ILC実験におけるトップ対生成過程を用いたトップクォークとゲージ粒子Z/γの異常結合探索手法の開発

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佐藤瑶

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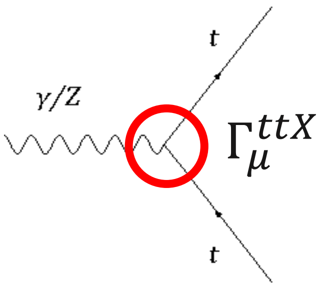
ILCにおけるttZ/γ結合の研究

◻ ttZ/γ結合は新物理探索のための重要なプローブ

(例)複合模型ではttZ/γ結合の形状因子F (結合定数g)が標準模型の値から10%程度ずれる



$$\Gamma_\mu^{ttX}(k^2, q, \bar{q}) = ie\left[\gamma_\mu\left(F_{1V}^X(k^2) + \gamma_5 F_{1A}^X(k^2)\right) + \frac{\sigma_{\mu\nu}}{2m_t}(q+\bar{q})^\nu\left(iF_{2V}^X(k^2) + \gamma_5 F_{2A}^X(k^2)\right)\right]$$

$$(X = Z, \gamma)$$

$$(g_L = F_{1V} - F_{1A}, g_R = F_{1V} + F_{1A})$$

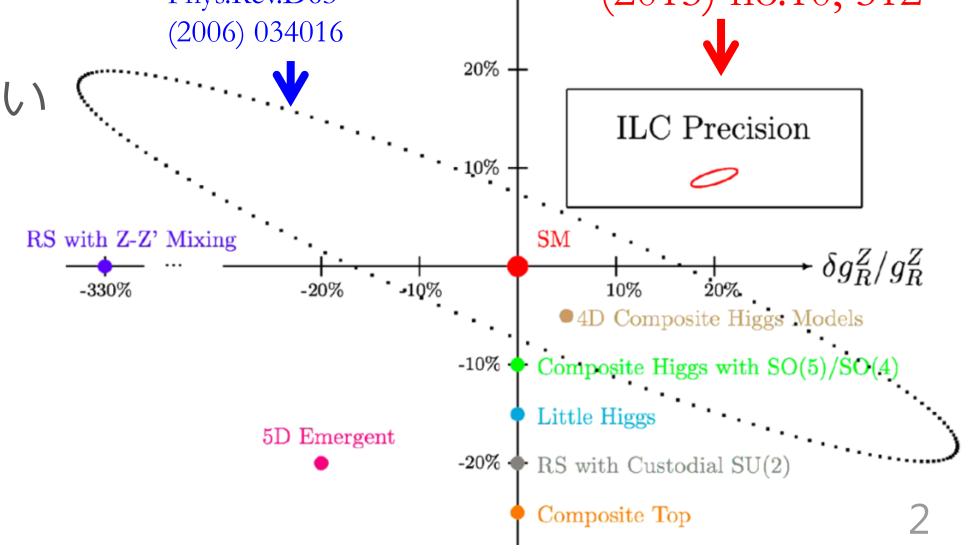
◻ ILCで期待される測定精度は

- LHCにおける精度より1-2桁高い
- 新物理模型の同定が可能

と期待される

LHC14, 3000 fb-1
Phys.Rev.D63
(2006) 034016

ILC, 500 fb-1
Eur.Phys.J. C75
(2015) no.10, 512



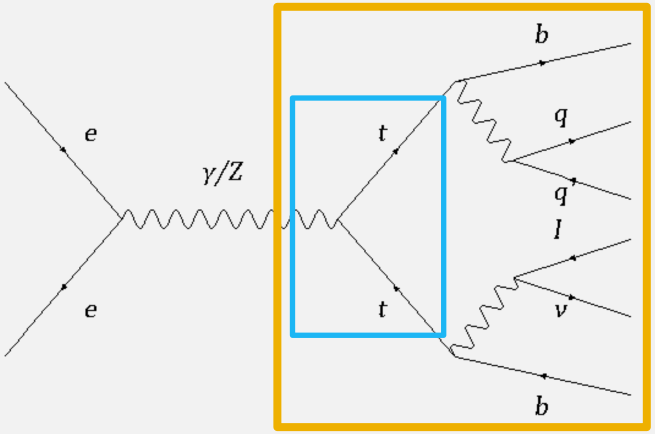
全角度情報による解析

先行研究 (Eur.Phys.J. C75 (2015) no.10, 512)

- 信号事象：Semi-leptonic過程 ($\sqrt{s}$ = 500 GeV)

$$e^-e^+ \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^- \rightarrow bq\bar{q}\bar{b}l^-\bar{\nu}$$

- 観測量：$\boldsymbol{\sigma, A_{FB}}$ x 2パターンのビーム偏極
- パラメータ：$(F_{1V}^{\gamma}, F_{1V}^{Z}, F_{1A}^{Z}), (F_{2V}^{\gamma}, F_{2V}^{Z})$ を測定



トップクォーク対の生成過程に関連した観測量を使用

トップクォークの崩壊過程もttZ/γ結合についての情報を持つ

- トップクォークはハドロン化する前に崩壊
- 崩壊粒子の角度分布がトップクォークのスピンに依存する

全角度情報を用いることでより高い測定精度が期待できる

本研究の目的

目的

ILD検出器のフルシミュレーション解析による
全角度情報を用いたttZ/γ異常結合探索手法の開発

シミュレーションのセットアップ

重心系エネルギー	$\sqrt{s}$	500 GeV
ビーム偏極	(P_{e^-}, P_{e^+})	(-0.8, +0.3) / (+0.8, -0.3) Left / Right
積分ルミノシティ	L	250 fb^{-1} / 250 fb^{-1}
イベント生成	WHIZARD, Pythia (SM-LOに準拠)	
検出器シミュレーション	Mokka, Marlin (TDR, DBDに準拠)	

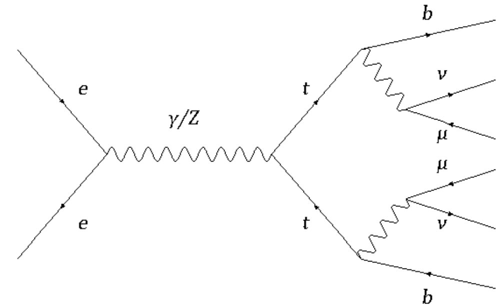
信号事象と背景事象

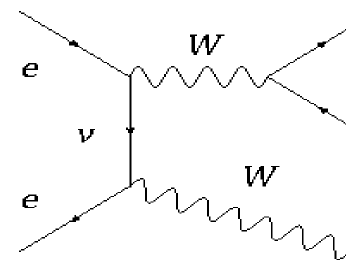
信号事象 : $e^-e^+ \to b\bar{b}\mu^-\mu^+\nu\bar{\nu}$

- **W粒子がどちらも $\mu\nu_\mu$**に崩壊する過程に注目

  $e^-e^+ \to t\bar{t} \to bW^+\bar{b}W^- \to b\mu^+\nu_\mu\bar{b}\mu^-\overline{\nu_\mu}$

  - Di-leptonic 過程は多くの観測量を持つ
  - その中でも再構成の精度が高い
- Single top production, ZWWなどを含む

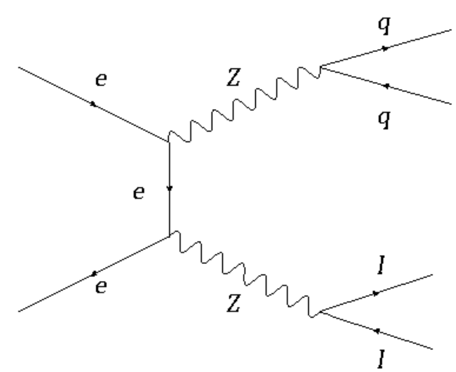




孤立ミューオンx2, bジェットx2 を要求

主な背景事象

- $e^-e^+ \to q\bar{q}l^-l^+$ (主に $e^-e^+ \to ZZ \to q\bar{q}l^-l^+$)
- $e^-e^+ \to b\bar{b}l^-l^+\nu\bar{\nu}$ ($b\bar{b}\mu^-\mu^+\nu\bar{\nu}$以外)



力学的再構成

力学的再構成

- $\vec{P}_\nu, \vec{P}_{\bar{\nu}}$ を以下の力学的制限を課すことで求める
  - 始状態の制限 : $(E_{\text{total}}, \vec{P}_{\text{total}}) = (500, \vec{0})$ [GeV]
  - 質量制限 : $m_{t,\bar{t}} = 174$ GeV, $m_{W^\pm} = 80.4$ GeV
- ビームからのγによって始状態の制限が崩れる
  γ の運動量(z軸成分のみ)を未知数に加える

未知数

$\vec{P}_\nu, \vec{P}_{\bar{\nu}}, P_{\gamma,z}$: 7

力学的制限

$E_{\text{total}}, \vec{P}_{\text{total}},$

$m_t, m_{\bar{t}}, m_{W^+}, m_{W^-}$: 8

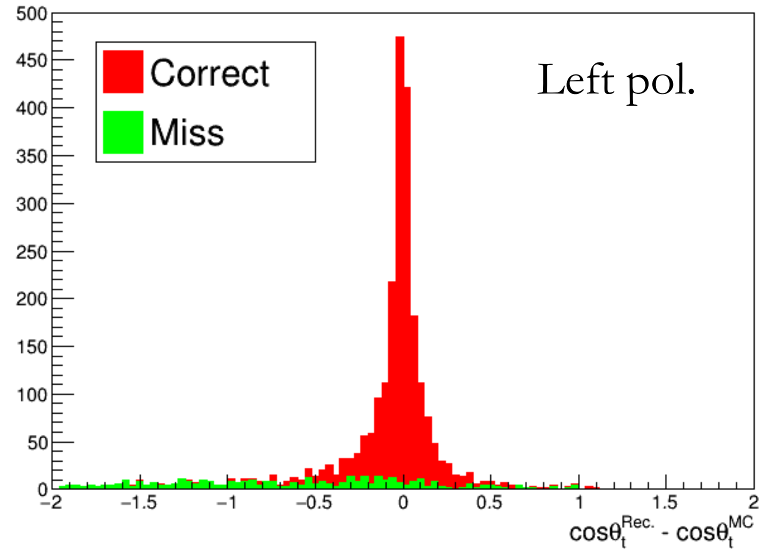
これらに対応する最尤関数 L を最大化することで再構成

再構成結果

(例) $\cos\theta_t$の再構成結果とMC Truth の差, $\Delta\cos\theta_t$

$\Delta\cos\theta_t = 0$にピーク

→ 角度情報が再構成出来ていることが確認できる



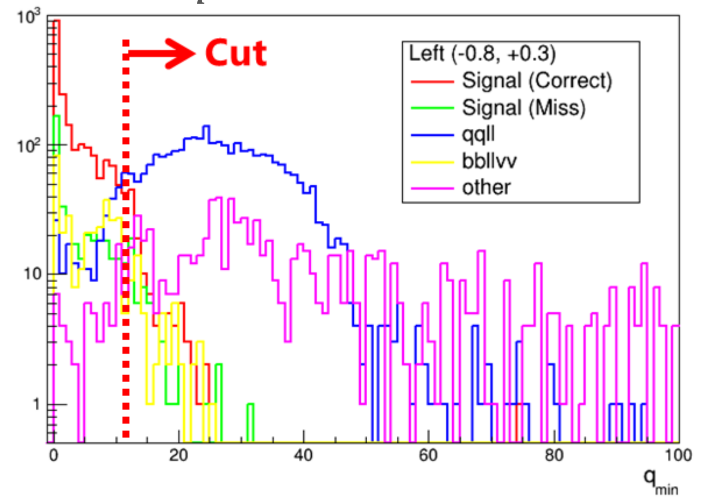
事象選別

事象選別

- L または $q = -2\log L$ (力学的再構成の精度)が背景事象の抑制に有効
- 信号有意度(S)が最大になるよう条件を決定

$$S = \frac{N_{signal}}{\sqrt{N_{signal} + N_{background}}}$$

q のヒストグラム



Left Polarization カット条件	信号事象 $bb\mu\mu\nu\nu$	全背景事象	$qqll$	$bbll\nu\nu$	S
No cut	2837	8410633	91478	23312	0.978
$N_{\mu^-} = 1$ & $N_{\mu^+} = 1$	2618	327488	13827	387	4.56
b-tag cut	2489	4143	2943	363	30.6
Quality cut ($q_{min} < 11.5$)	**2396**	**624**	**258**	**313**	**43.6**

振幅 $|M|^2$の展開

Di-leptonic 過程の振幅 $|M|^2$は、9つの角度の関数として書かれる

$$|M|^2(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; F)$$

観測量は非常に豊富 → 9つの角度情報をどうやって同時に取り扱うか

振幅$|M|^2$を形状因子Fについて標準模型の値の周りで展開

$$|M|^2(\Phi; F) = \left(1 + \sum_i \omega_i(\Phi)\delta F_i + \sum_{ij} \tilde{\omega}_{ij}(\Phi)\delta F_i \delta F_j\right)|M^{SM}|^2\left(\Phi; F^{\mathrm{SM}}\right)$$

$$\omega_i = \frac{1}{|M|^2(\Phi)} \frac{\partial |M|^2(\Phi)}{\partial F_i}\bigg|_{\delta F=0}, \tilde{\omega}_{ij} = \frac{1}{|M|^2(\Phi)} \frac{\partial^2 |M|^2(\Phi)}{\partial F_i \partial F_j}\bigg|_{\delta F=0}, \delta F_i = F_i - F_i^{\mathrm{SM}}$$

Φは角度、 Fは形状因子のベクトル

$\omega, \tilde{\omega}$ は角度情報Φを形状因子Fの測定に最適化した表示

Binned likelihood fitによるFの測定

ω分布をBinned likelihoodでフィットする

- Full simulationによってω分布を作成しFの依存性を評価
- 以下のχ^2を用いてFull simulationの分布を実験データ(*)にフィットする

$$\chi^2(F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\mathrm{Data}} - n_i^{\mathrm{Sim.}}(F)}{\sqrt{n_i^{\mathrm{Data}}}} \right)^2$$

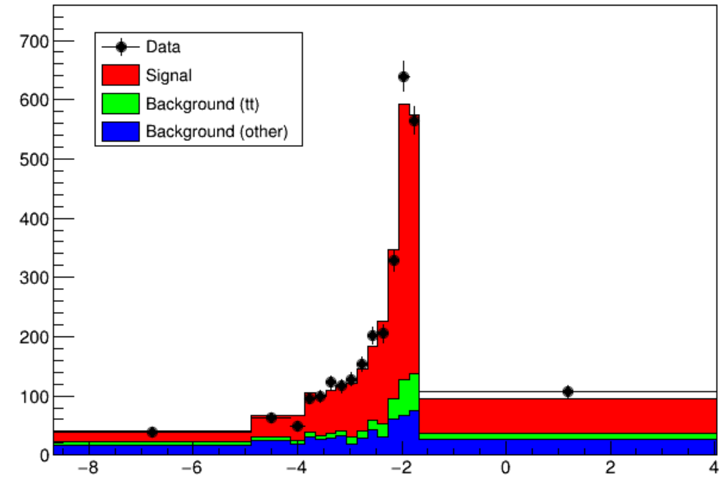
(*) 本研究ではFull simulation によるサンプル

各形状因子Fのシングルパラメータ測定

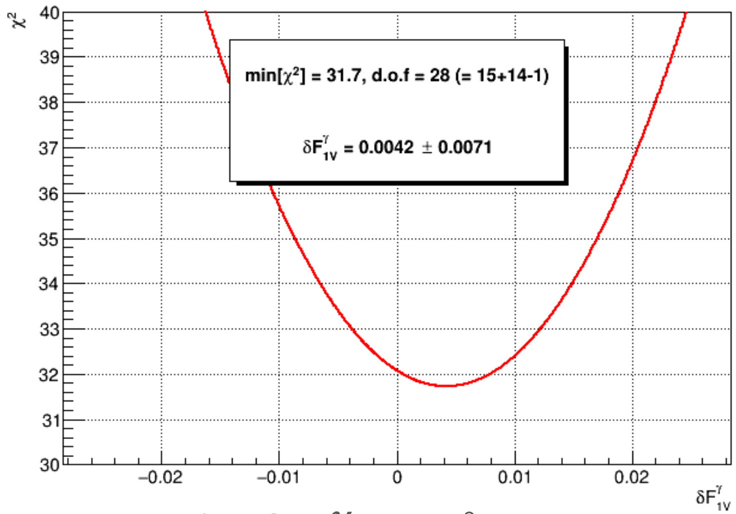
(e.g.) $\delta F_{1V}^{\gamma} = 0.0042 \pm 0.0071$

C.L. = 28.7 %

測定結果が入力と一致することを確認



F_{1V}^{γ}に対するωの分布 (Left pol.)



関数 $\chi^2(F_{1V}^{\gamma})$のプロット

先行研究との比較

形状因子, F	先行研究(1) Semi-lep	本研究 $bb\mu\mu\nu\nu$
F_{1V}^{γ}	± 0.002	± 0.0071
F_{1V}^{Z}	± 0.003	± 0.0128
F_{1A}^{γ}	---	± 0.0162
F_{1A}^{Z}	± 0.007	± 0.0262
F_{2V}^{γ}	± 0.001	± 0.0058
F_{2V}^{Z}	± 0.002	± 0.0102

形状因子, F	先行研究(2) Semi-lep	本研究 $bb\mu\mu\nu\nu$
ReF_{2A}^{γ}	± 0.005	± 0.0238
ReF_{2A}^{Z}	± 0.007	± 0.0351
ImF_{2A}^{γ}	± 0.006	± 0.0223
ImF_{2A}^{Z}	± 0.010	± 0.0394

信号事象数の比

$$\frac{N_{\text{semi-lep}}}{N_{bb\mu\mu\nu\nu}} \simeq \frac{\frac{6}{9} \times \frac{2}{9} \times 2}{\frac{1}{9} \times \frac{1}{9}} = 24$$

→ 統計誤差に5倍程度の差が予想される

- **先行研究と矛盾しない結果**(*)
- Semi-leptonic過程に本手法を応用することで、精度を改善する可能性がある

(*) 先行研究はマルチフィットをしているものもあるが相関は小さい

(1) Eur.Phys.J. C75 (2015) no.10, 512

(2) arXiv:1710.06737 [hep-ex].

まとめ

目的　ILD検出器のフルシミュレーション解析による
全角度情報を用いたttZ/γ異常結合探索手法の開発

- 力学的再構成によってDi-leptonic過程の全終状態を再構成
- ω分布による全角度情報を用いた、形状因子Fの測定手法を開発
- 得られた結果は先行研究と無矛盾
  一部のFについては先行研究の測定精度を改善する可能性

今後の展望

- 他の終状態に本手法の応用
- 複数の形状因子Fのマルチパラメータフィット

Backup

ILC (International Linear Collider)

TDR (Technical Design Report), 2013

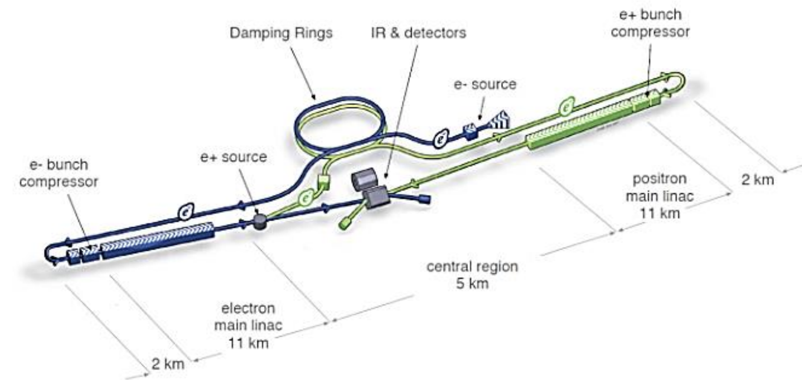
- $\sqrt{s}$ = 250-500 GeV → 1 TeV
- Length : 31 km → 50 km

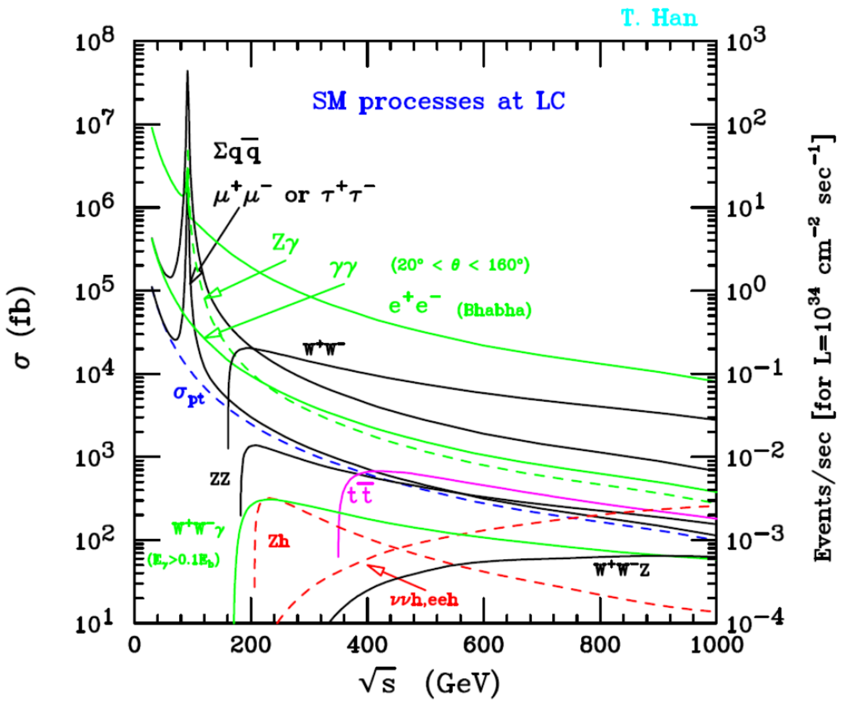
ILC250 (Staging Plan), 2017

- $\sqrt{s}$ = 250 GeV
- Length : 20 km

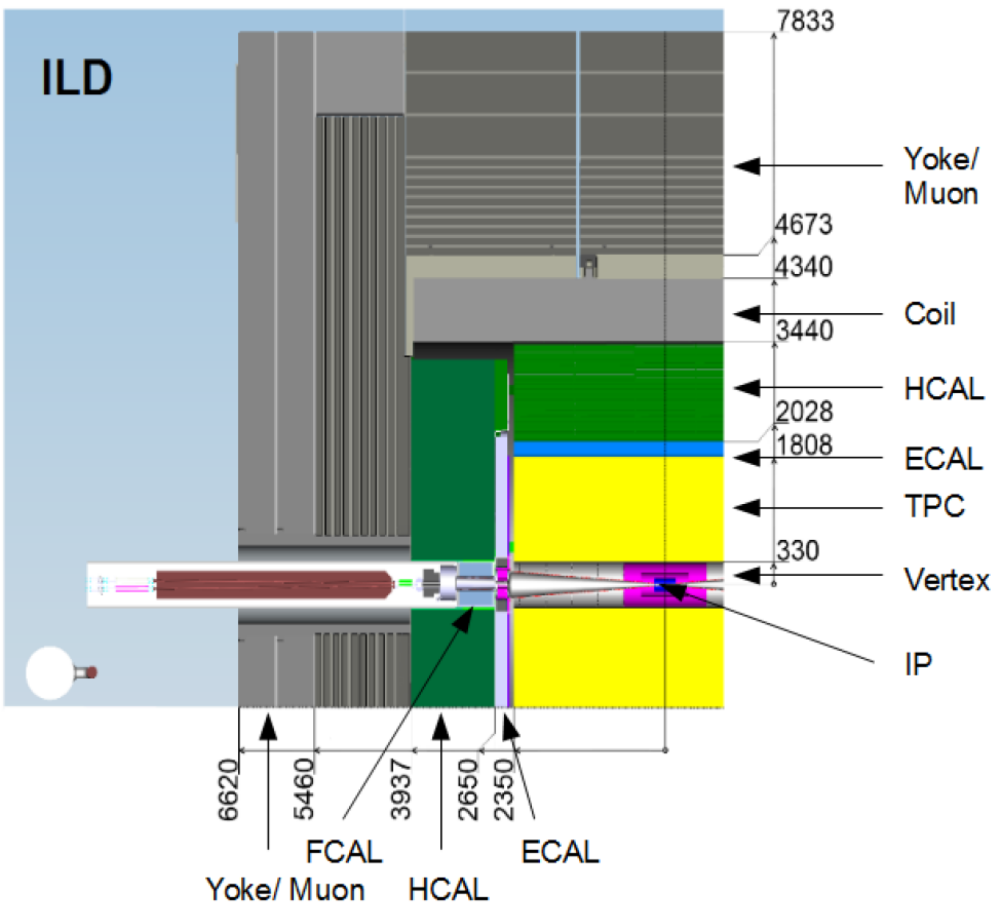
Physics Motivation

- Precise measurement of Higgs boson and Top quark
- New physics search





ILD (International Large Detector)



The ILD is composed of

- Vertex detector
- TPC
- ECAL
- HCAL
- Yoke / Muon detector
- Forward detectors

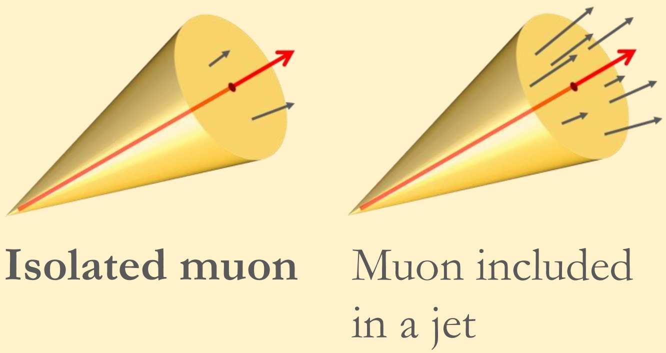
The reconstruction process uses all aspects of the ILC

Reconstruction Process

Reconstruct all final state particles, $b\bar{b}\mu^-\mu^+\nu\bar{\nu}$.

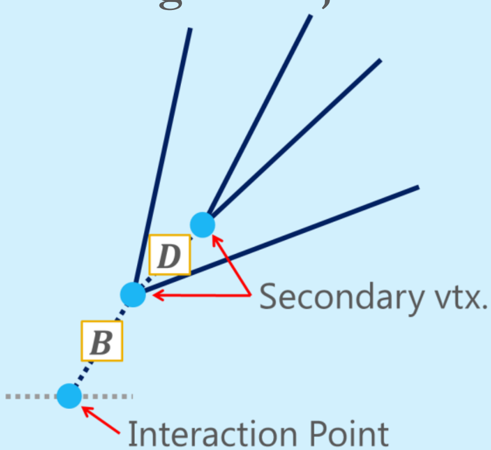
1. Selection of μ^+ and μ^-

- μ^-, μ^+ are isolated from other particles
- Extract isolated muons as final state muons



2. Jet clustering and b-tagging

- Cluster jet particles corresponding to $b, \bar{b}$
- B, D meson moves ~100 μm before the decay
- Assess the "b-likeness" from the vertex information (such as # of vtx. and distance between IP and vtx.)



Algorithm of the Kinematical Reconstruction

- Introduce 4 free parameters : $\vec{P}_\nu, P_{\gamma,z}$

$\vec{P}_{\bar{\nu}}$ can be computed using the initial momentum constraints

$$\vec{P}_{\bar{\nu}} = -\vec{P}_{\mathrm{vis.}} - \vec{P}_\nu - \vec{P}_\gamma, \qquad \left(\vec{P}_{\mathrm{vis.}} = \vec{P}_b + \vec{P}_{\bar{b}} + \vec{P}_{\mu^+} + \vec{P}_{\mu^-}\right)$$

- Define the likelihood function :

$$L_0\left(\vec{P}_\nu, P_{\gamma,z}\right) = BW(m_t)BW(m_{\bar{t}})BW(m_{W^+})BW(m_{W^-})Gaus(E_{\mathrm{total}})$$

- To correct the energy resolution of b-jets, add 2 parameters, $E_b, E_{\bar{b}}$, with the resolution functions to L_0 :

$$L\left(\vec{P}_\nu, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = L_0 \times Res(E_b, E_b^{\mathrm{meas.}})Res\left(E_{\bar{b}}, E_{\bar{b}}^{\mathrm{meas.}}\right)$$

Define $q\left(\vec{P}_\nu, P_{\gamma,z}, E_b, E_{\bar{b}}\right) = -2\log L + \mathrm{Const.}$

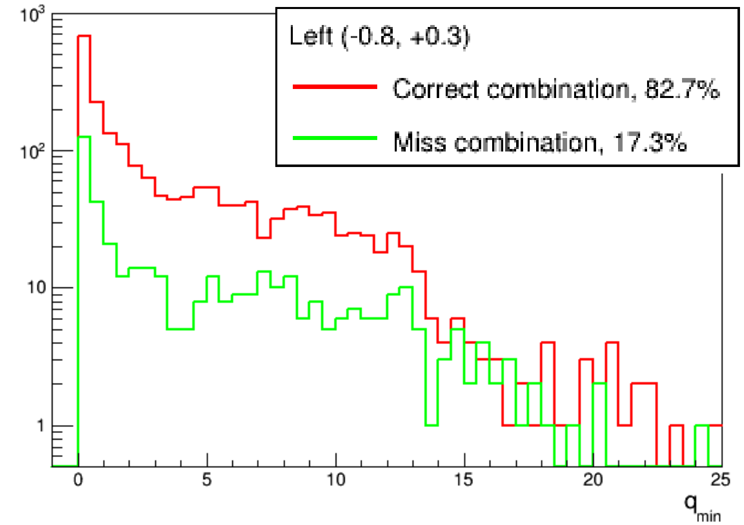
(scaled as the minimum of each component ($BW(m_t)$, etc) is equal to 0)

Combination of μ and b-jet

Choice of a combination of μ and b-jet

There are two candidates for the combination

- Select one having smaller q, defined as $\boldsymbol{q}_{\mathbf{min}}$
- Fraction of correct combination is ~83%

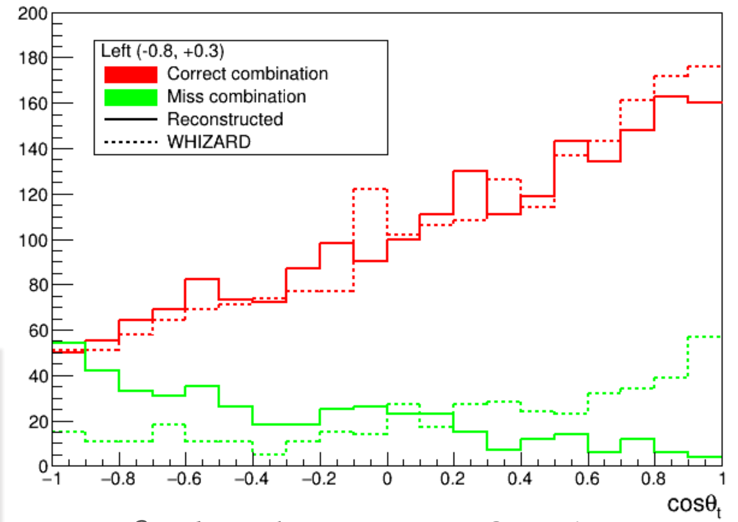


q_{min} distribution (Left polarization)

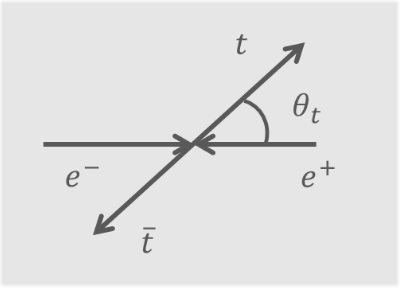
$\cos\theta_t$ distribution (Rec vs. MC Truth)

- Correct combination: OK !
- Miss combination: Disagree with the MC truth.

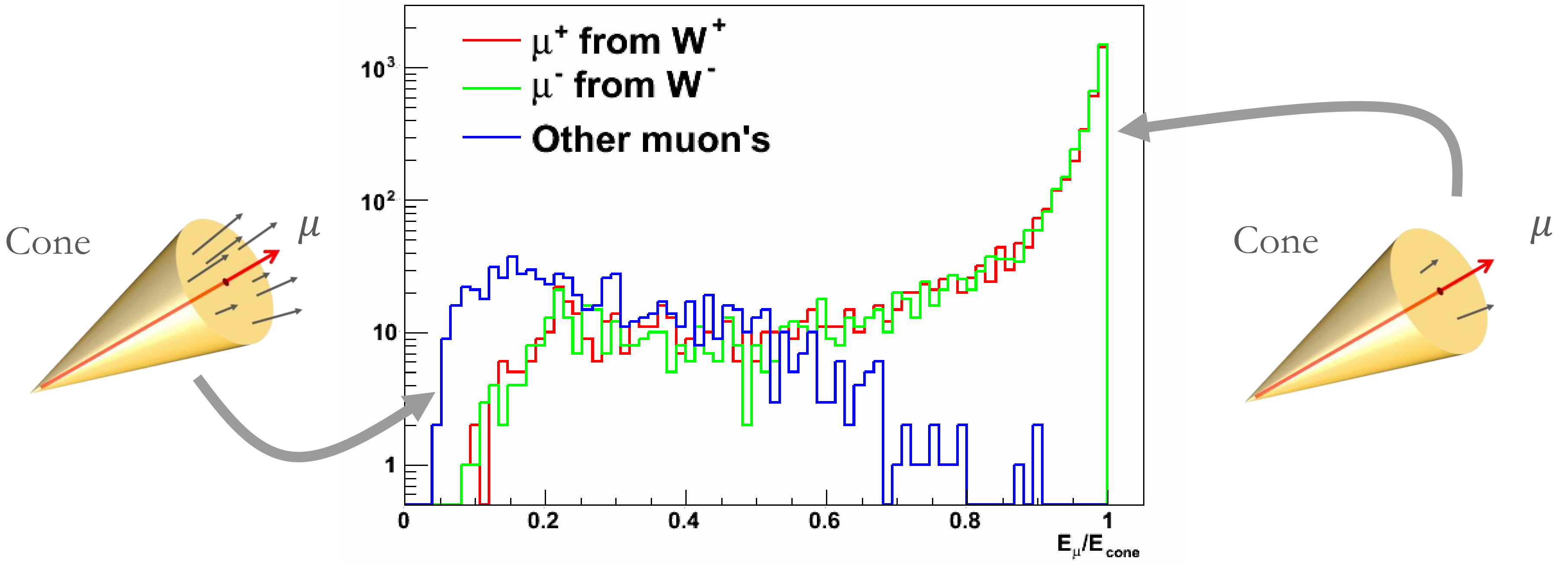
Need to estimate an effect of the miss combination for the analysis.



$\cos\theta_t$ distribution (Left polarization)



Isolated muon finder



Energy ratio between μ and a cone

$R = E_\mu / E_{cone}$ is a quantity to evaluate how isolated the muon is.

(E_{cone} : total energy of particles in the cone)

μ from W boson is more isolated than other μ

Isolated muon finder

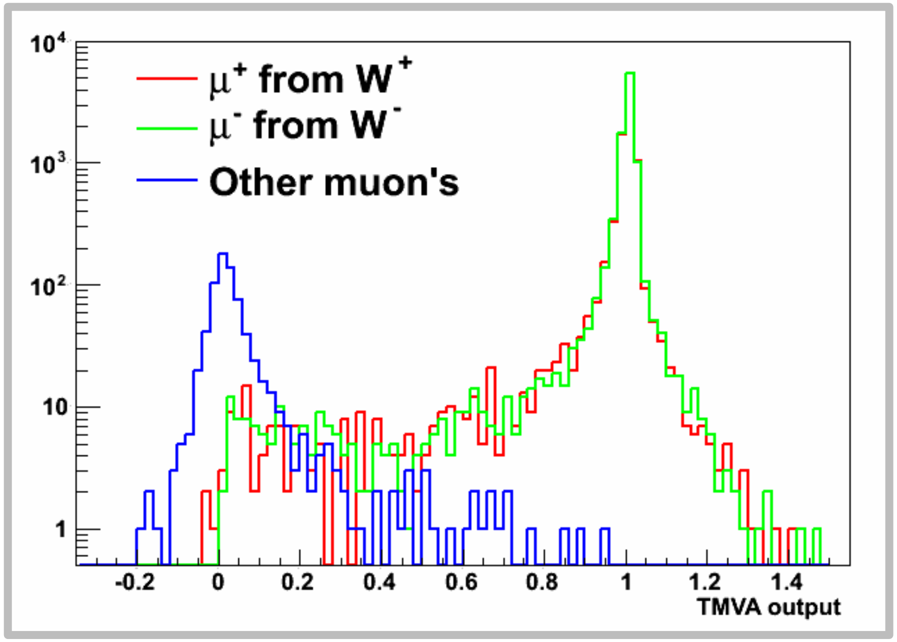
Quantities

$R = E_\mu / E_{cone}$, $E_{cone,neutral}$, $E_{cone,charged}$

$\cos\theta = \frac{P_\mu \cdot P_{cone}}{|P_\mu| \times |P_{cone}|}$, ΔE_{ECAL}, ΔE_{Yoke}, …

TMVA

Multi variable analysis tool



Jet clustering

General strategy

Merge a pair of particles whose "**Distance**" is the smallest until a condition meets "**Criteria**"

"**Distance**"

Durham algorithm : $Y_{ij} = 2\frac{\min\left[E_i^2, E_j^2\right](1-\cos\theta_{ij})}{E_{vis}^2}$, θ_{ij} : angle between P_i and P_j

k_t algorithm : $d_{ij} = \min\left[{p_T}_i^2, {p_t}_j^2\right]\frac{R_{ij}}{R}$ or $d_{iB} = {p_t}_i^2, R_{ij}^2 = (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$

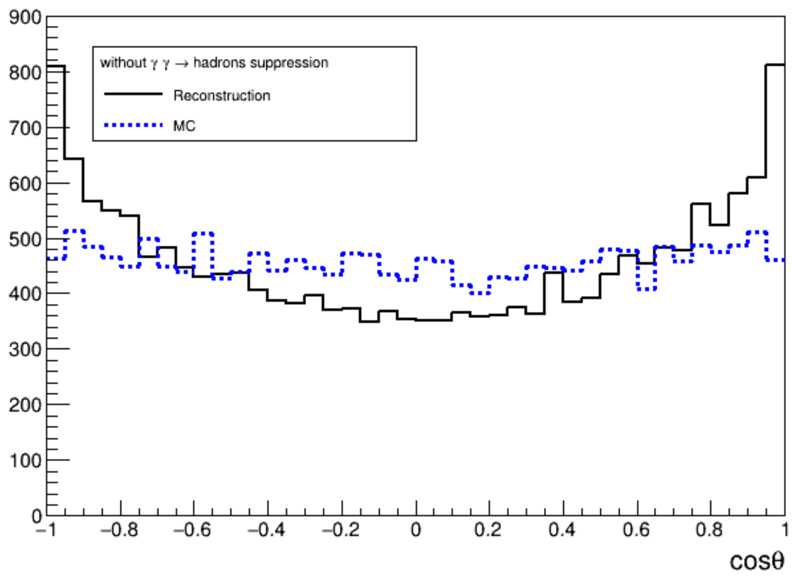
η : pseudo rapidity, ϕ azimuthal angle

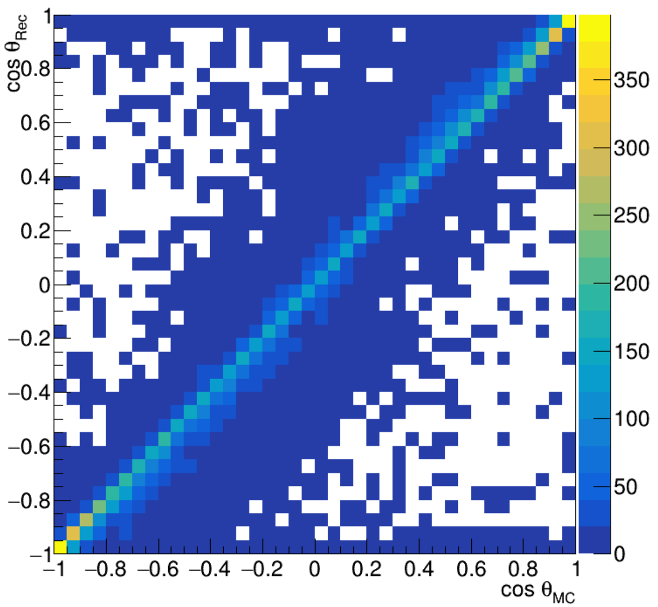
"**Criteria**"

- Number of remaining particles is equal to N_{Req}
- The smallest distance is smaller than D_{Req}

$\gamma\gamma \to$ hadrons rejection

$b, \bar{b}$ are reconstructed from the rest of particles with LCFIPlus





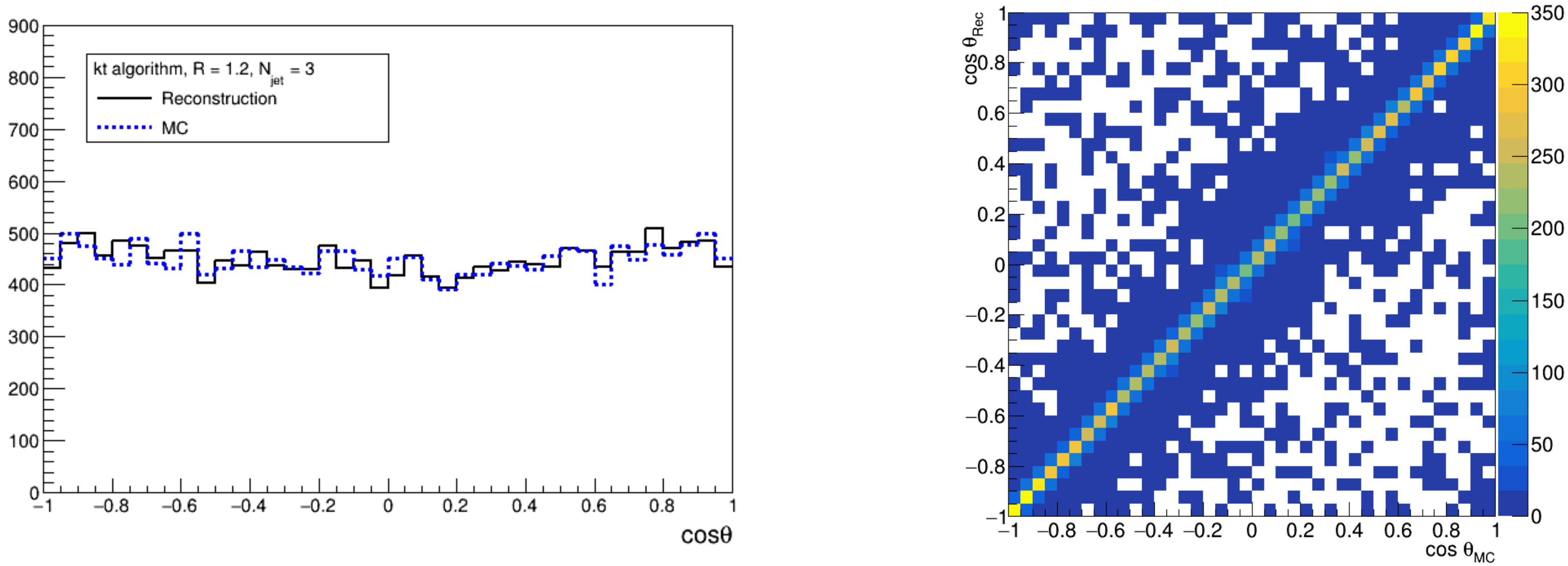
$\cos\theta_{jet}$ distribution

Strongly peaked at very forward region by mistake

$\gamma\gamma \to$ hadrons are emitted along the beam direction

$\gamma\gamma \rightarrow$ hadrons rejection

Eliminate particles close to beam direction rather than other particles with kt algorithm.



$\cos\theta_{jet}$ distribution

Good agreement between Rec and MC

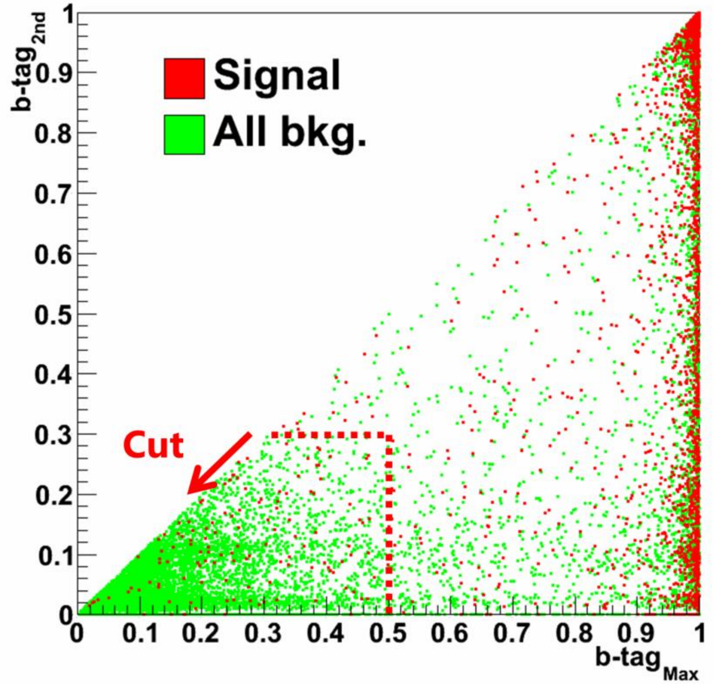
b-tagging with LCFIPlus

b-tag is TMVA output indicating “b-likeness” of a jet obtained by the LCFIPlus(*).

- $b\text{-}tag_{Max}$: the largest b-tag
- $b\text{-}tag_{2nd}$: the 2nd largest b-tag

- Signal has large $b\text{-}tag_{Max}$
- Many of bkg. have small $b\text{-}tag_{Max}$ and $b\text{-}tag_{2nd}$

$$b - tag_{Max} > 0.5 \text{ or } b - tag_{2nd} > 0.3$$



b-tag value (Left polarization)

(*) A software package of Marlin for the multi-jet analysis.

Kinematical Reconstruction

$$BW(x;m,\Gamma) \propto \frac{1}{1+\left(\frac{x^2-m^2}{m\Gamma}\right)^2}$$

$$Gaus(x;\mu,\sigma) \propto \exp\left[-\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)^2\right]$$

Detail definition of L_0 is

$$L_0\left(\vec{P}_\nu, P_{\gamma,Z}\right) = BW(m_t;174{,}5)BW(m_{\bar{t}};174{,}5) \cdot BW(m_{W^+};80.4{,}5)BW(m_{W^-};80.4{,}5)Gaus(E_{\mathrm{total}};500{,}0.39)$$

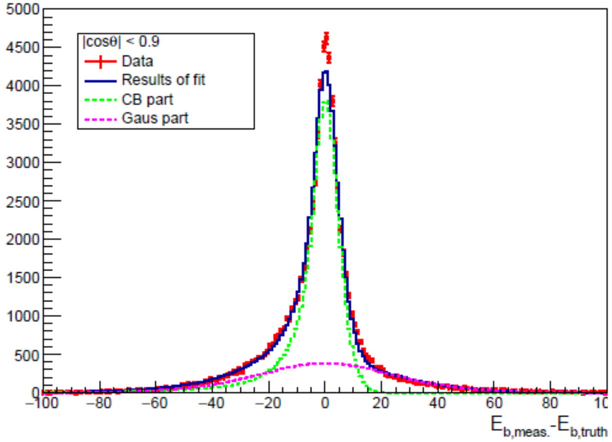
- Larger value for Γ than theoretical value is set because of detector effects
- σ is caused by the Beam energy spread.

Energy resolution of b-jet

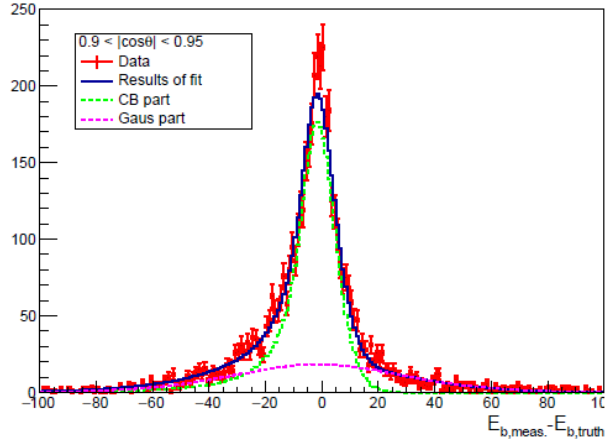
Estimate the energy resolution of b-jet with the following $Res(E_b, E_b^{\text{meas.}})$;

$$Res(E_b, E_b^{\text{meas.}}) = (1-f)CB(\Delta E_b; \alpha, n, \mu_{CB}, \sigma_{CB}) + f * Gaus(\Delta E_b; \mu_{Gaus}, \sigma_{Gaus})$$

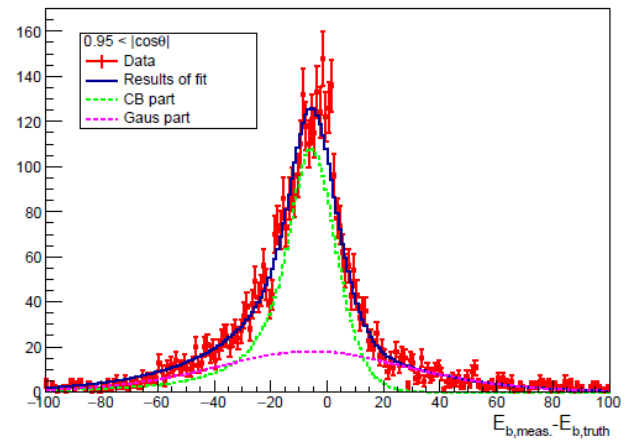
Divide into 3 regions ; $|\cos\theta| = (0, 0.9), (0.9, 0.95), (0.95, 1)$



(a) $|\cos\theta| < 0.9$



(b) $0.9 < |\cos\theta| < 0.95$



(c) $0.95 < |\cos\theta|$

Crystal Ball function

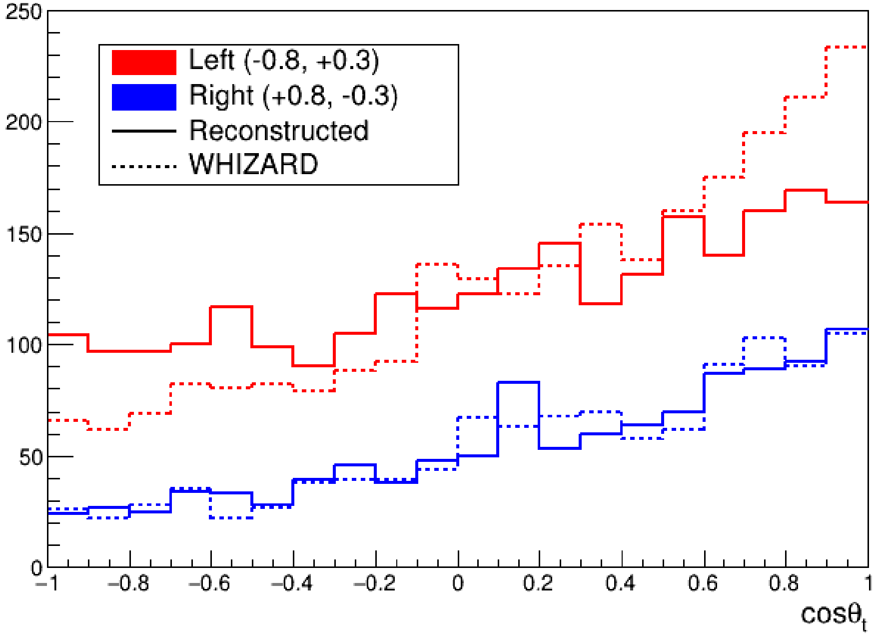
Crystal Ball function consists of a Gaussian core portion and power-law tail.

$$CB(x;\alpha,n,\bar{x},\sigma) = N \cdot \begin{cases} \exp\left(-\frac{(x-\bar{x})^2}{2\sigma^2}\right) & \frac{x-\bar{x}}{\sigma} > -\alpha \\ A \cdot \left(B - \frac{x-\bar{x}}{\sigma}\right)^{-n} & \frac{x-\bar{x}}{\sigma} \leq -\alpha \end{cases}$$

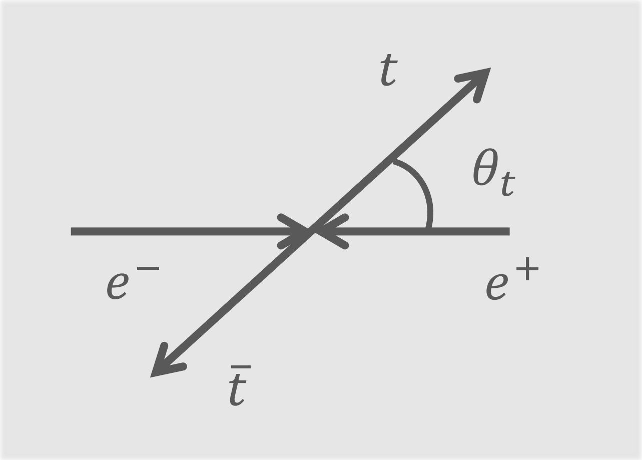
$$\begin{aligned} A &= \left(\frac{n}{|\alpha|}\right)^n \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ B &= \frac{n}{|\alpha|} - |\alpha| \\ N &= \frac{1}{\sigma(C+D)} \\ C &= \frac{n}{|\alpha|} \cdot \frac{1}{n-1} \cdot \exp\left(-\frac{|\alpha|^2}{2}\right) \\ D &= \sqrt{\frac{\pi}{2}}\left(1 + \mathrm{erf}\left(\frac{|\alpha|}{\sqrt{2}}\right)\right) \end{aligned}$$

Results of Reconstruction

Top quark polar angle distribution, $\cos\theta_t$



Definition of θ_t

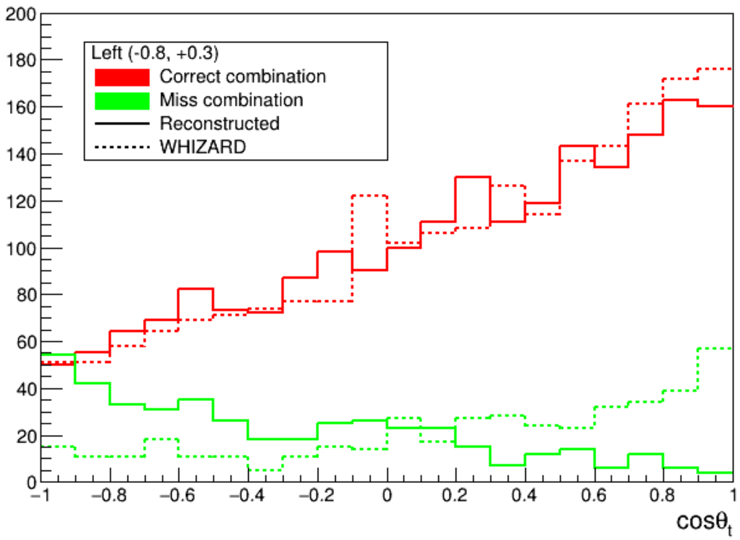


Considerable migration occurs in the Left polarization case

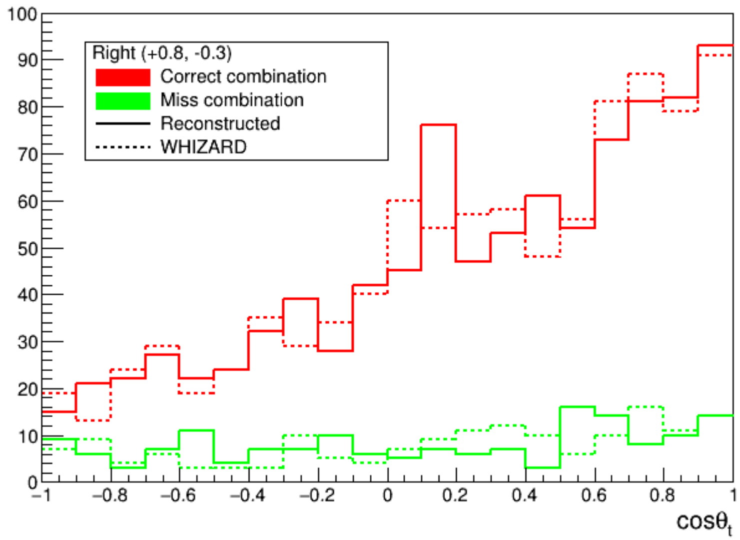
Some events pass from forward to backward because of the miss combination of μ and b-jet.

Dependence from the beam polarization

$\cos\theta_t$ distribution (Left polarization)



$\cos\theta_t$ distribution (Right polarization)



Left polarization

Reconstructed distribution of miss combination is very different from the MC truth.

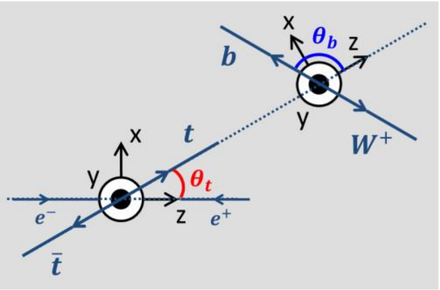
Right polarization

Similar distribution can be reconstructed even when the miss combination is selected.

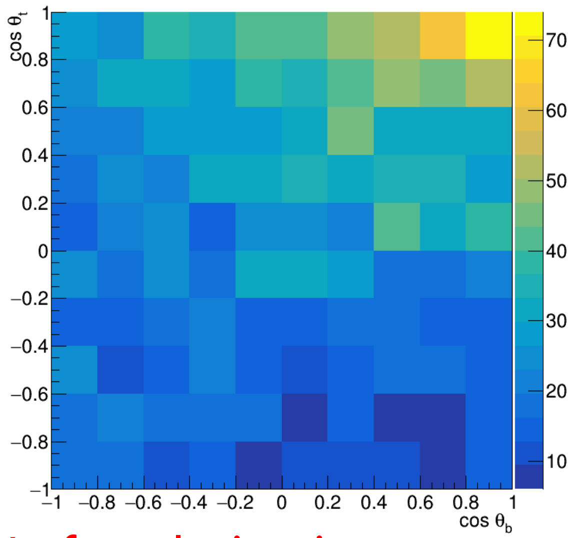
Dependence from the beam polarization

$\cos\theta_b \simeq 1$ → b-jets are energetic

→ **Migration effect is strong**



$\cos\theta_t$ vs. $\cos\theta_b$ (Left polarization)

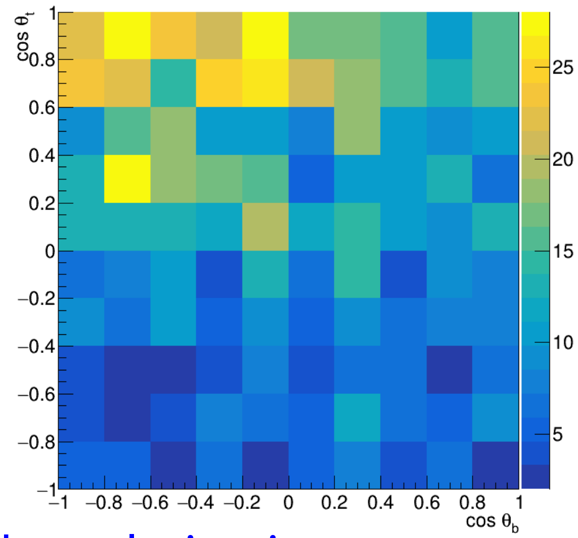


Left polarization

Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq 1$

→ Migration is asymmetry

$\cos\theta_t$ vs. $\cos\theta_b$ (Right polarization)



Right polarization

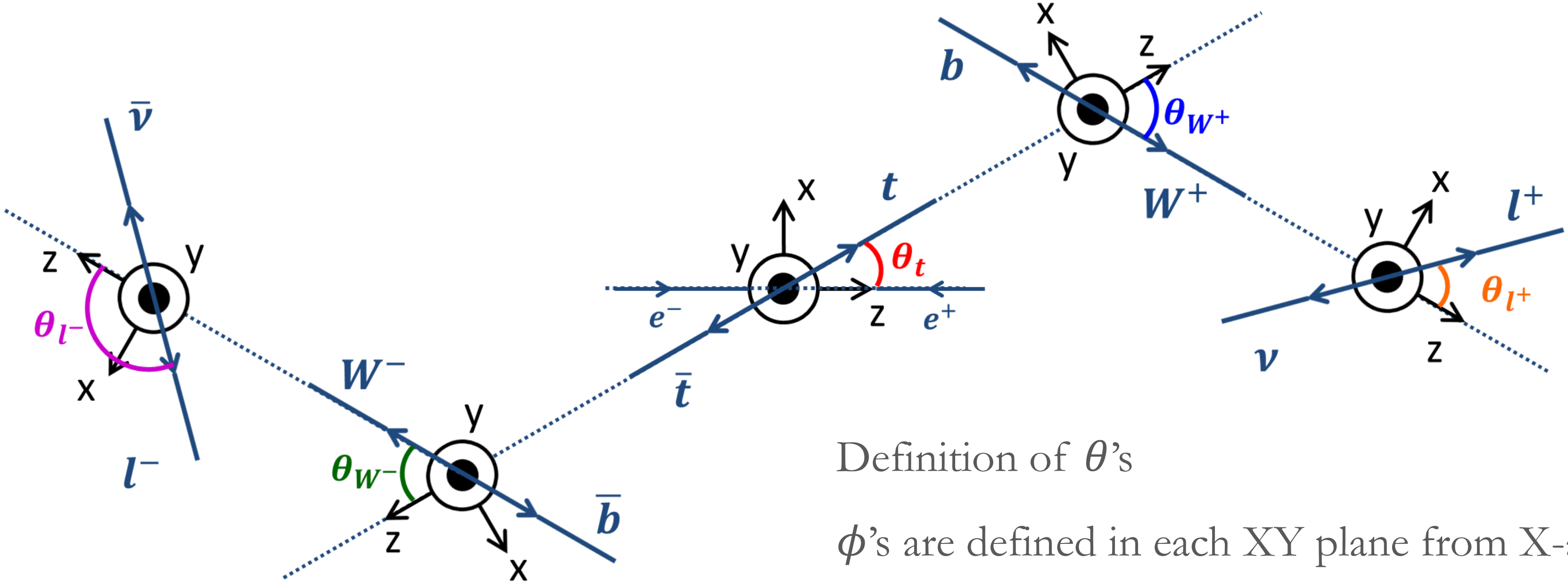
Peak at $\cos\theta_t \simeq 1$ & $\cos\theta_b \simeq -1$

→ Migration is symmetry

The amplitude of the di-leptonic process

The amplitude of the di-leptonic process is a function of 9 angles.

$$|M|^2(\cos\theta_t, \cos\theta_{W^+}, \phi_{W^+}, \cos\theta_{W^-}, \phi_{W^-}, \cos\theta_{l^+}, \phi_{l^+}, \cos\theta_{l^-}, \phi_{l^-}; F)$$



Definition of θ's

ϕ's are defined in each XY plane from X-axis

It is difficult to handle the 9-dimention phase space

→ Expand the amplitude in the form factors, F

Reweighting (Template-like) Technique

Binned likelihood method : $\chi^2(\delta F) = \sum_{i=1}^{N_{bin}} \left(\frac{n_i^{\text{Data}} - n_i^{\text{Sim.}}(\delta F)}{\sqrt{n_i^{\text{Data}}}} \right)^2$

$n_i^{\text{Sim.}}(\delta F)$ is obtained from the large full simulation

Reweighting technique :

Produce a sample using SM value, then change the weight of events.

$$\begin{aligned} n_i^{\text{Sim.}}(\delta F) &= n_i^{\text{Sim.,sig}}(\delta F) + n_i^{\text{Sim.,bkg}} \\ &= n_i^{\text{Sim.,sig}}(0)(1\langle\omega\rangle_i \delta F + \langle\widetilde{\omega}\rangle_i \delta F^2) + n_i^{\text{Sim.,bkg}} \\ &\simeq n_i^{\text{Sim.,sig}}(0)(1 + \langle\omega\rangle_i \delta F) + n_i^{\text{Sim.,bkg}} \end{aligned}$$

Template technique : Produce many samples using different parameters

Di-leptonic process

Apply this method to the di-leptonic process, $e^-e^+ \to b\bar{b}l^-l^+\nu\bar{\nu}$.

Single Parameter fit

- Precision becomes twice better because the number of signal events is about 4 times larger.

Form factor	$bb\mu\mu\nu\nu$	$bbll\nu\nu$
F_{1V}^{γ}	±0.0071	±0.0034
F_{1V}^{Z}	±0.0128	±0.0061
F_{1A}^{γ}	±0.0162	±0.0082
F_{1A}^{Z}	±0.0262	±0.0133
F_{2V}^{γ}	±0.0058	±0.0028
F_{2V}^{Z}	±0.0102	±0.0049

Form factor	$bb\mu\mu\nu\nu$	$bbll\nu\nu$
ReF_{2A}^{γ}	±0.0238	±0.0123
ReF_{2A}^{Z}	±0.0351	±0.0180
ImF_{2A}^{γ}	±0.0223	±0.0110
ImF_{2A}^{Z}	±0.0394	±0.0194

Di-leptonic process

Multi-Parameter fit

- ω's are not correlated → Each parameter can be measured independently.
  (10 Parameters can be separated into top 6, middle 2 and bottom 2)
- ω's are correlated or there is a structure in a 2D histogram
  → The 2D histogram is needed for the multi-parameter fit
- ω's are strongly correlated.
  → The 1D histogram can measure multi parameter (not optimal way)

(eg) Use a 1D histogram of ω of F_{1V}^{γ}

Form factor	Single	Multi
F_{1V}^{γ}	± 0.0034	± 0.0038
F_{1V}^{Z}	± 0.0061	± 0.0068
F_{1A}^{γ}	± 0.0082	± 0.0099
F_{1A}^{Z}	± 0.0133	± 0.0155

Correlation matrix

$$\begin{pmatrix} +1.000 & -0.232 & -0.183 & +0.380 \\ -0.232 & +1.000 & +0.328 & -0.123 \\ -0.183 & +0.328 & +1.000 & -0.280 \\ +0.380 & -0.123 & -0.280 & +1.000 \end{pmatrix}$$

Di-leptonic process

(eg) Use a 2D histogram of $\omega's$ of $F_{1V}^{\gamma}, F_{2V}^{\gamma}$

Form factor	Single	Multi
F_{1V}^{γ}	± 0.0034	± 0.0264
F_{1V}^{Z}	± 0.0061	± 0.0437
F_{1A}^{γ}	± 0.0082	± 0.0102
F_{1A}^{Z}	± 0.0133	± 0.0157
F_{2V}^{γ}	± 0.0028	± 0.0210
F_{2V}^{Z}	± 0.0049	± 0.0344

Correlation matrix

$$\begin{pmatrix} +1.000 & -0.237 & -0.098 & +0.296 & -0.990 & +0.232 \\ -0.237 & +1.000 & +0.322 & -0.058 & +0.231 & -0.989 \\ -0.098 & +0.322 & +1.000 & -0.310 & +0.070 & -0.276 \\ +0.296 & -0.058 & -0.310 & +1.000 & -0.246 & +0.034 \\ -0.990 & +0.231 & +0.070 & -0.246 & +1.000 & -0.231 \\ +0.232 & -0.989 & -0.276 & +0.034 & -0.231 & +1.000 \end{pmatrix}$$