L^{signal} + N_R^{signal}} \frac{1 + \langle P_{e-} \rangle \langle P_{e+} \rangle}{\langle P_{e-} \rangle + \langle P_{e+} \rangle} = 0.21947$$

Statistical Error of A_{LR}

$$A_{LR} = \frac{\sigma_L^{meas} - \sigma_R^{meas}}{\sigma_L^{meas} + \sigma_R^{meas}} \frac{1 + \langle P_{e-} \rangle \langle P_{e+} \rangle}{\langle P_{e-} \rangle + \langle P_{e+} \rangle}$$

$$= \frac{N_L - N_R \cdot r_L}{N_L + N_R \cdot r_L} \frac{1 + \langle P_{e-} \rangle \langle P_{e+} \rangle}{\langle P_{e-} \rangle + \langle P_{e+} \rangle}$$

$$r_L = \frac{\text{L run integrated luminosity}}{\text{R run integrated luminosity}}$$

Suppose the statistical error of the magnitude of polarization is small enough to be negligible,

$$\Delta A_{LR} \approx \sqrt{\left(\frac{\partial A_{LR}}{\partial N_L}\right)^2 (\Delta N_L)^2 + \left(\frac{\partial A_{LR}}{\partial N_R}\right)^2 (\Delta N_R)^2} = \frac{2N_L N_R \cdot r_L}{(N_L + N_R)^2} \left(\frac{1}{\sqrt{N_L}} + \frac{1}{\sqrt{N_R}}\right) \frac{1 + \langle P_{e-} \rangle \langle P_{e+} \rangle}{\langle P_{e-} \rangle + \langle P_{e+} \rangle}$$

The statistical error of the signal events can be regarded as the square root of that of all events

$$A_{LR} = 0.21947 \pm 0.00038 \quad (500 \text{ fb}^{-1})$$

$$\Rightarrow \sin^2 \theta_W = 0.22223 \pm 0.00005 \quad (500 \text{ fb}^{-1})$$

Fit

In order to remove MC modeling uncertainties for the number of background events, fitting is applied for the total x distribution.

Signal : **three Gaussian functions**

Background : **Gaussian function and third-order polynomial function**

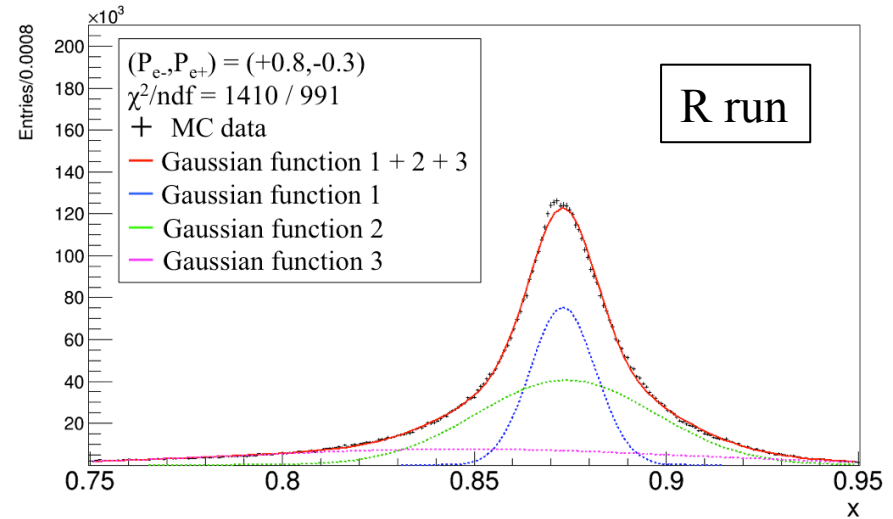
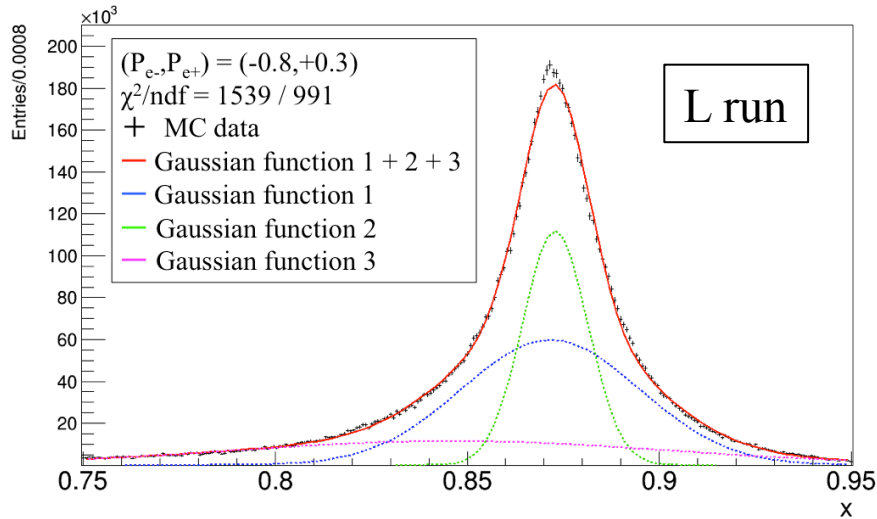


Fitting the x distribution for the total events

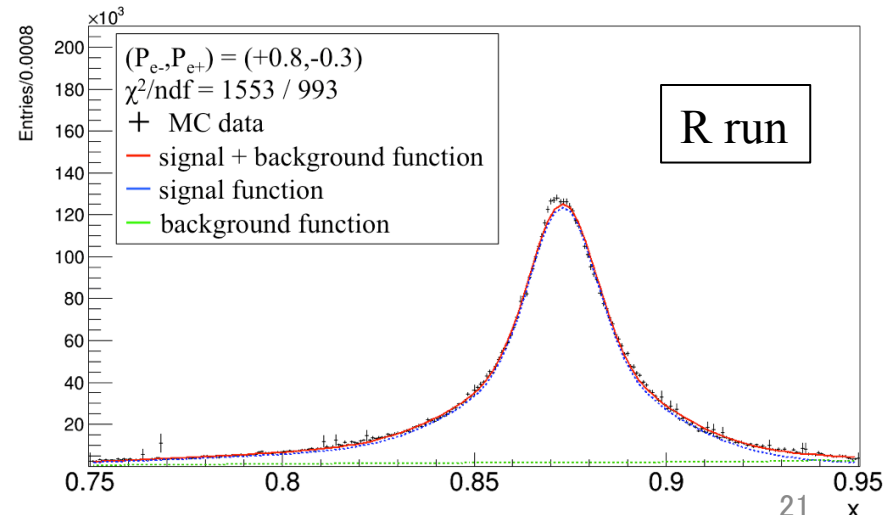
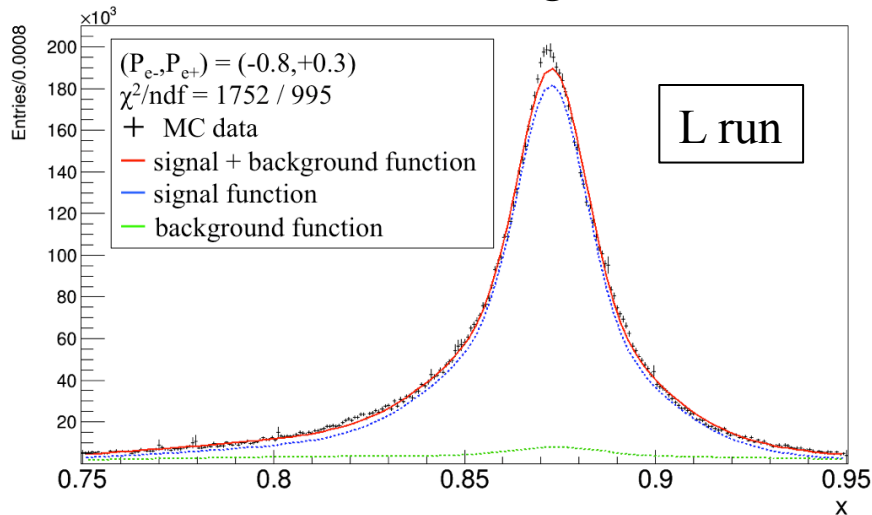
- The shape of the signal function are fixed
- All parameters for the background function is floated

Result for the fit

Fitting for the x distribution of the signal events



Fitting for the x distribution of the total events



Result for the fit

	N_L (250 fb ⁻¹)	N_R (250 fb ⁻¹)	A_{LR}
All Events	10216660	6655036	0.23797
Signal Events	9314017	6277252	0.21956
Background Events	902643	377784	0.46208

In the same way as before, the statistical error is obtained by

$$A_{LR} = 0.21956 \pm 0.00040 \text{ (500 fb}^{-1}\text{)}$$

With the full-data at 250 GeV of 2000 fb⁻¹, the statistical error can be reduced by a factor of two

$$A_{LR} = 0.21956 \pm 0.00020 \text{ (2000 fb}^{-1}\text{)}$$

Summary

- Evaluated of the statistical error of the A_{LR} at the ILC with the center-of-mass energy of 250 GeV
- Reconstructed the x value for only the events satisfying $0.75 < x < 0.95$
- Estimated the statistical errors from the fit for the x distribution
- At the ILC, the relative statistical error of A_{LR} can be reduced to $\sim 0.1\%$ (previous value $\sim 1.5\%$) with the full-running at 250 GeV

Backup


A_{LR}

$$\sigma^{meas} = \frac{N_L^{e^-} N_R^{e^+} \sigma_L + N_R^{e^-} N_L^{e^+} \sigma_R}{(N_R^{e^-} + N_L^{e^-})(N_R^{e^+} + N_L^{e^+})}$$

$$= \frac{1}{4}(1 - P_{e^-})(1 + P_{e^+})\sigma_L + \frac{1}{4}(1 + P_{e^-})(1 - P_{e^+})\sigma_R$$

$$\sigma_L^{meas} = \frac{1}{4}(1 + \langle P_{e^-} \rangle)(1 + \langle P_{e^+} \rangle)\sigma_L + \frac{1}{4}(1 - \langle P_{e^-} \rangle)(1 - \langle P_{e^+} \rangle)\sigma_R$$

$$\sigma_R^{meas} = \frac{1}{4}(1 - \langle P_{e^-} \rangle)(1 - \langle P_{e^+} \rangle)\sigma_L + \frac{1}{4}(1 + \langle P_{e^-} \rangle)(1 + \langle P_{e^+} \rangle)\sigma_R$$


$$\sigma_L^{meas} - \sigma_R^{meas} = \frac{1}{2}(\langle P_{e^-} \rangle + \langle P_{e^+} \rangle)(\sigma_L - \sigma_R)$$

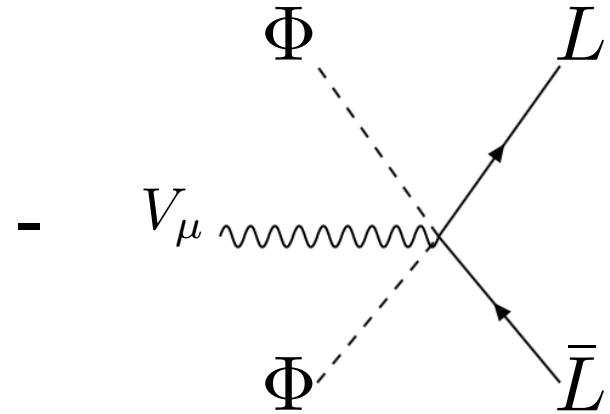
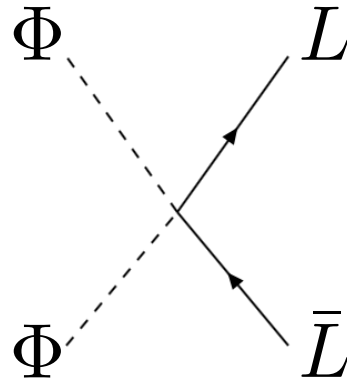
$$\sigma_L^{meas} + \sigma_R^{meas} = \frac{1}{2}(1 + \langle P_{e^-} \rangle \langle P_{e^+} \rangle)(\sigma_L + \sigma_R)$$

$$A_{LR} = \frac{\sigma_L^{meas} - \sigma_R^{meas}}{\sigma_L^{meas} + \sigma_R^{meas}} \frac{1 + \langle P_{e^-} \rangle \langle P_{e^+} \rangle}{\langle P_{e^-} \rangle + \langle P_{e^+} \rangle}$$

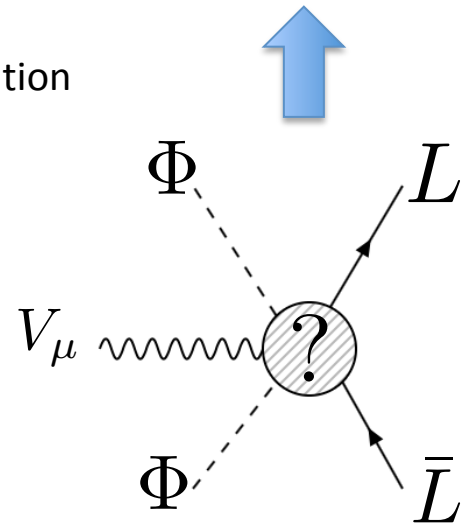
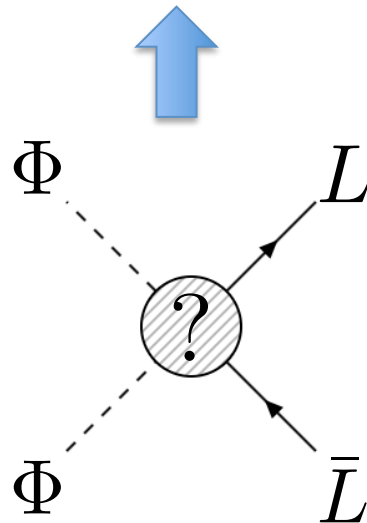
Effective Field Theory for Higgs precision measurement

$$(\Phi^\dagger D^\mu \Phi)(\bar{L}\gamma_\mu L)$$

$$D^\mu = \partial^\mu - V^\mu$$

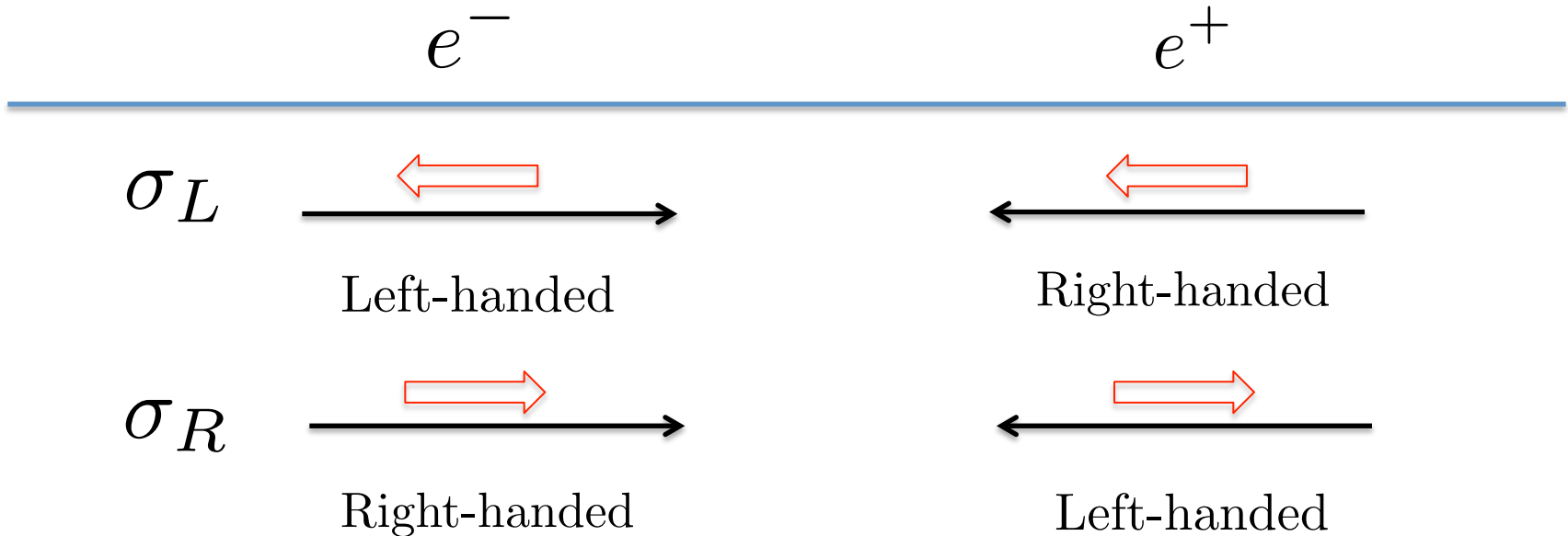


approximation

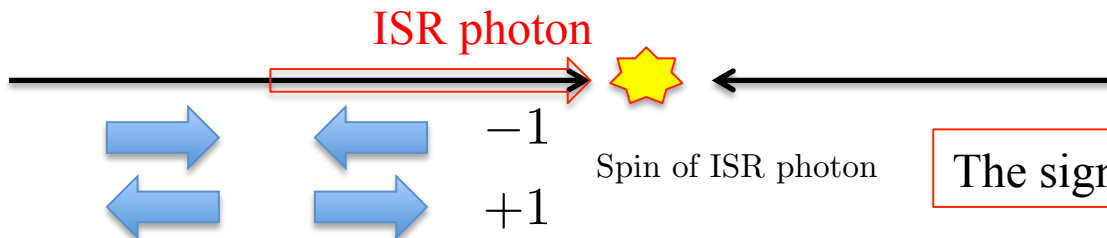


Helicity

Helicity is the projection of the spin vector on the direction of motion

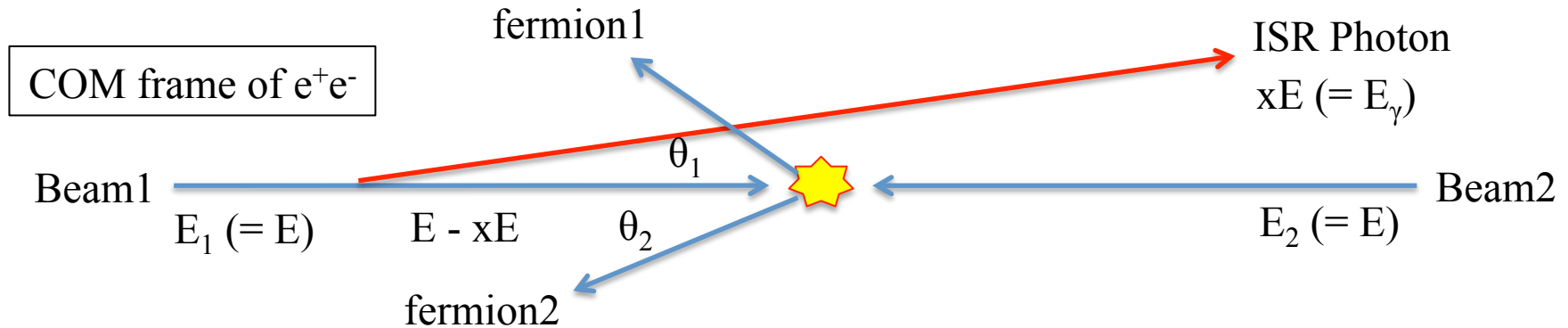


if $E_{kin} \gg E_0 \rightarrow m_e \approx 0$



The sign of the beam helicity is flipped

Reconstruction of $x \equiv E_\gamma/E_{\text{beam}}$



COM Energy with no ISR : $\sqrt{s} = \sqrt{4E_1E_2} = \sqrt{4E^2}$

COM Energy with ISR : $\sqrt{s'} = \sqrt{4E_1E_2} = \sqrt{4E^2(1-x)} = \sqrt{s(1-x)}$

$\sqrt{s} = 250 \text{ GeV}, \sqrt{s'} = M_Z \rightarrow x = 0.8670$

Reconstruction of $x \equiv E_\gamma/E_{\text{beam}}$

$$|\beta| = \frac{P_{tot}}{E_{tot}} = \frac{E_\gamma}{E_1 + E_2} = \frac{x E}{E + E(1 - x)} = \frac{x}{2 - x}$$

β : the velocity of the recoil system

$$x = \frac{2|\beta|}{1 + |\beta|} \dots (1)$$

COM frame

Particle1

$$\begin{aligned} E_1 &= E \\ P_{1x} &= P \cos\theta \\ P_{1y} &= P \sin\theta \end{aligned}$$

Particle2

$$\begin{aligned} E_2 &= E \\ P_{2x} &= -P \cos\theta \\ P_{2y} &= -P \sin\theta \end{aligned}$$

Lorentz
transformation



Rest frame of the ISR

Particle1

$$\begin{aligned} E_1' &= \gamma E + \eta P \cos\theta \\ P_{1x}' &= \eta E + \gamma P \cos\theta \\ P_{1y}' &= P \sin\theta \end{aligned}$$

Particle2

$$\begin{aligned} E_2' &= \gamma E - \eta P \cos\theta \\ P_{2x}' &= \eta E - \gamma P \cos\theta \\ P_{2y}' &= -P \sin\theta \end{aligned}$$

$$\sin\theta_1 = \frac{|P'_{1y}|}{E'_1} \quad \sin\theta_2 = \frac{|P'_{2y}|}{E'_2}$$

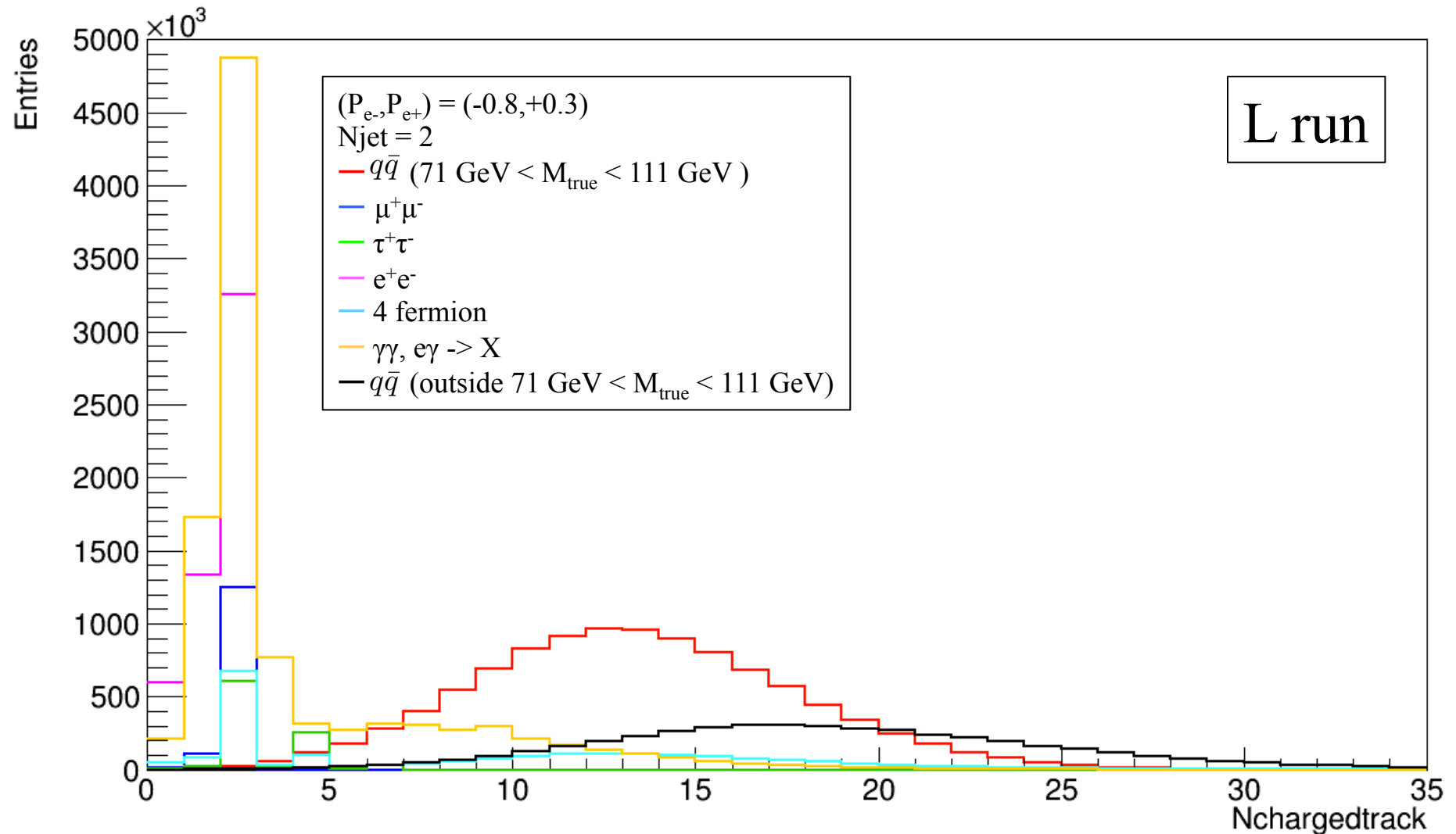
$$\cos\theta_1 = \frac{P'_{1x}}{E'_1} \quad \cos\theta_2 = \frac{P'_{2x}}{E'_2}$$



$$\frac{|\sin(\theta_1 + \theta_2)|}{\sin\theta_1 + \sin\theta_2} = \frac{\eta}{\gamma} = |\beta| \dots (2)$$

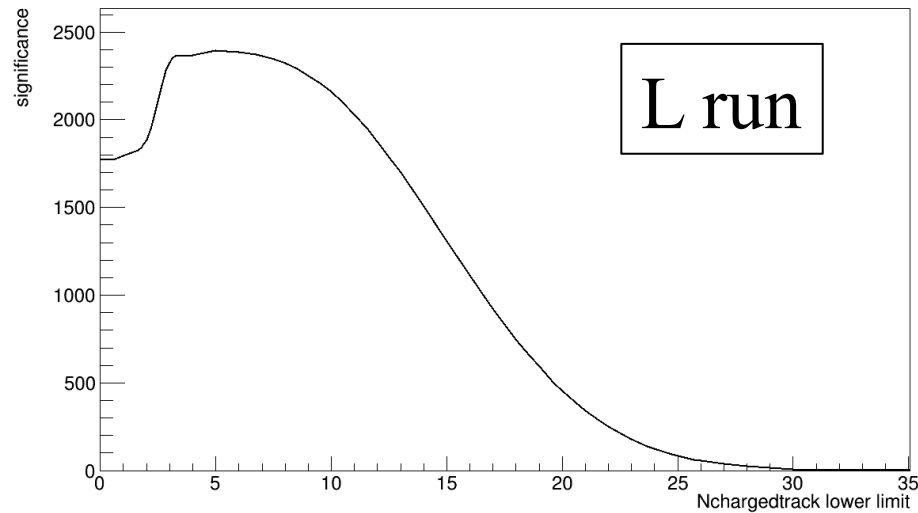
Nchargedtrack

Nchargedtrack distribution

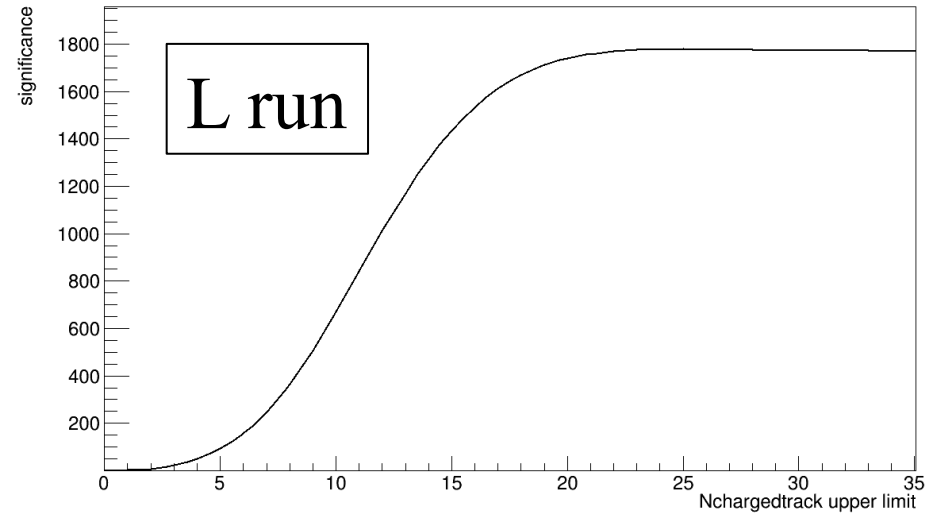


Nchargedtrack

The dependence of the significance on Nchargedtrack



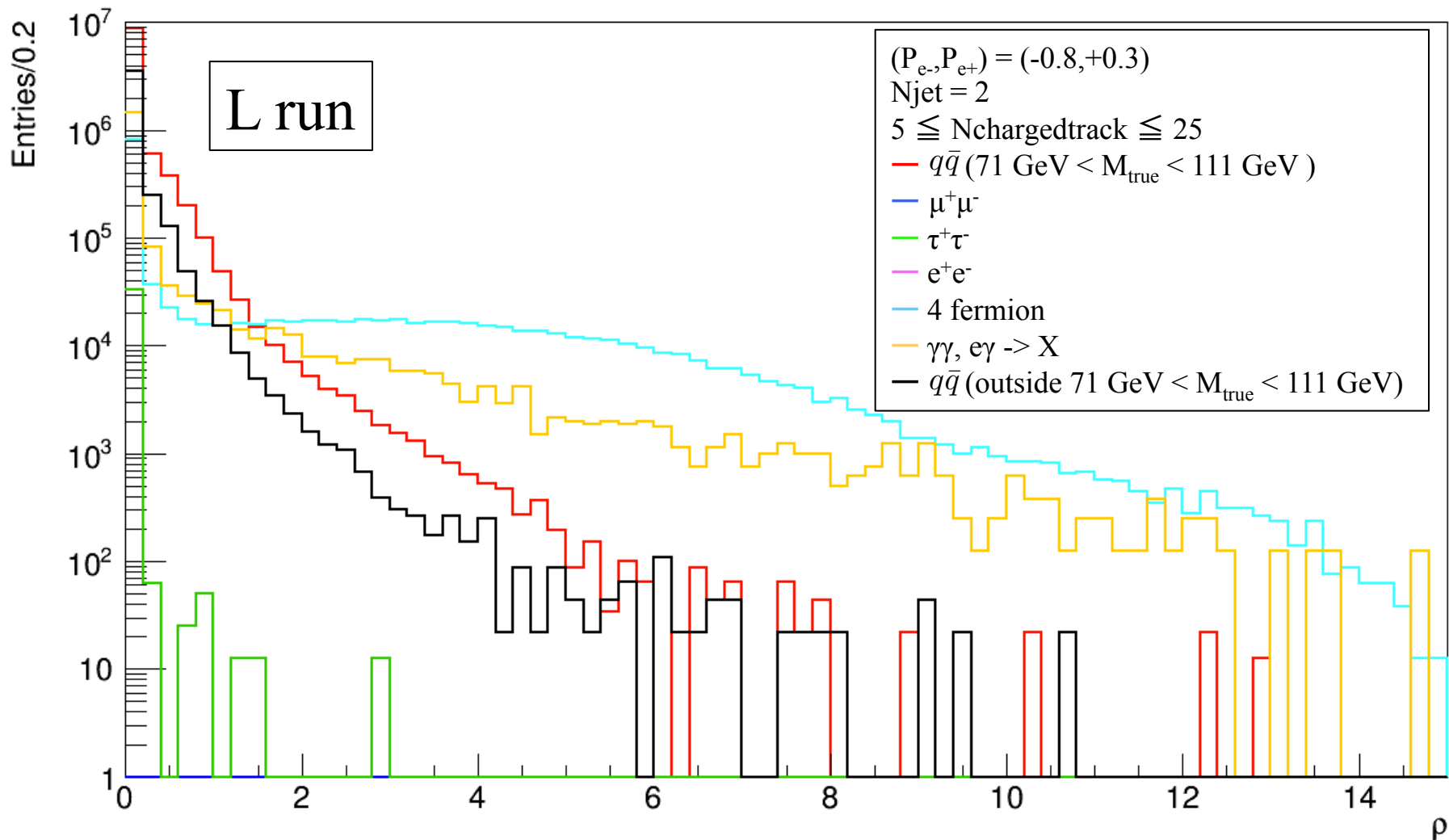
Lower Limit



Upper Limit

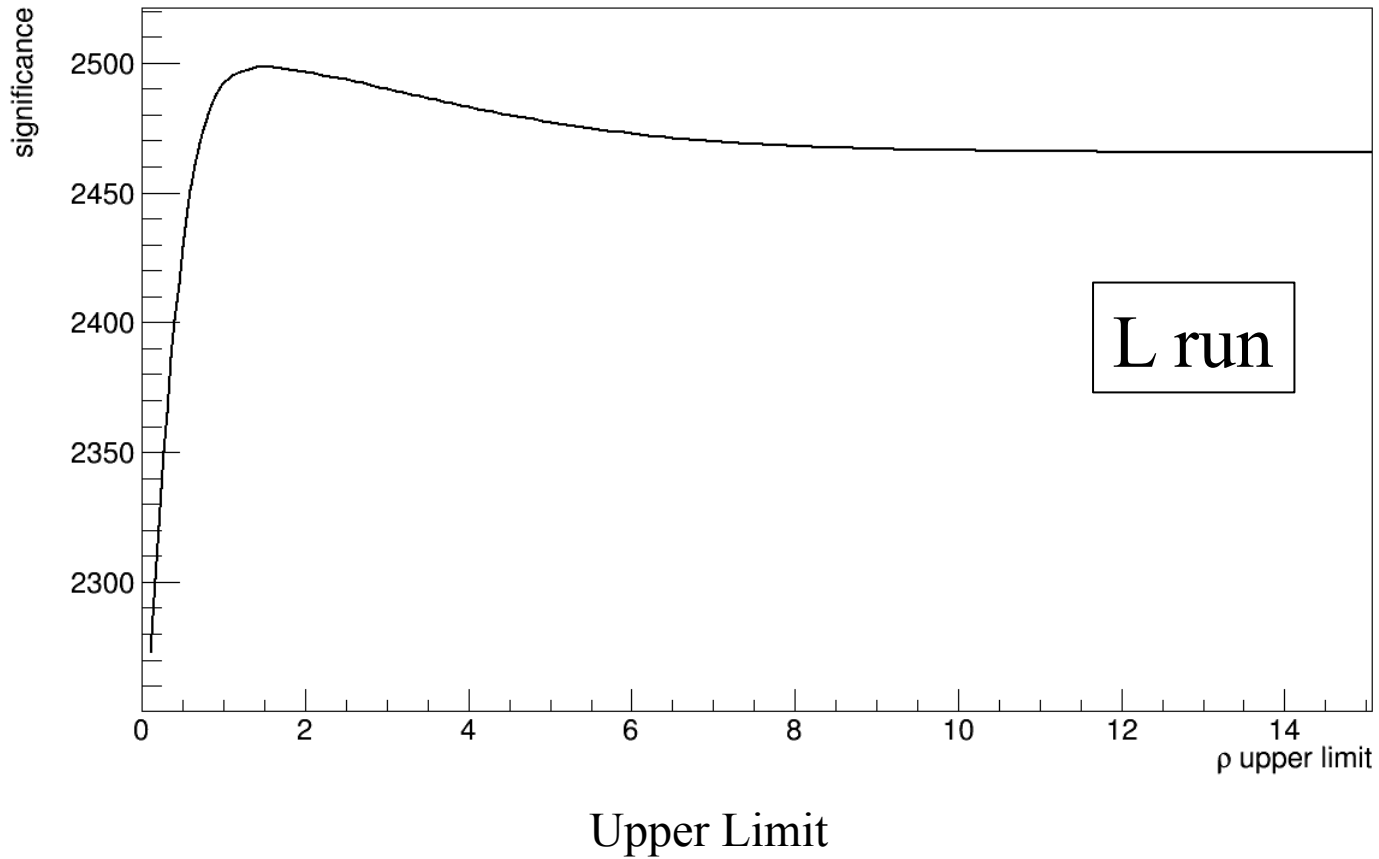
ρ

ρ distribution



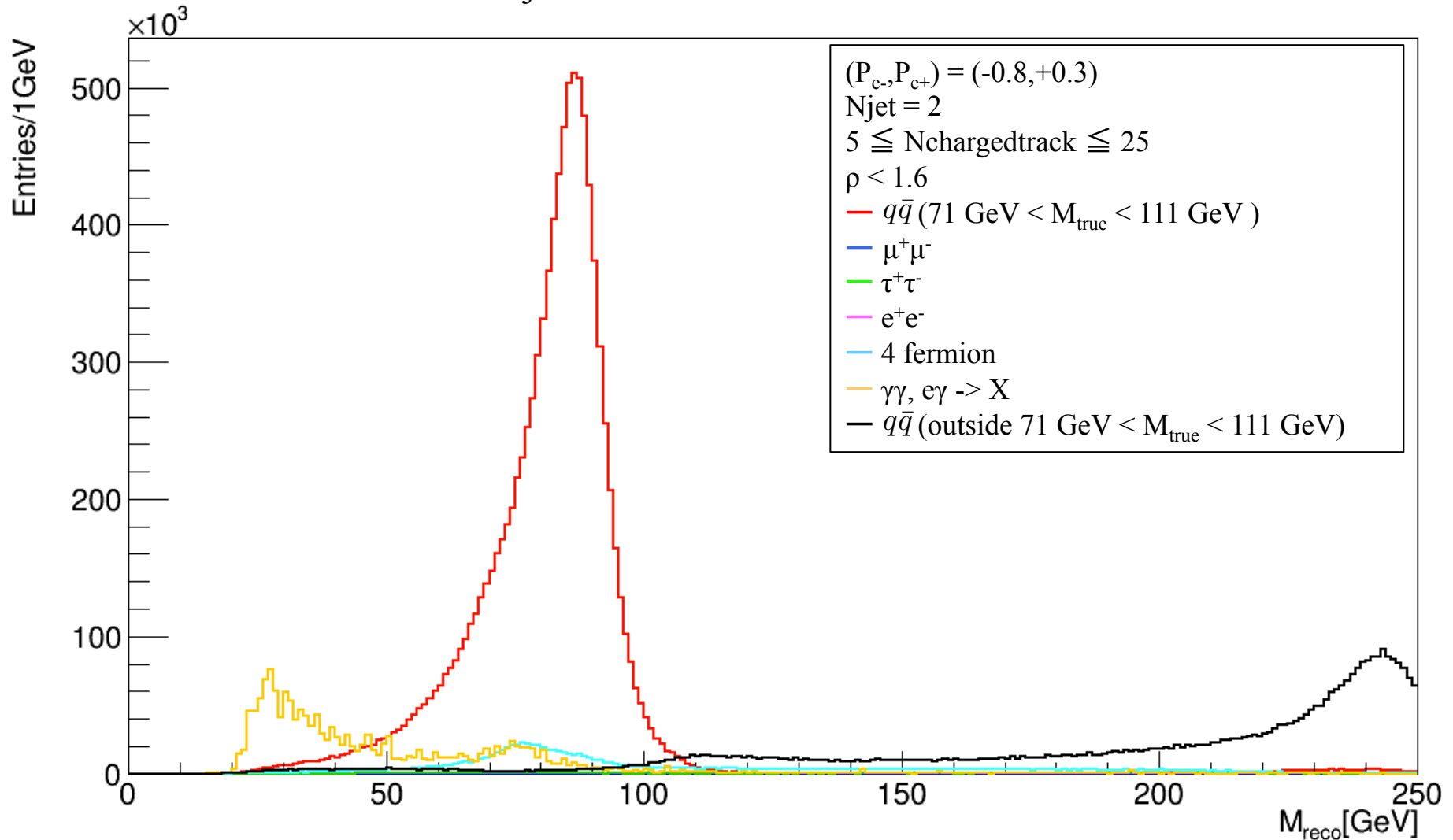
ρ

The dependence of the significance on ρ



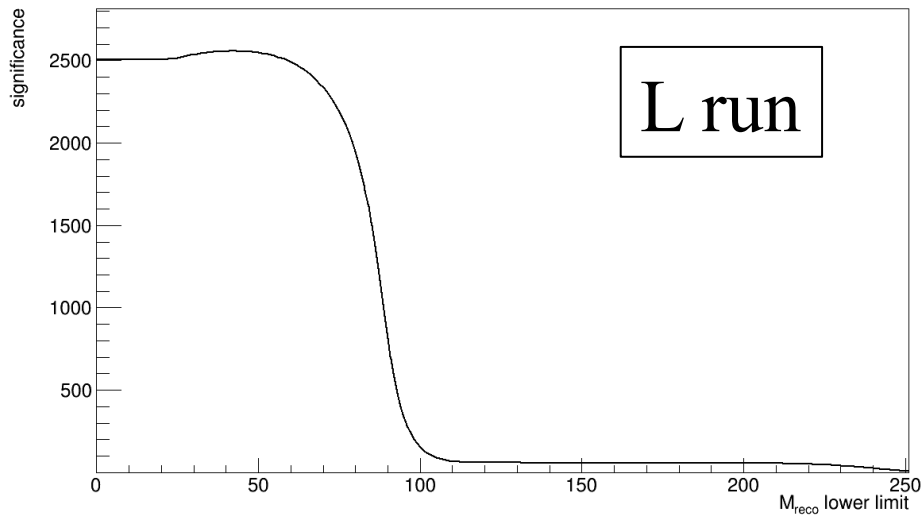
$M_{2\text{jet}}(\text{reco})$

$M_{2\text{jet}}(\text{reco})$ distribution

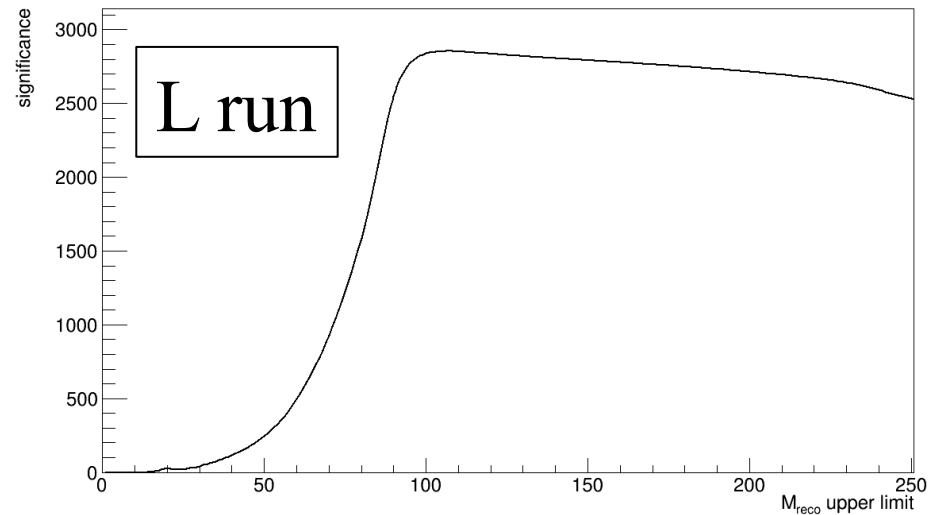


$M_{2\text{jet}}(\text{reco})$

The dependence of the significance on $M_{2\text{jet}}(\text{reco})$



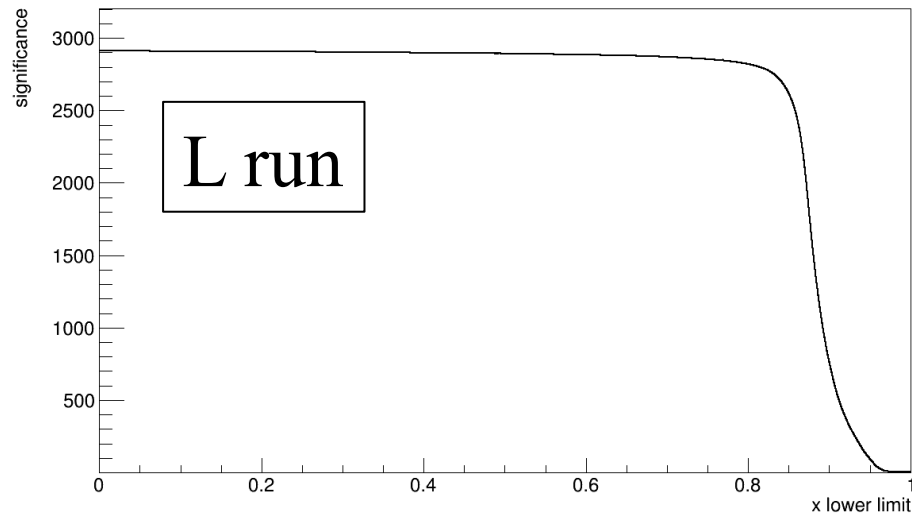
Lower Limit



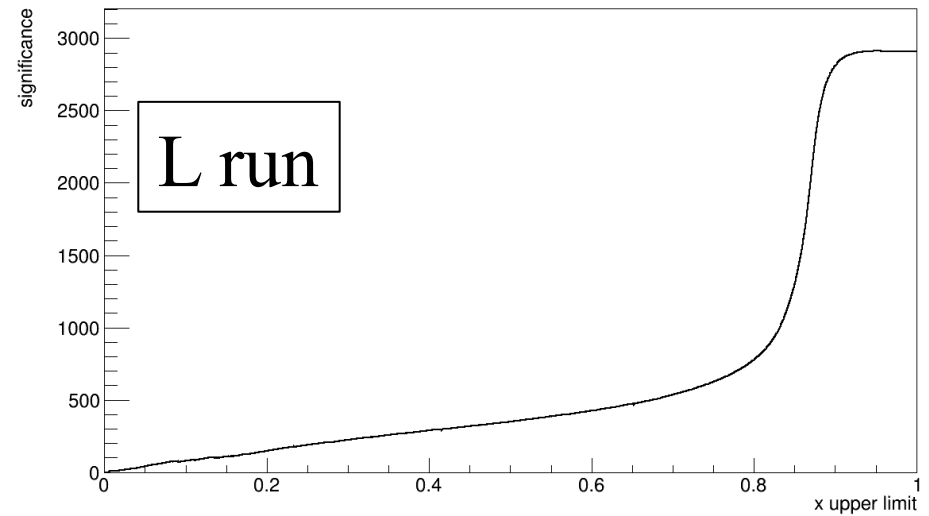
Upper Limit

X

The dependence of the significance on x



Lower Limit



Upper Limit