

CPV and Angles of UT

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Symmetries

Nature and its Law: \sim Symmetry = Beauty

P, C, T : most Fundamental Symmetry

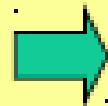
P : Parity = Space inversion

C : Charge conjugate (Particle \leftrightarrow Anti-particle; Quantum #)
[Lagrangian \rightarrow Hermitian conjugate]

T : Time reversal [c-number \rightarrow complex conjugate]

CPT Theorem

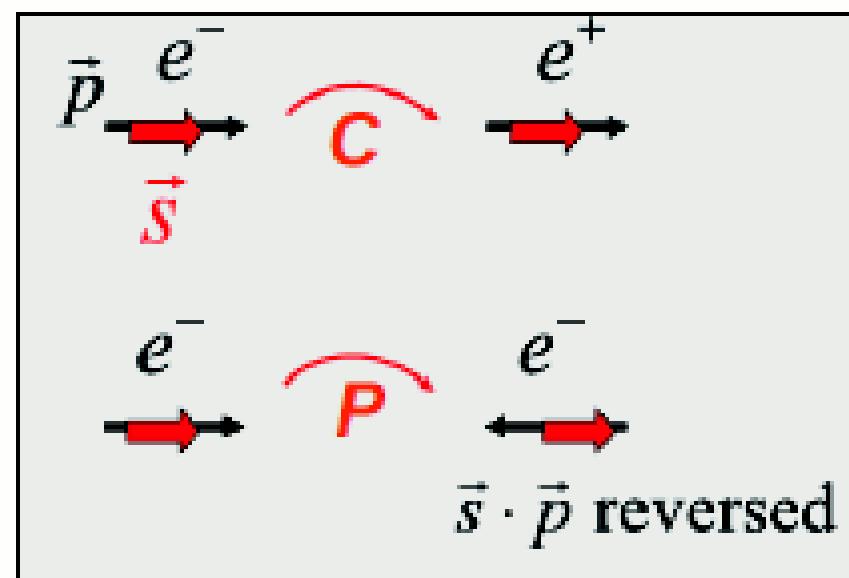
Lorentz invariant local quantum field theory \rightarrow CPT symmetry



Particle \leftrightarrow Anti-particle: Mass and Lifetime are identical

Charge conjugation, Parity, CP

- **CP = P and C applied consecutively**
- Charge conjugation (**C**)
 - Change of all the charge quantum numbers into their opposite
 - transforms a particle into its anti-particle
- Parity (**P**)
 - Inversion of the spatial coordinates.
 $x \rightarrow -\vec{x}$ “Image in a mirror”



CP Violation \Leftrightarrow physics is not symmetric under CP transformation
 \Leftrightarrow Experimental results in CP conjugate systems are not the same

In the Standard Model of Particle Physics (SM):

- C and P are symmetries of strong and electromagnetic interactions.
- C and P symmetries are violated by weak interaction
- CP symmetry is slightly violated by weak interaction

CP Violation with Escher's images

CP (anti-matter in a mirror)

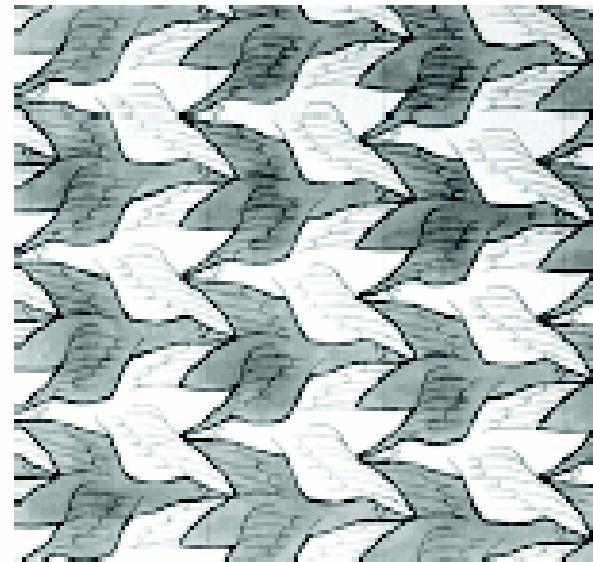
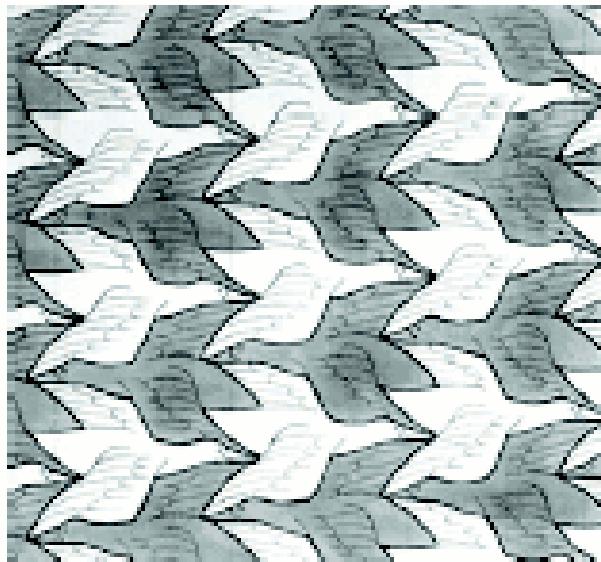
P (mirror)

C (anti-matter)

White geese fly right

White geese fly left

White geese fly right



Slight breaking of CP (look at the tails...)

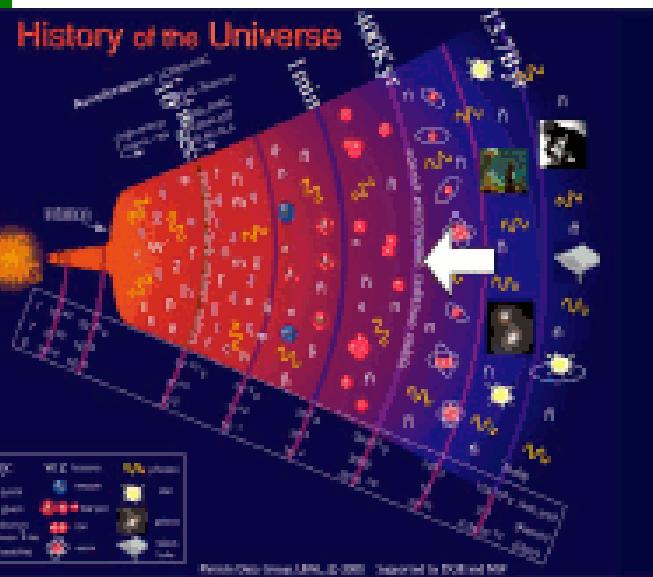
Analogy to weak interaction in the Standard Model.

Why CPV is important ?

Difference between particle & anti-particle (matter & anti-matter)

Universe: almost “matter” only (no anti-matter)

Big-Bang \rightarrow $N(\text{particles}) = N(\text{anti-particles})$



Sakhalov's 3 conditions (1967):

1. baryon number violation
2. **CP violation**
3. existence of non-equilibrium

CPV is a key for Existence of Universe & us !

Andrei Sakharov (1921-1989)



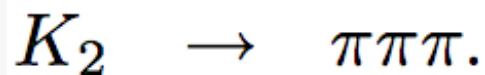
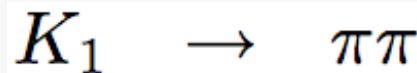
But the CKM source of CPV orders of magnitude too small
to explain the observed state of the universe!

discovery of CP violation

With simple quantum mechanics, one can show that in absence of CP violation:

$$\begin{aligned} CP|K_1\rangle &= \frac{1}{\sqrt{2}}(CP|K^0\rangle + CP|\bar{K}^0\rangle) = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle) = +|K_1\rangle \\ CP|K_2\rangle &= \frac{1}{\sqrt{2}}(CP|K^0\rangle - CP|\bar{K}^0\rangle) = \frac{1}{\sqrt{2}}(|\bar{K}^0\rangle - |K^0\rangle) = -|K_2\rangle \end{aligned}$$

Final states CP eigenvalues are $+1$ ($\pi^+ \pi^-$) and -1 ($\pi^+ \pi^- \pi^0$). If CP is a conserved quantity, one then should have:



which well identify as K_S^0 and K_L^0 respectively.

measuring K_L^0 decays into two pions ?

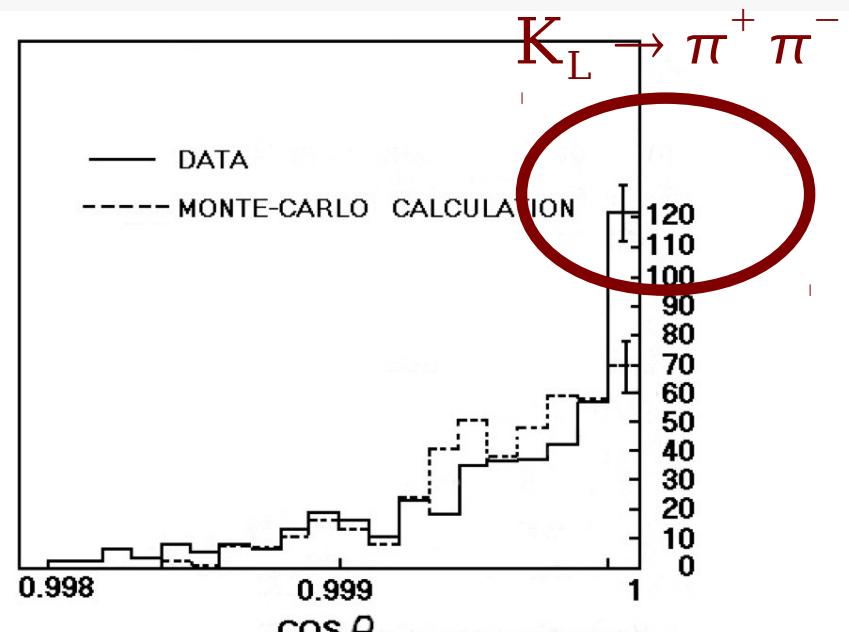
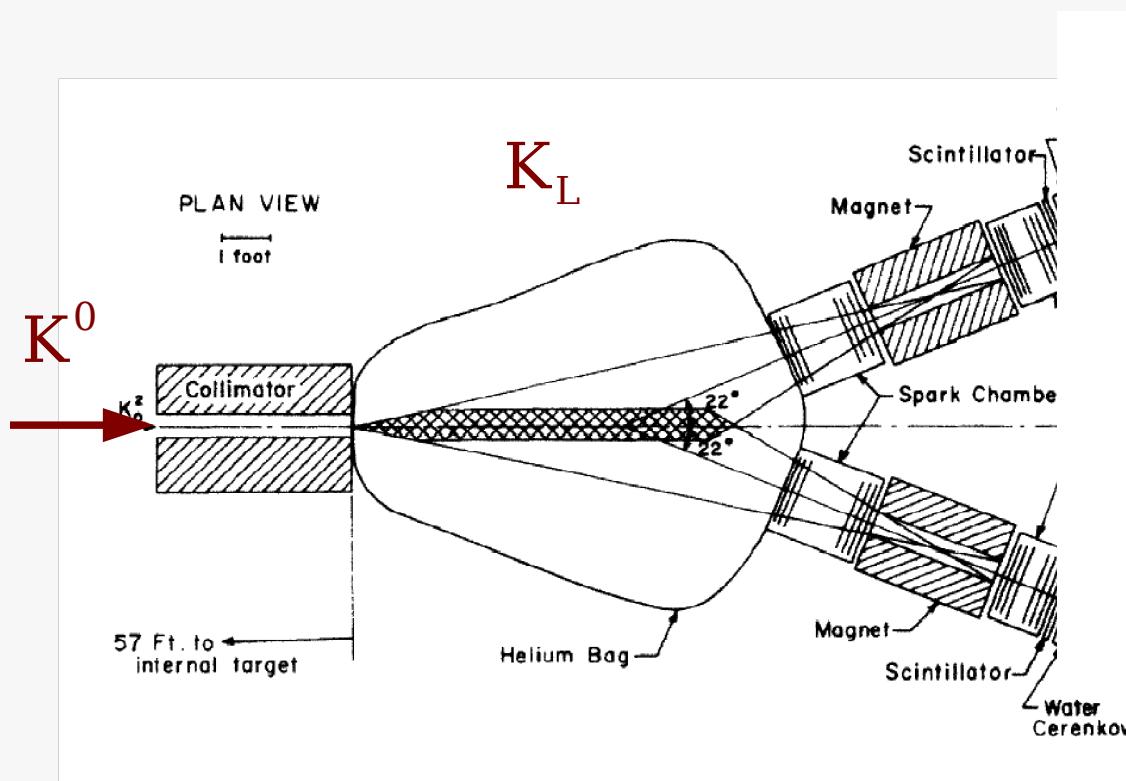
\Rightarrow proof that CP asymmetry is violated in weak interaction

discovery of CP violation

The CP violation in kaon system:

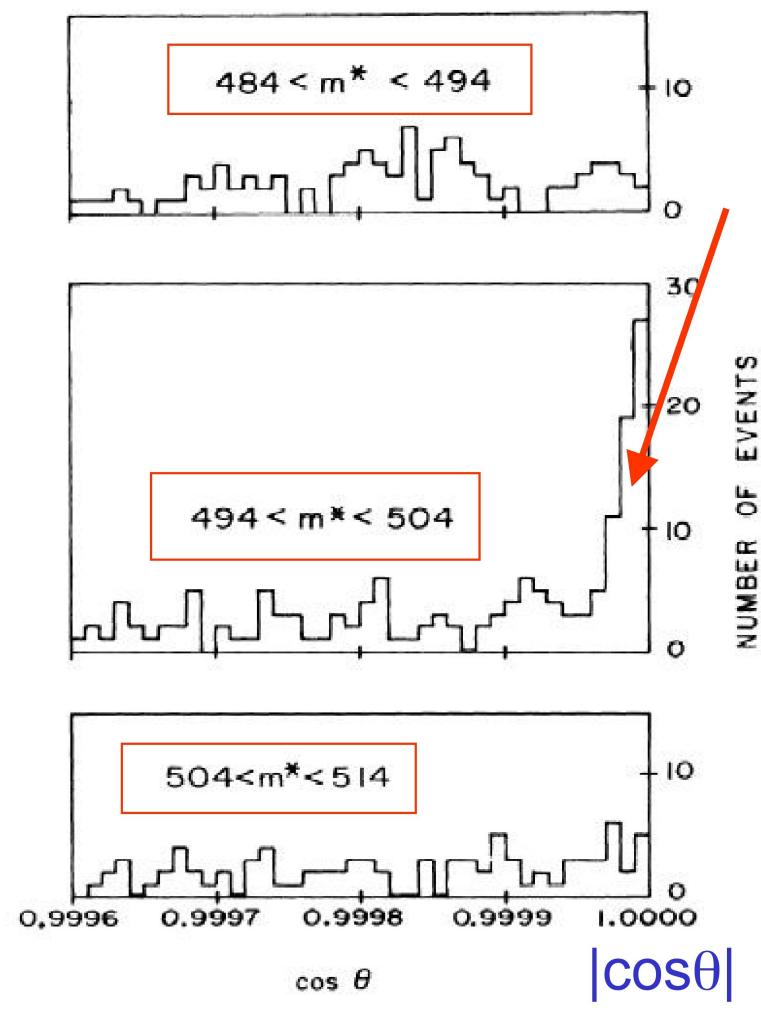
Christenson, Cronin, Fitch, Turlay, Phys. Rev. Lett. 13 (1964) 138.

Far after the target, only K_L survives. They measured:



$$|\eta_{+-}| = \frac{A(K_L^0 \rightarrow \pi\pi)}{A(K_S^0 \rightarrow \pi\pi)}$$

discovery of CP violation



- Two body decay : in the K^0 center of mass system the two π are back to back: $|\cos \theta|=1$
- Today's more precise measurement for the ratio of amplitudes:

$$|\eta_{+-}| = \frac{A(K_L^0 \rightarrow \pi\pi)}{A(K_S^0 \rightarrow \pi\pi)} = (2.271 \pm 0.017)10^{-3}.$$

The unitarity triangle: introduction

The Higgs boson gives mass to bosons and fermions (quarks and leptons) through the Yukawa couplings but this is not the end of the story :

$$\mathcal{L}_{cc}^{\text{quarks}} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \left[\sum_{ij} \bar{u}_i(q_2) \gamma^\mu (1 - \gamma^5) V_{ij} d_j \right] + \text{h.c}$$

Once the mass matrix is diagonalized, it determines also how the mass and weak eigenstates are related. This is the CKM matrix. As for the masses, nothing is predicted except the mass matrix must be unitary and complex

$$\begin{pmatrix} u \\ s \\ b \end{pmatrix}_{EW} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} u \\ s \\ b \end{pmatrix}_{MASS}$$

The unitarity triangle: introduction

- Weak eigenstates are therefore a mixture of mass eigenstates , by the Cabibbo-Kobayashi-Maskawa elements V_{ij} : flavour changing charged currents between quark generations.
- This matrix is a 3×3 unitary , complex , and hence described by means of four parameters: 3 rotation angles and a phase. The latter makes possible the CP symmetry violation in the Standard Model.
- These four parameters are free parameters of the SM. As for electroweak precision tests , they must be measured with some redundancy and the SM hypothesis is to be falsified by a consistency test. We will review in this lecture this overall test. But let's define first the parameters.

Origin of CPV ?

Kobayashi-Maskawa Ansatz (1973)

Complex phase in the quark mixing matrix

→ source of CPV in Weak Interactions

KM-phase

► Requires 3 (or more) generation of quarks

- only 3 quarks (u, d, s) were known at that time !
- All 6 quarks are now discovered

Essential ingredient of
the Standard Model (SM)



The unitarity triangle: parameterization

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Consider the Wolfenstein parameterization as in EPJ C41: 1-131, 2005: unitary-exact and phase convention independent

$$\lambda^2 = \frac{|V_{us}|^2}{|V_{ud}|^2 + |V_{us}|^2}, \quad A^2 \lambda^4 = \frac{|V_{cb}|^2}{|V_{ud}|^2 + |V_{us}|^2} \quad \text{and} \quad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*}$$

- λ is measured from $|V_{ud}|$ and $|V_{us}|$ in superallowed beta decays and semileptonic kaon decays, respectively.
- A is further determined from $|V_{cb}|$, measured from semileptonic charmed B decays.
- The last two parameters are to be determined from angles and side measurements of the CKM unitarity triangle

The unitarity triangle: representation

Unitarity ($U^\dagger U$) prescribes 6 complex equations...

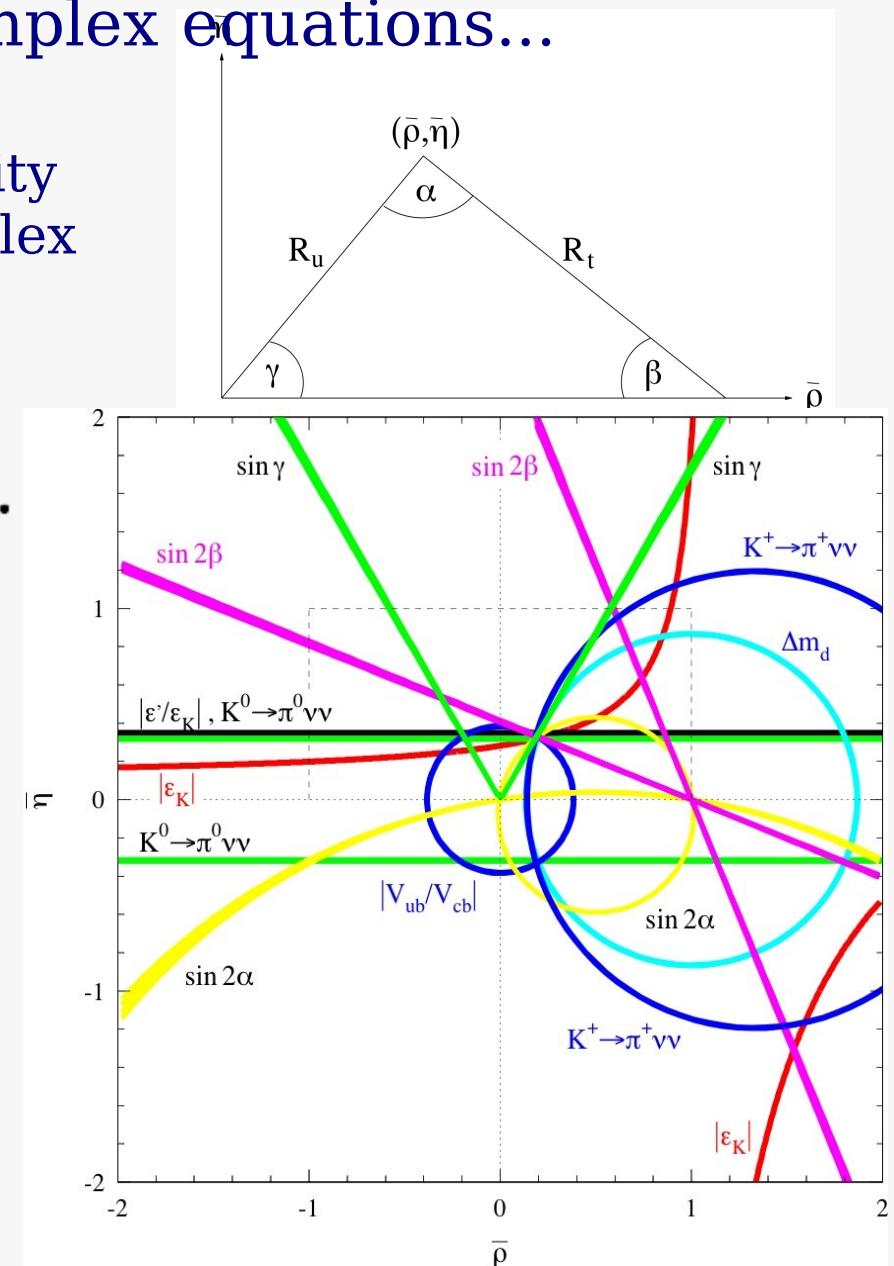
An elegant way to represent the unitarity relations is to display them in the complex plane as the sum of three vectors:

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{cd}V_{cb}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0.$$

The area of the triangle is half the Jarlkog invariant and measures the magnitude of the CP violation

$$J \sum_{\sigma\gamma=1}^3 \epsilon_{\mu\nu\sigma} \epsilon_{\alpha\beta\gamma} = \text{Im}(V_{\mu\alpha} V_{\nu\beta} V_{\mu\beta}^* V_{\nu\alpha}^*) ,$$

$$J = A^2 \lambda^6 \eta (1 - \lambda^2/2) \simeq 10^{-5}$$



The unitarity triangle: definitions

- Sides and angles of the unitarity triangle
- Normalization given by the matrix element V_{cb}

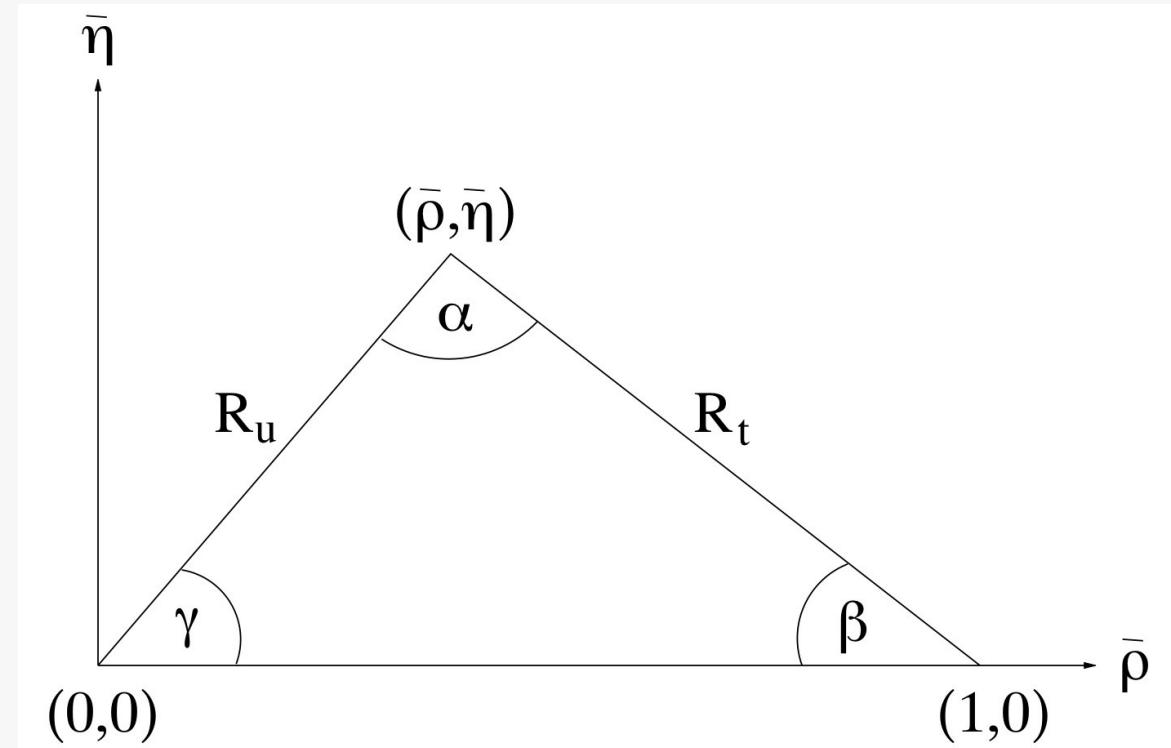
$$R_u = \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2},$$

$$R_t = \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}.$$

$$\alpha = \arg \left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right),$$

$$\beta = \pi - \arg \left(\frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right),$$

$$\gamma = \arg \left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right).$$

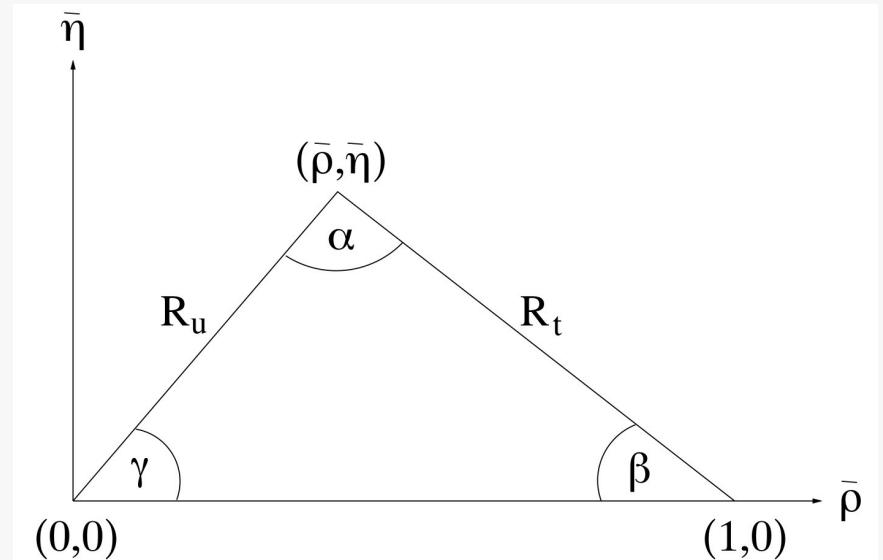


The unitarity triangle: definitions

- Sides of the unitarity triangle

Towards the experimental constraints:

$$\begin{aligned} R_u &= \left| \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right| = \sqrt{\bar{\rho}^2 + \bar{\eta}^2}, \\ R_t &= \left| \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right| = \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2}. \end{aligned}$$



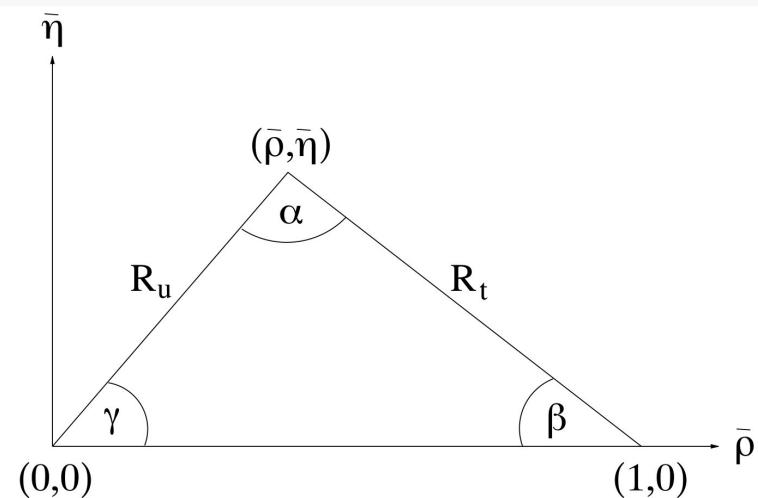
- R_u is measured by the matrix elements V_{ub} and V_{cb} extracted from the semileptonic decays of b-hadrons.
- R_t implies the matrix element V_{td} and hence can be measured from the mixing of B^0 mesons.

The unitarity triangle: definitions

- Angles of the unitarity triangle

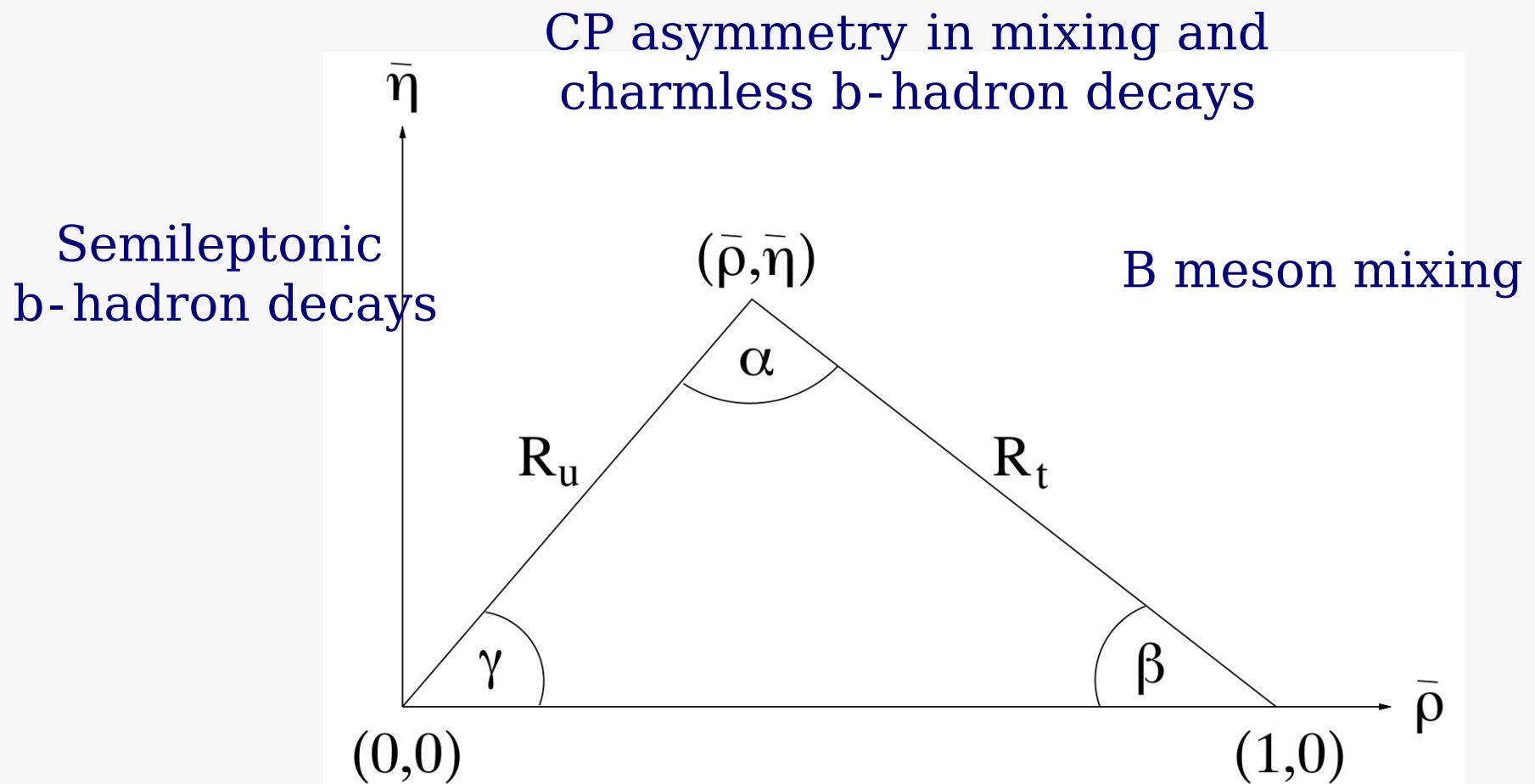
Towards the experimental constraints:

$$\alpha = \arg \left(-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*} \right),$$
$$\beta = \pi - \arg \left(\frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right),$$
$$\gamma = \arg \left(-\frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right).$$



- The angle β/ϕ_1 is directly the weak mixing phase of B^0 mixing
- The angle γ is the weak phase at work in the charmless decays of b-hadrons
- The angle α is nothing else than $(\pi - \beta - \gamma)$ and can be exhibited in processes where both charmless decays and mixing are present

The unitarity triangle: measurements



CP asymmetry in charmless
b-hadron decays

Overall normalization
given by $|V_{cd} V_{cb}^*|$,
hence semileptonic b decays

CP asymmetry in
mixing processes

Angles

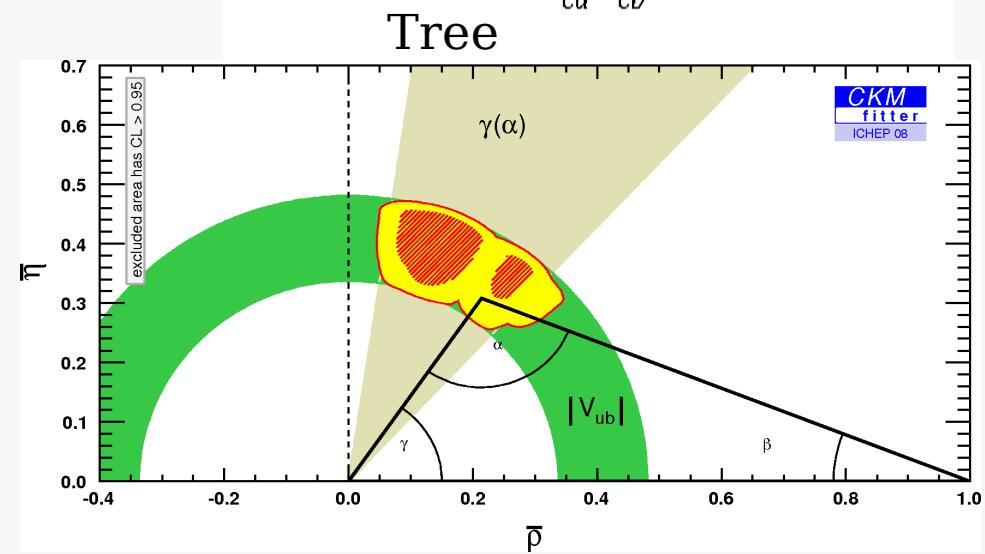
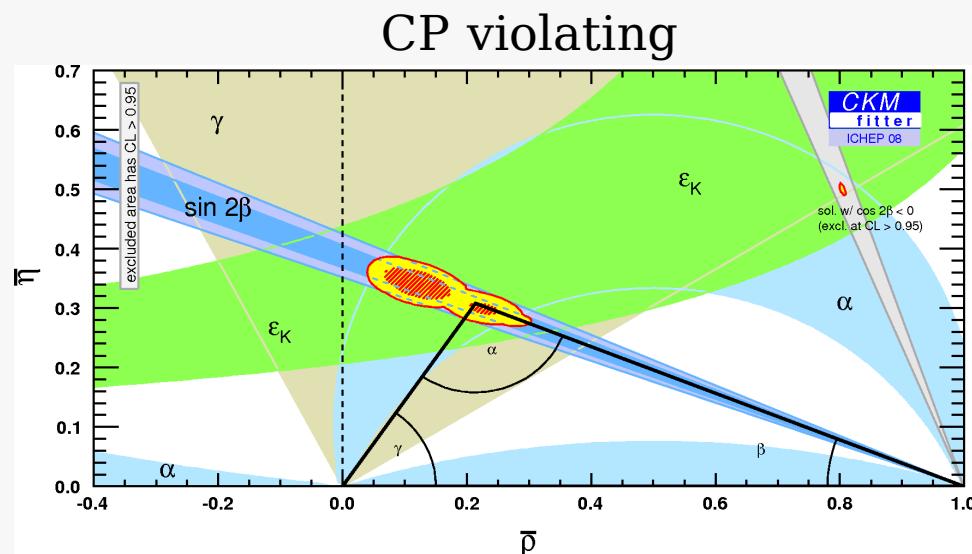
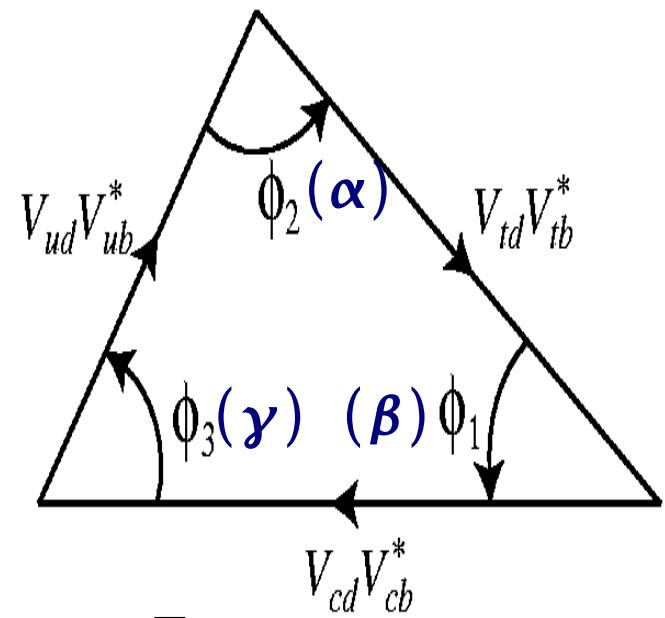
β , α and γ measurements
 ϕ_1 , ϕ_2 and ϕ_3 measurements

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Motivations

- Overconstrain the CKM matrix: measure fundamental parameters, constrain new physics effects
- Measure the 4 free parameters in various ways:
 - CP conserving $\{|V_{us}|, |V_{cb}|, |V_{td}|, |V_{ub}|\}$
 - CP violating $\{\epsilon_K, \phi_s, \beta, \gamma\}$
 - Tree level $\{..., ..., |V_{ub}|, \gamma\}$
 - Loop level $\{..., ..., |V_{td}|, \beta\}$
 - ...

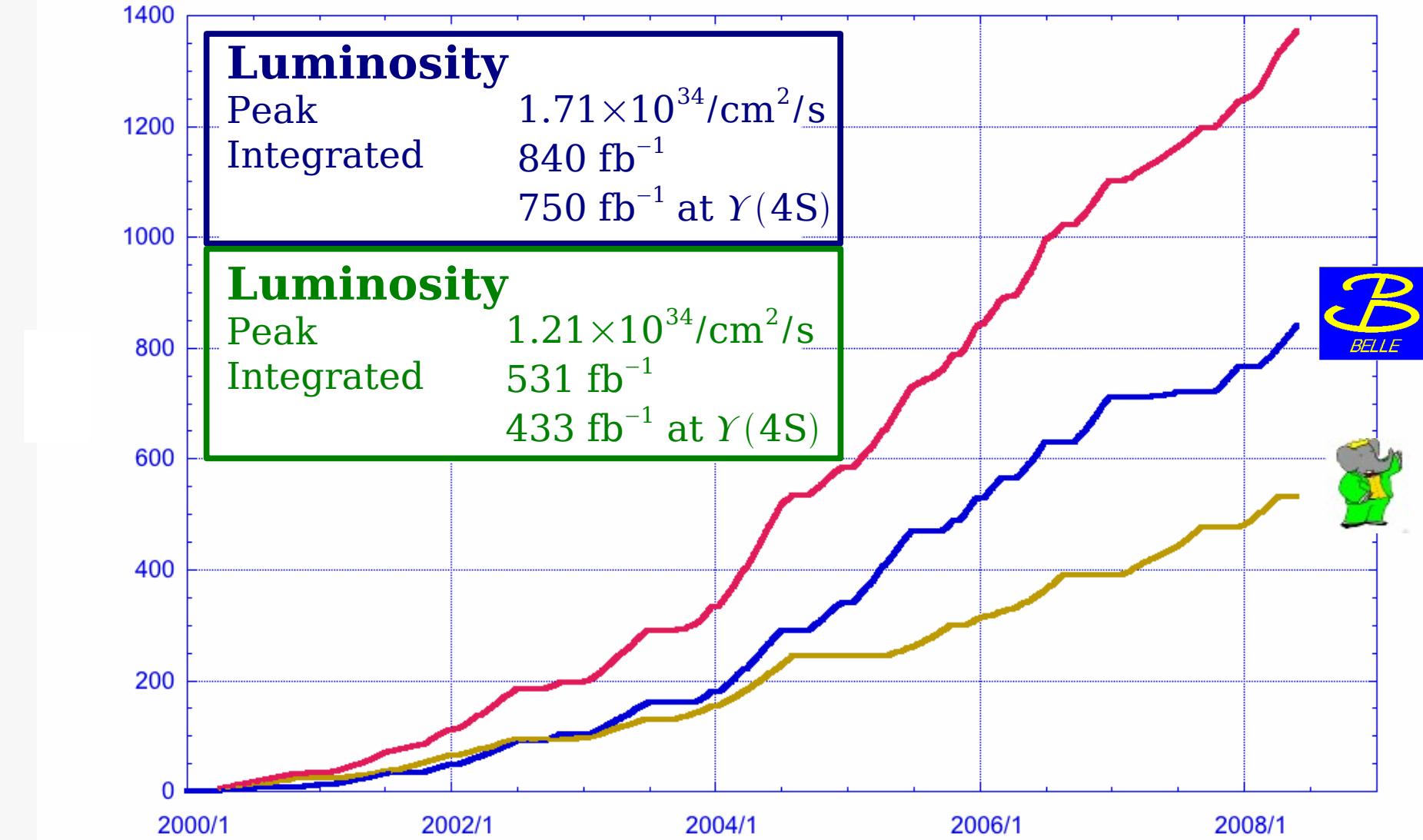


B factories: BaBar and Belle

⇒ experiments designed for ϕ_1 extraction !

Luminosity (fb^{-1})

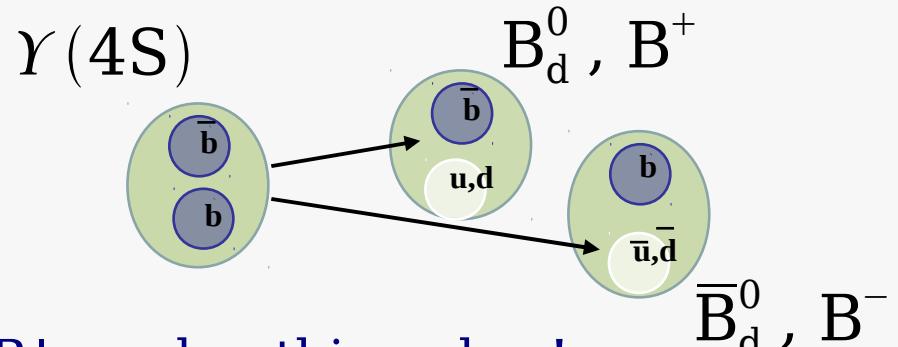
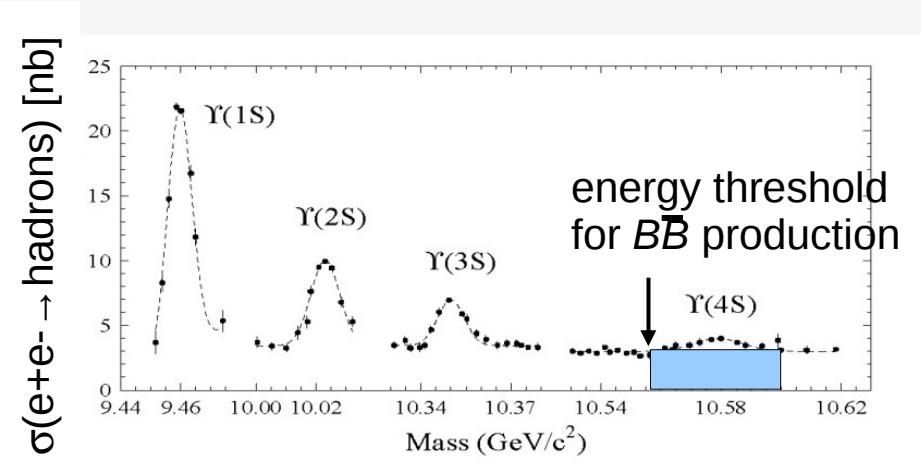
cumulated stat: $\sim 1400 \text{ fb}^{-1}$!!



BaBar: $\sim 465 \times 10^6 B\bar{B}$ pairs = final sample

Belle: $\sim 657 \times 10^6 B\bar{B}$ pairs = max. current sample (final sample will probably be $\sim 800 \times 10^6 B\bar{B}$ pairs)

$\Upsilon(4S)$ B-factory



- 2 B's and nothing else !
- 2 B mesons are created simultaneously in a $L=1$ coherent state
⇒ before first decay, the final states contains a B and a \bar{B}

"on resonance" production

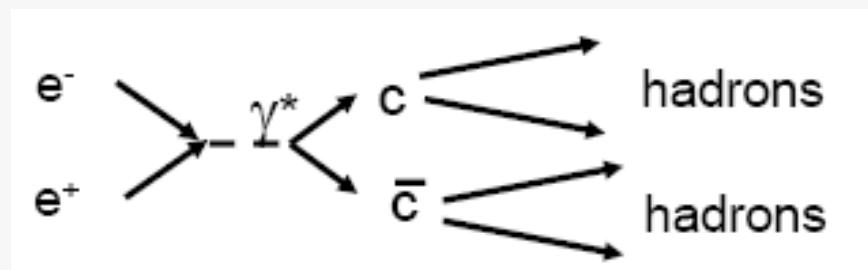
$$e^+e^- \rightarrow \Upsilon(4S) \rightarrow B_d^0 \bar{B}_d^0, B^+ B^-$$

$$\sigma(e^+e^- \rightarrow B\bar{B}) \simeq 1.1 \text{ nb} (\sim 10^9 B\bar{B} \text{ pairs})$$

"continuum" production

$$\sigma(e^+e^- \rightarrow c\bar{c}) \simeq 1.3 \text{ nb} (\sim 1.3 \times 10^9 X_c \bar{Y}_c \text{ pairs})$$

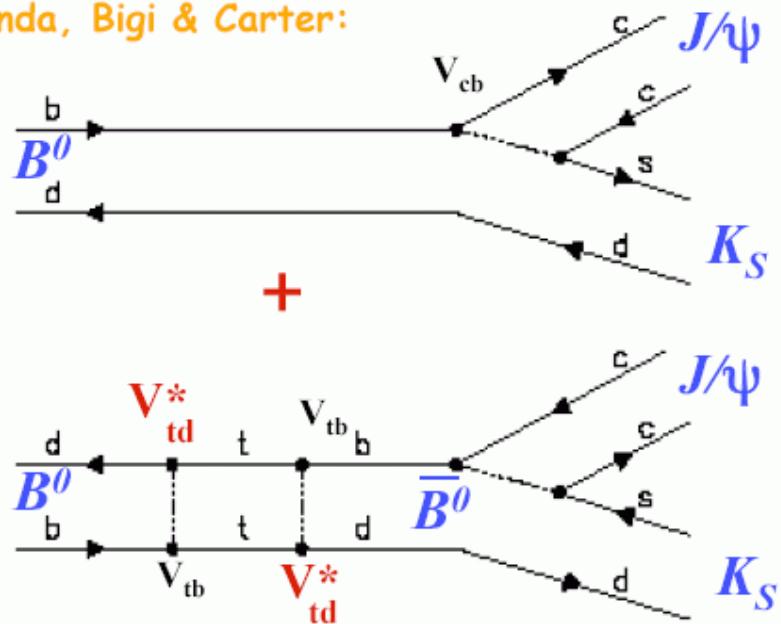
$\tau\tau$ production also !



Time-dependent CP asymmetries in decays to CP eigenstates

$\sin 2\phi_1$ from $B \rightarrow f_{CP} + B \leftrightarrow \bar{B} \rightarrow f_{CP}$ interf.

Sanda, Bigi & Carter:



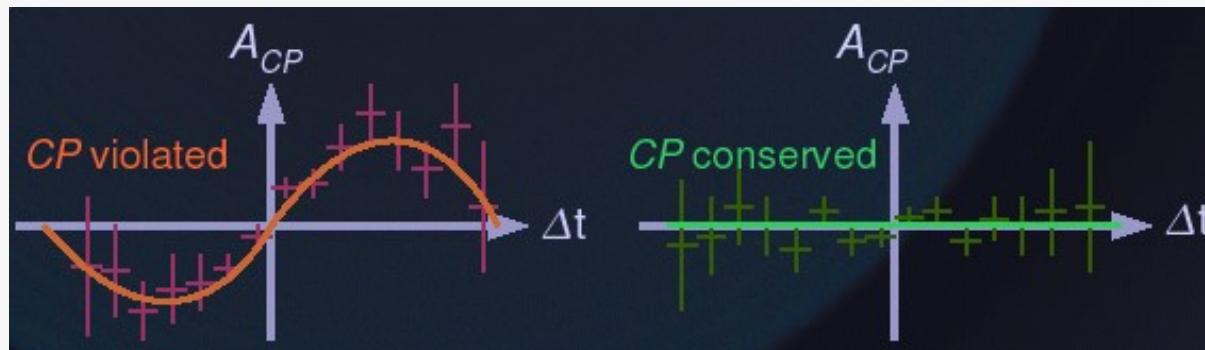
$$A_{CP}(f; t) = \frac{N(\bar{B}^0(t) \rightarrow f) - N(B^0(t) \rightarrow f)}{N(\bar{B}^0(t) \rightarrow f) + N(B^0(t) \rightarrow f)}$$

$$= \mathbf{S} \sin \Delta m_d t + \mathbf{A} \cos \Delta m_d t$$

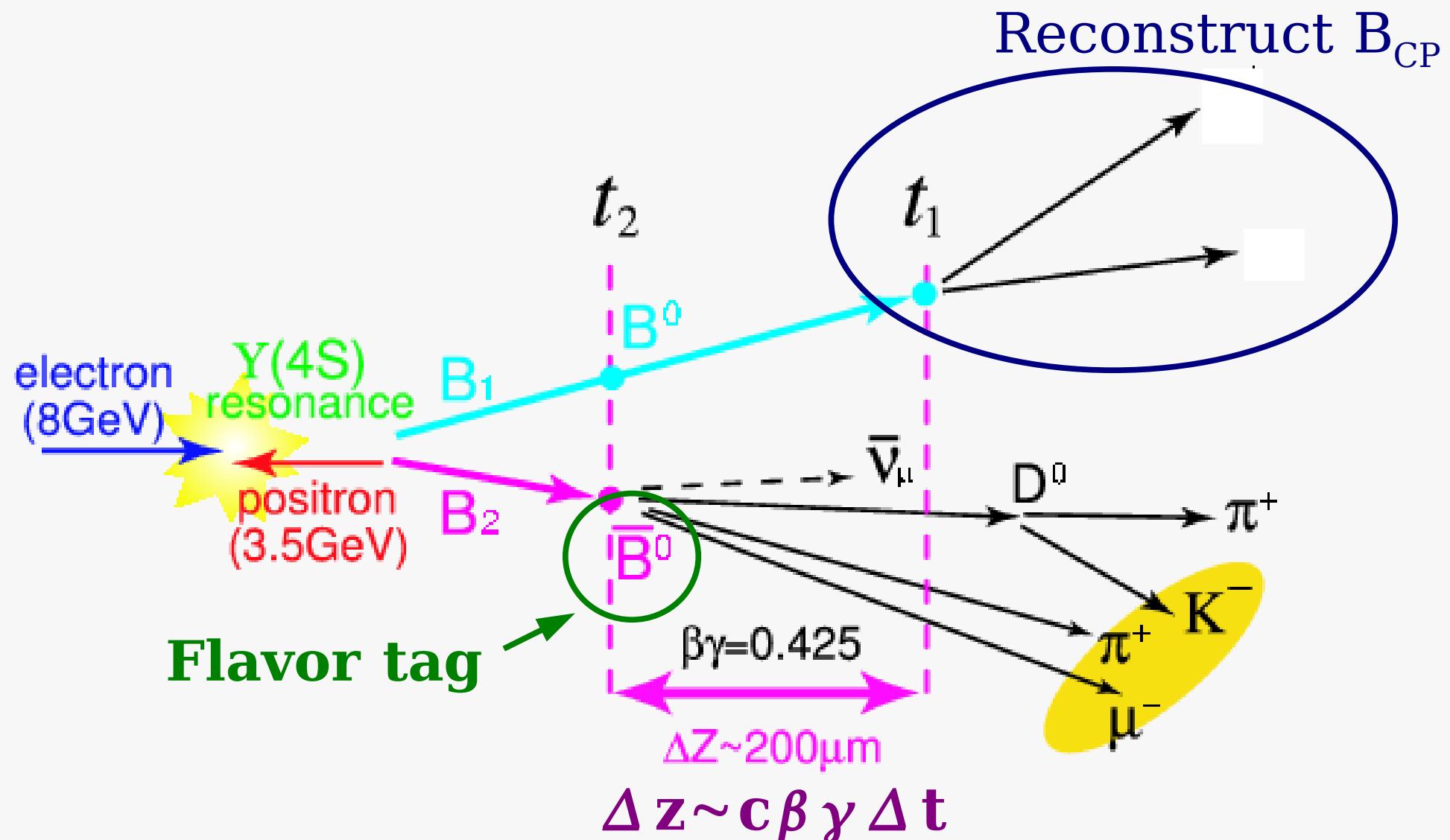
$$= \frac{2 \operatorname{Im} \lambda}{|\lambda|^2 + 1} \sin \Delta m_d t + \frac{|\lambda|^2 - 1}{|\lambda|^2 + 1} \cos \Delta m_d t$$

$$\lambda = \frac{q}{p} \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)} = e^{-i 2 \phi_i} \frac{\bar{A}_f}{A_f}$$

- $\mathbf{A} = 0$ and $\mathbf{S} = -\xi_f \sin 2\beta$ for $(c\bar{c})K_{S/L}$ ($\xi_f = \mp 1$)
- $\mathbf{A} = 0$ and $\mathbf{S} = \sin 2\alpha$ for $\pi^+ \pi^-$ (if tree only)



Measuring the CP parameters **S** and **A**

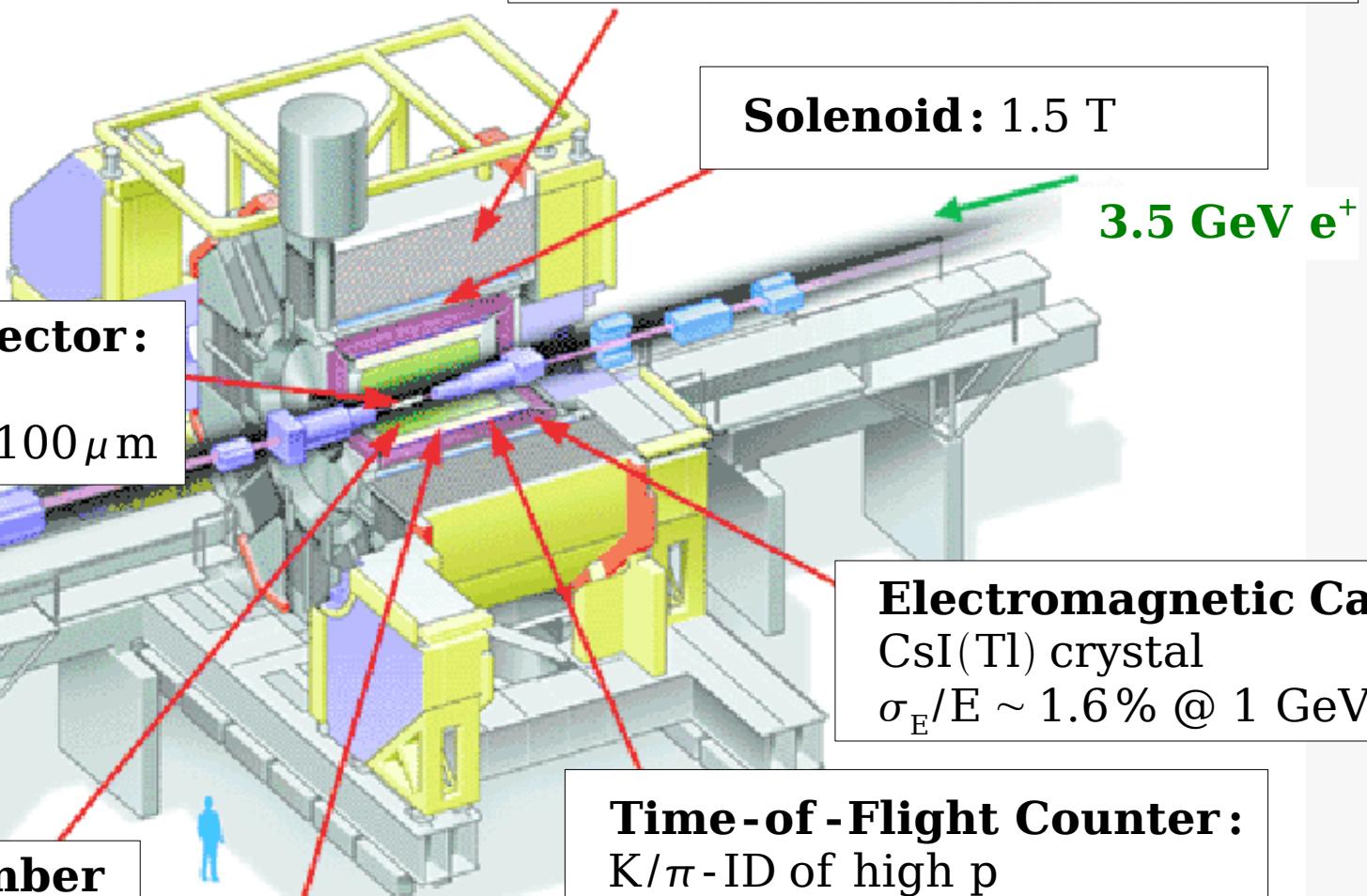


$$\frac{dP_{\text{sig}}}{dt}(\Delta t, \mathbf{q}) = \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B} (1 + \mathbf{q}(\mathbf{S} \sin(\Delta m_d \Delta t) + \mathbf{A} \cos(\Delta m_d \Delta t)))$$

Belle in a nutshell



KLM ($K_L\mu$) Detector: Sandwich of 14 RPCs and 15 iron plates



Silicon Vertex Detector:
3/4 detection layers
Vertex resolution $\sim 100 \mu m$

Solenoid: 1.5 T

3.5 GeV e^+

Central Drift Chamber
8,400 sense wires
PID with dE/dx

Electromagnetic Cal:
CsI(Tl) crystal
 $\sigma_E/E \sim 1.6\% @ 1 \text{ GeV}$

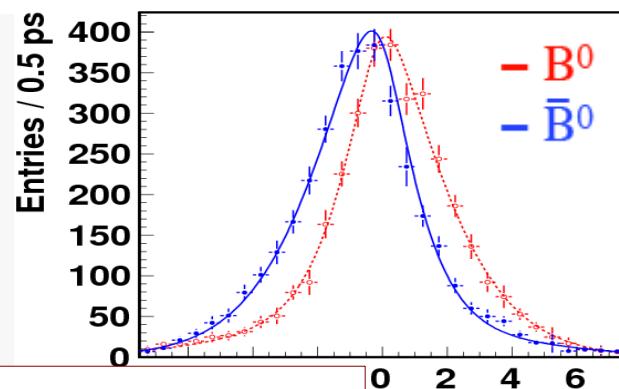
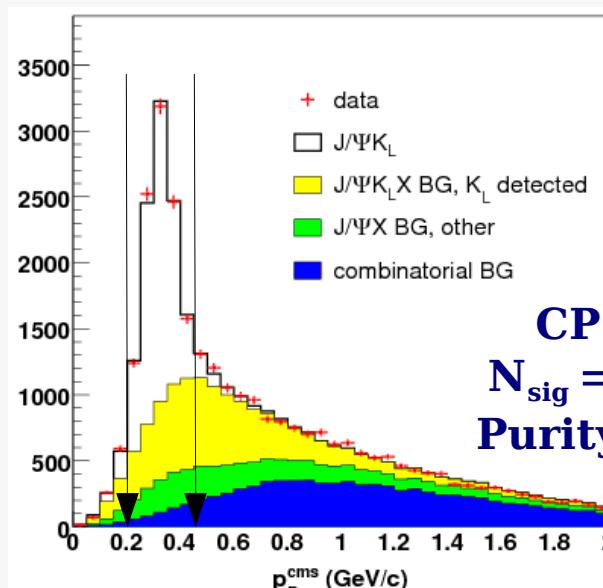
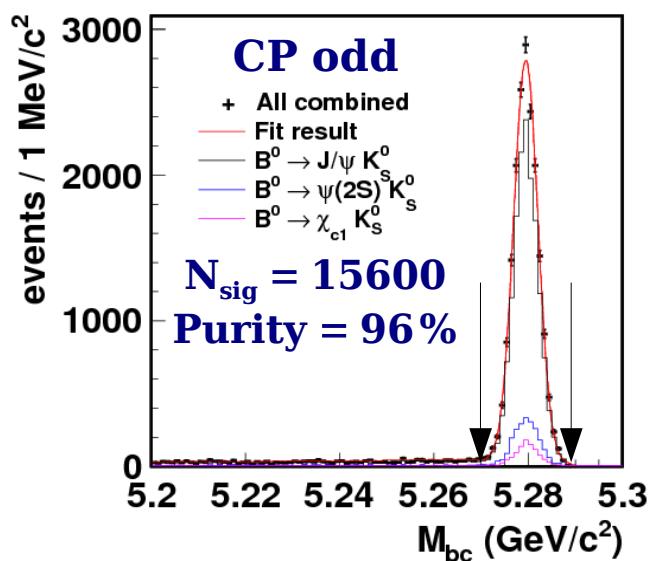
Time-of-Flight Counter:
 K/π -ID of high p

Aerogel Cerenkov Counter:
Refractive index $n=1.01-1.03$
 K/π of middle p

very stable detector, good particle identification, (kaon, pion, electron, muon),
 e^+e^- is a clean environment: excellent tracking, triggering, tagging...

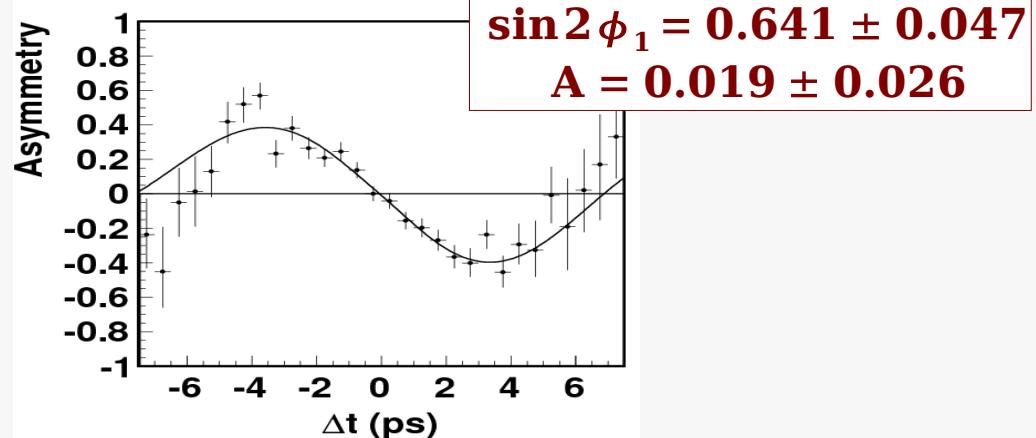
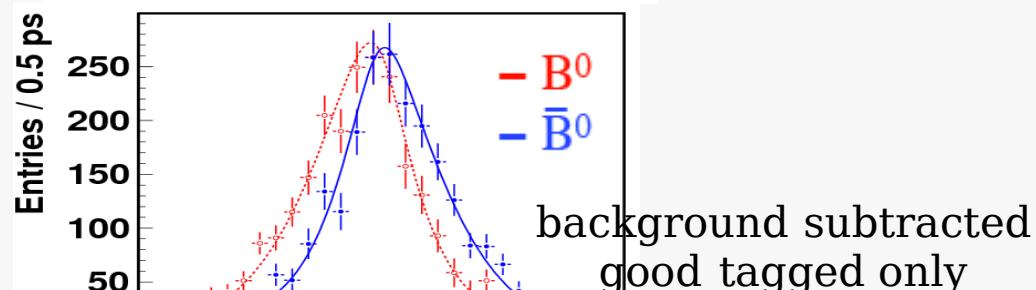
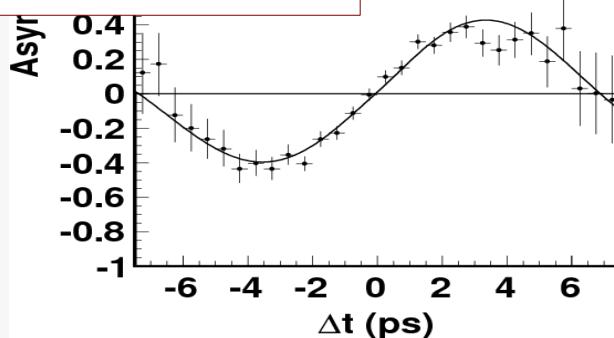
$c\bar{c}$ K_S and J/ψ K_L

772×10^6 $B\bar{B}$ pairs

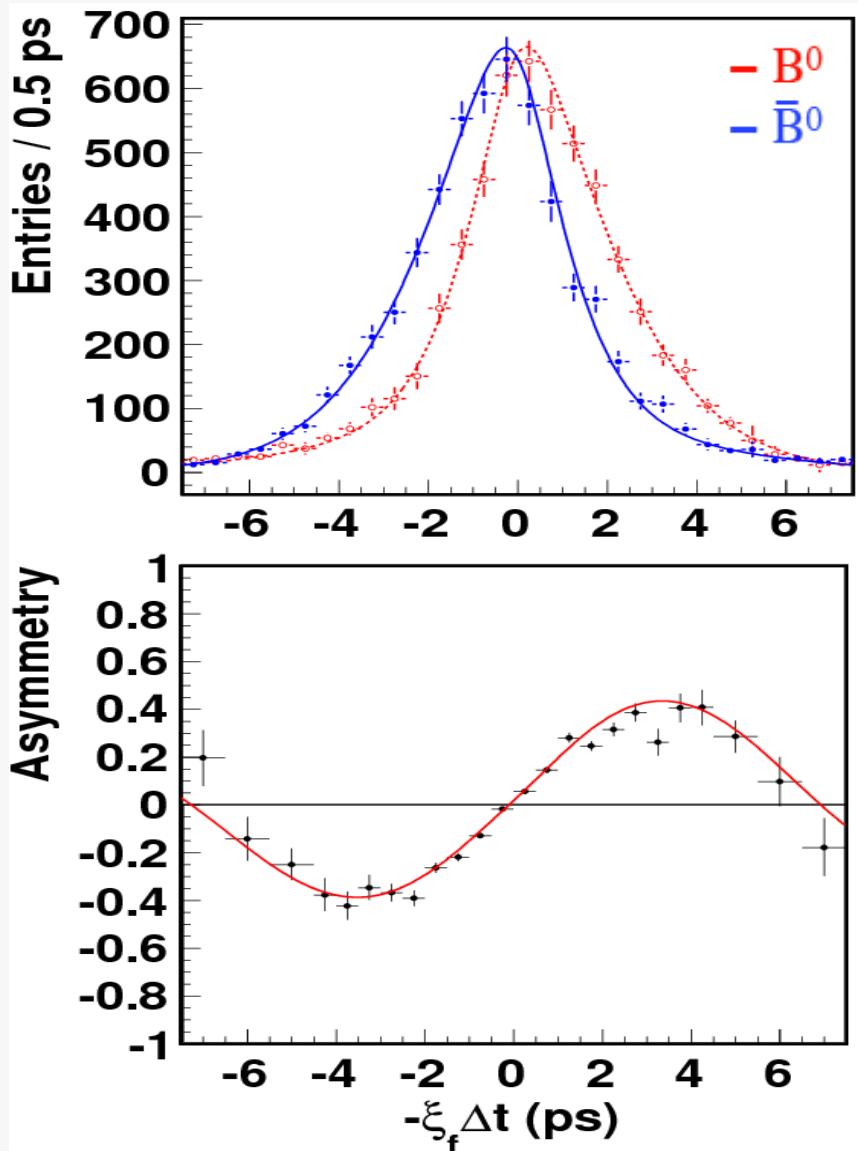


$$\sin 2\phi_1 = 0.671 \pm 0.029$$

$$A = -0.014 \pm 0.021$$



$\sin 2\phi_1$ in $(c\bar{c})K^0 \dots$ $772 \times 10^6 B\bar{B}$ pairs



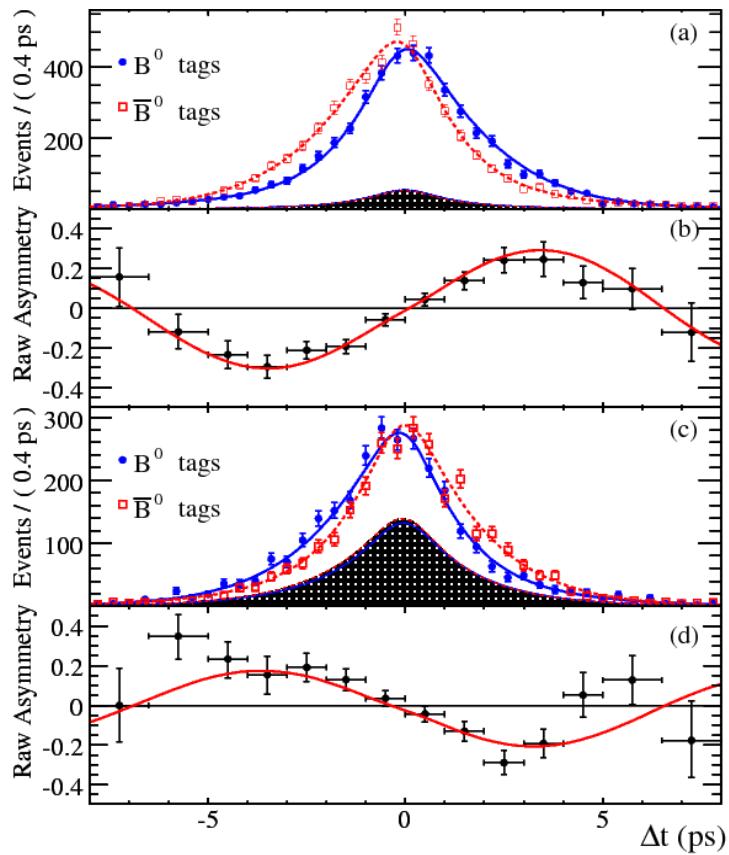
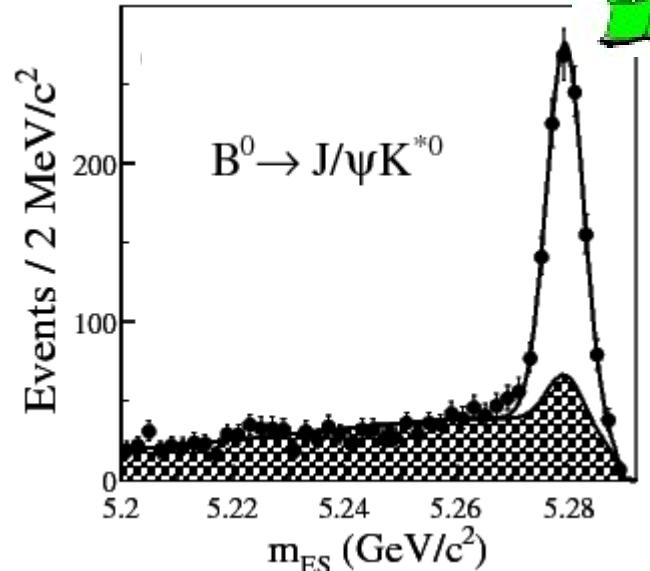
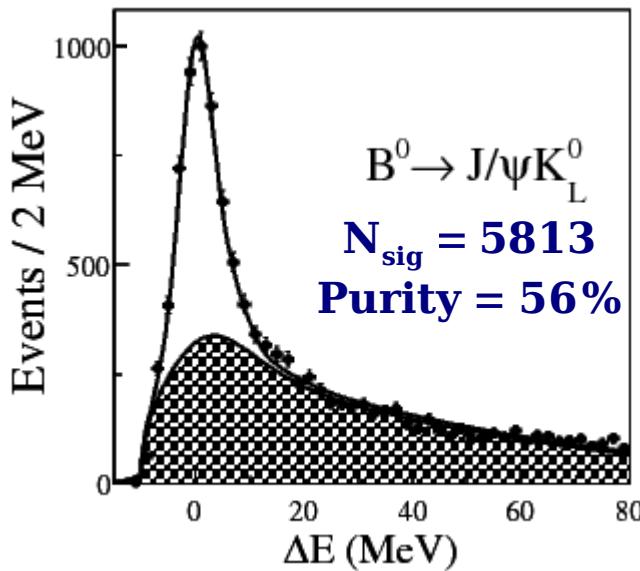
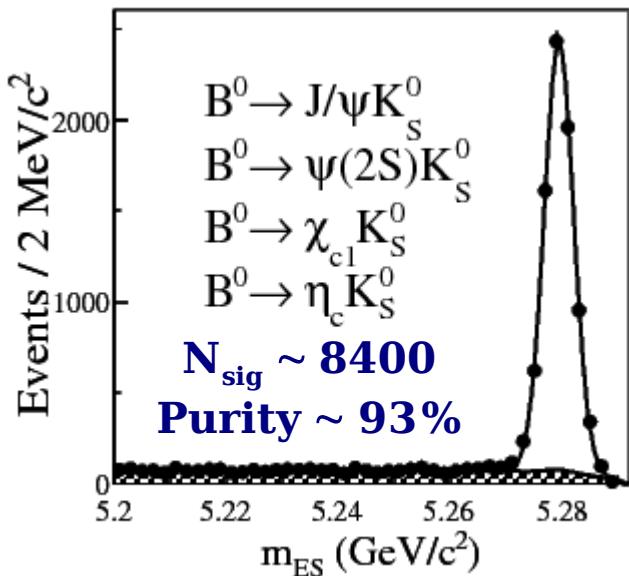
$$\sin 2\phi_1 = 0.668 \pm 0.023 \pm 0.013$$

$$A = 0.007 \pm 0.016 \pm 0.013$$

- World's most precise measurements
- anchor point of the SM
- still statistically limited !

$\sin 2\beta$ in $(c\bar{c}) K^{(*)0}$

$465 \times 10^6 B\bar{B}$ pairs
[ArXiv:0902.1708]



Mode	$\sin 2\beta$
$J/\psi K_S$	$0.657 \pm 0.036 \pm 0.012$
$J/\psi K_L$	$0.694 \pm 0.061 \pm 0.031$
$J/\psi K^0$	$0.666 \pm 0.031 \pm 0.013$
$\psi(2S) K_S$	$0.897 \pm 0.100 \pm 0.036$
$\chi_{c1} K_S$	$0.614 \pm 0.160 \pm 0.040$
$\eta_c K_S$	$0.925 \pm 0.160 \pm 0.057$
$J/\psi K^{*0}$	$0.601 \pm 0.239 \pm 0.087$
$c\bar{c} K^{(*)0}$	$0.687 \pm 0.028 \pm 0.012$

Charmonium K⁰ Systematics



Preliminary!

Systematic errors:

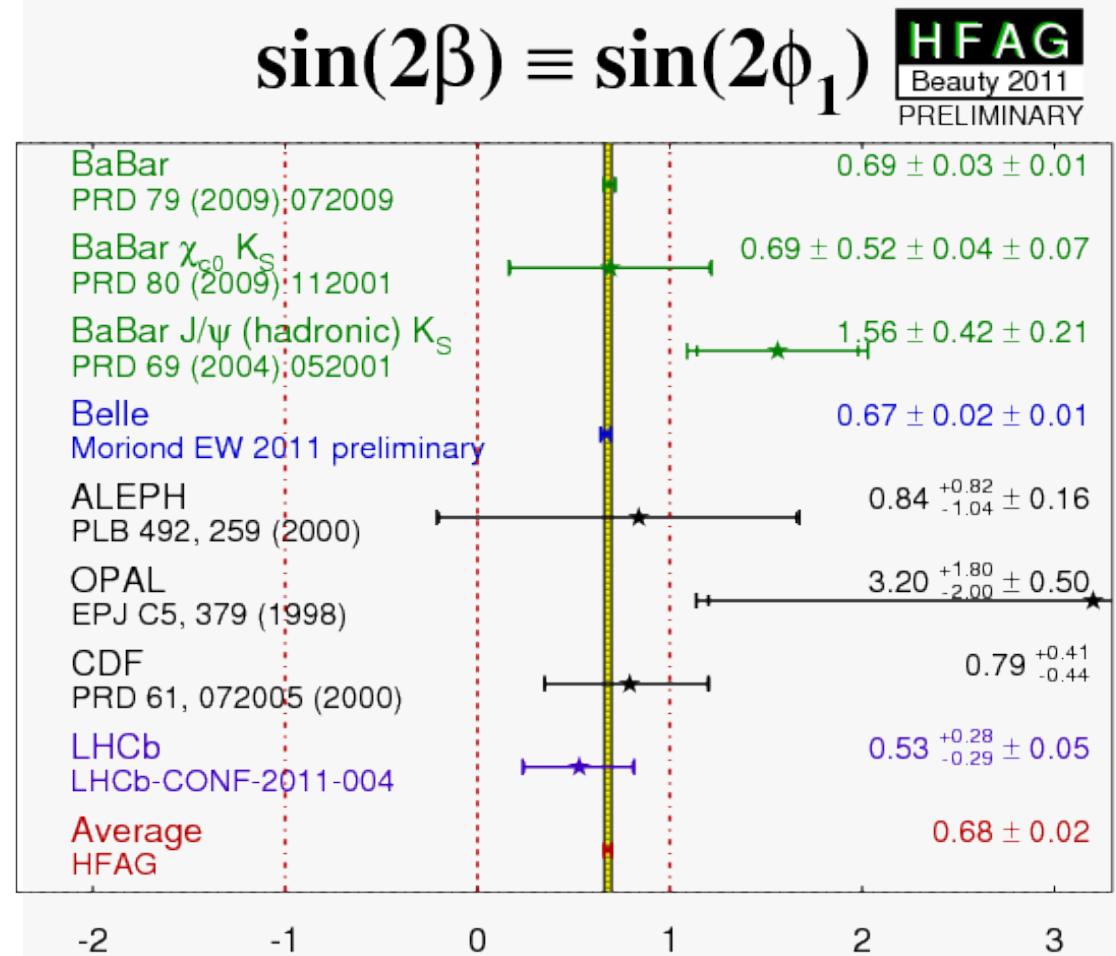
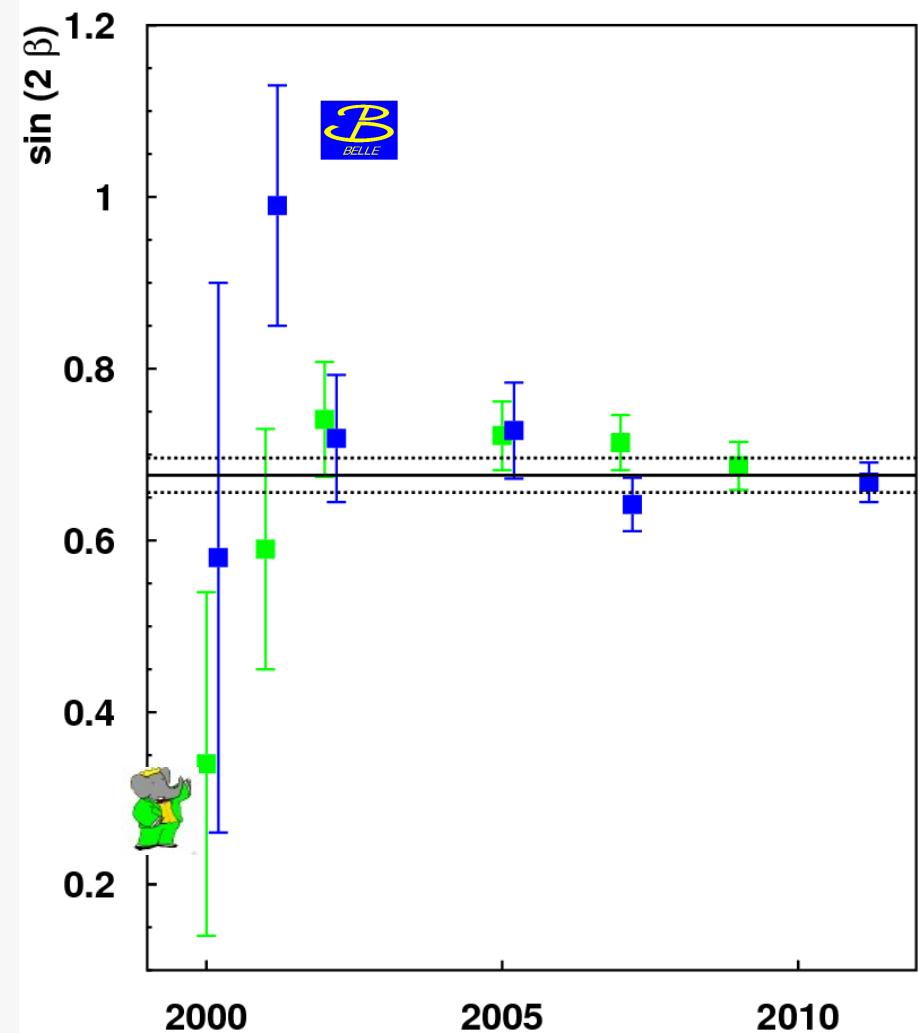
	ΔS	ΔA
Vertexing	+0.006 -0.009	± 0.008
Flavor tagging	+0.004 -0.003	± 0.003
Resolution function	± 0.007	± 0.001
Physics parameters	± 0.001	< 0.001
Fit bias	± 0.004	± 0.005
$J/\psi K_S^0$ signal fraction	± 0.002	± 0.001
$J/\psi K_L^0$ signal fraction	± 0.004	± 0.000 ± 0.002
$\psi(2S)K_S^0$ signal fraction	< 0.001	< 0.001
$\chi_{c1}K_S^0$ signal fraction	< 0.001	< 0.001
Background Δt	± 0.001	< 0.001
Tag-side interference	± 0.001	± 0.008
Total	± 0.013	± 0.013

2011

2006

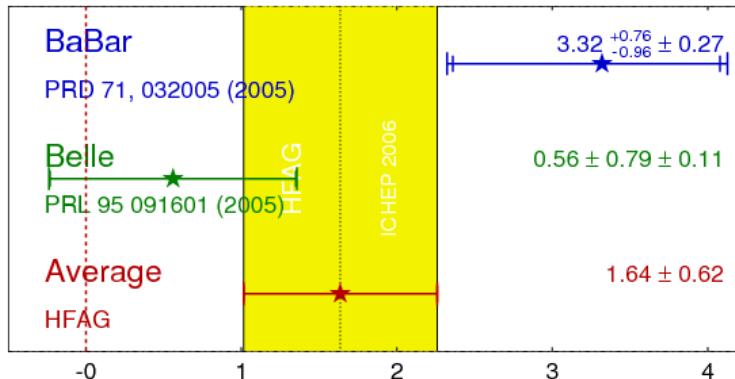
	ΔS	ΔA
vertexing	0.012	0.009
(K_S : 0.013)	0.021	
Flv tag	0.004	0.003
Res. func.	0.006	0.001
Phys.	0.001	0.001
Fit bias	0.007	0.004
K_S frac.	0.003	0.001
K_L frac.	0.005	0.002
BG dt	0.001	0.001
T.S.I.	0.001	0.009
total	<hr/>	<hr/>
	0.017	0.014

La raison d'être of the B factories

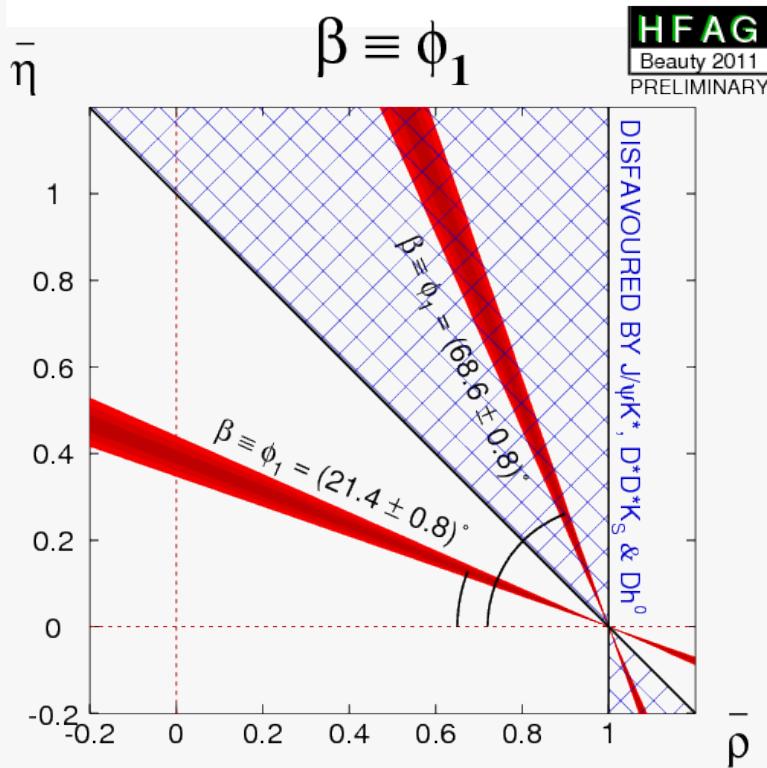
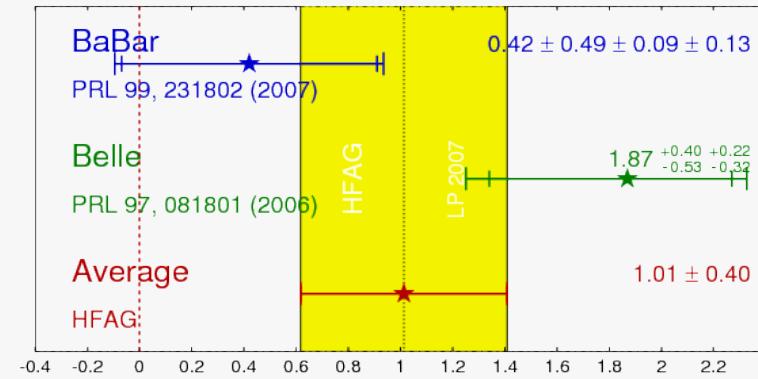


La raison d'être of the B factories

$J/\psi K^* \cos(2\beta) \equiv \cos(2\phi_1)$ **HFAG**
ICHEP 2006
PRELIMINARY



$D^{(*)} h^0 \cos(2\beta) \equiv \cos(2\phi_1)$ **HFAG**
LP 2007
PRELIMINARY



$$\beta = (21.4 \pm 0.8)^\circ$$

What is the source of CP violation ?
The Kobayashi-Maskawa phase is the source

Checking the quality of gold

$B^0 \rightarrow J/\psi K_S$ is a golden mode for $\sin 2\phi_1$

LP2011, T.Gershon

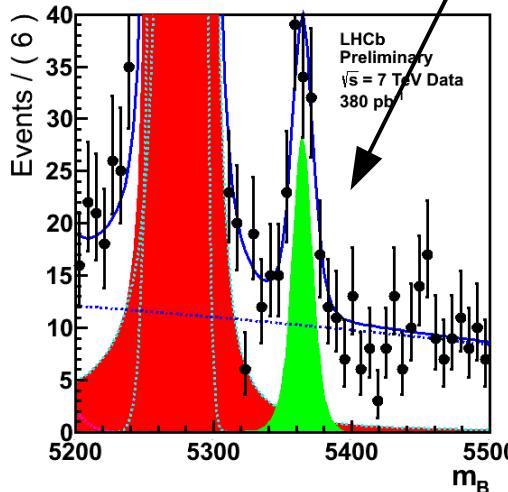
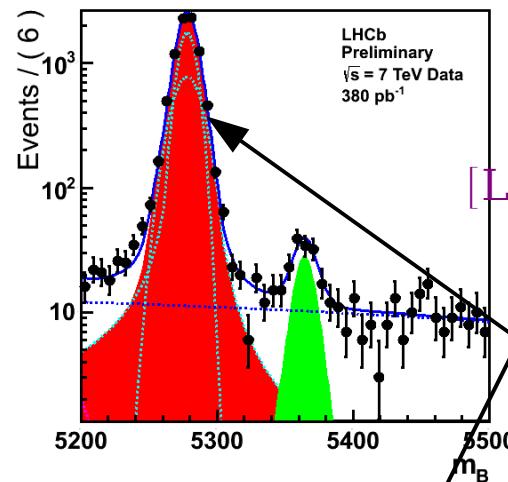
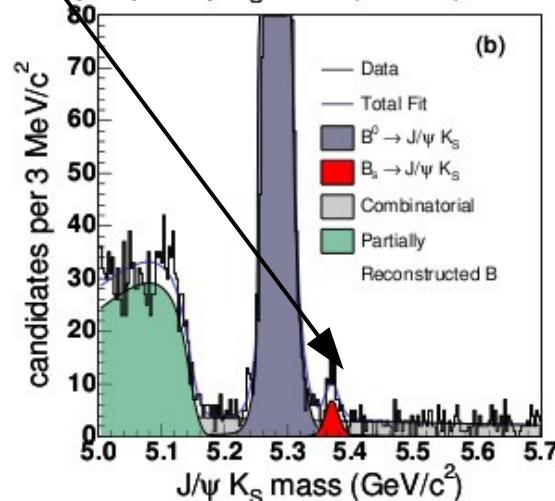
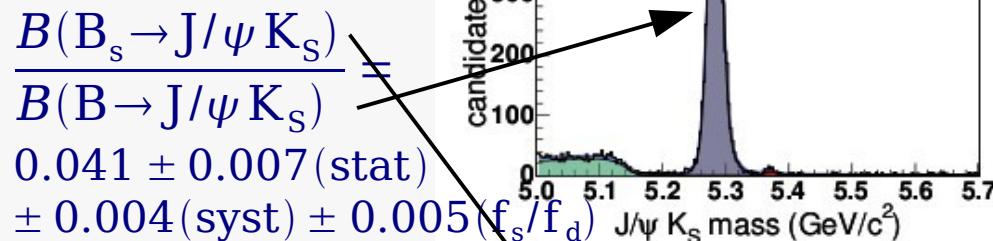
Can check purity using flavour symmetries

- $B^0 \rightarrow J/\psi \pi^0$ (related by SU(3))
- $B_s^0 \rightarrow J/\psi K_S$ (related by U spin)



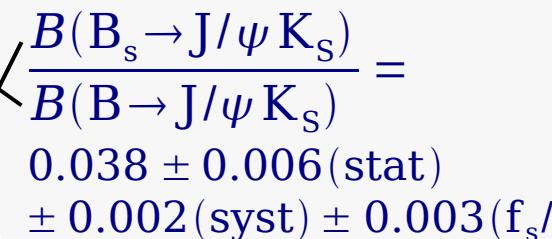
CDF

[PRD 83(2011)052012]



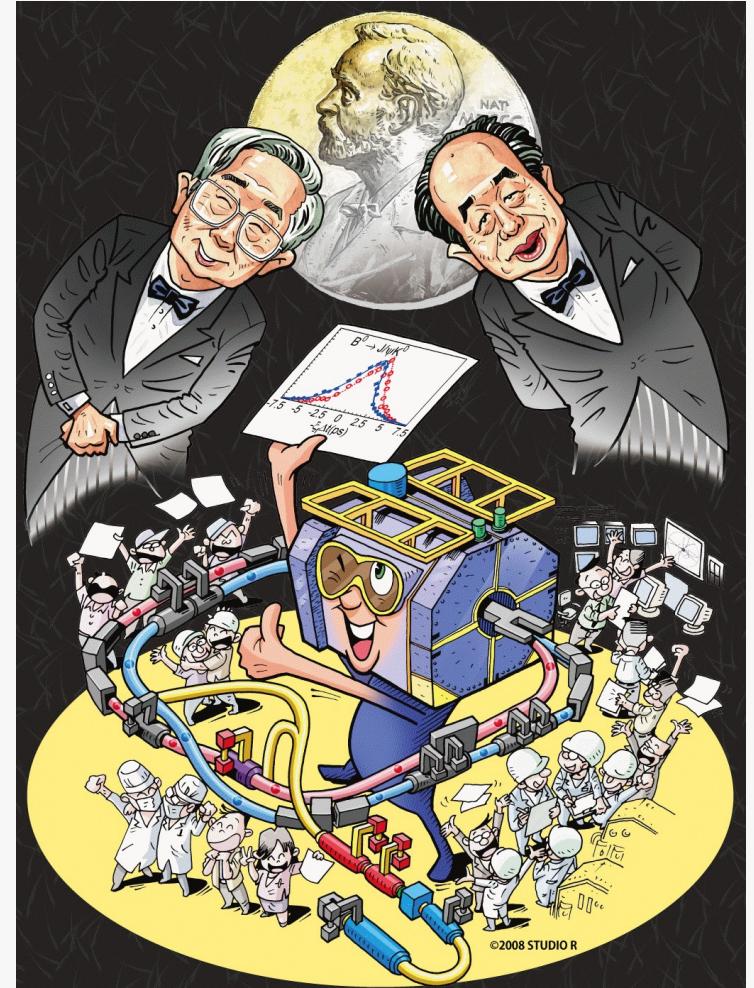
LHCb

[LHCb-CONF-2011-048]

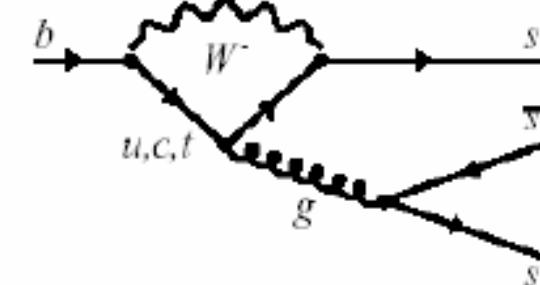
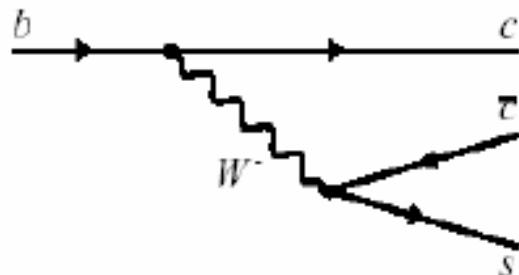


Critical role of the B factories in
the verification of the KM hypothesis

**A single irreducible phase in the
weak interaction matrix accounts
for most of the CPV observed
in kaons and B's**



β in other modes



$J/\psi K_S^0, \psi(2S)K_S^0, \chi_{c1}K_S^0,$
 $\eta_c K_S^0, J/\psi K_L^0,$
 $J/\psi K^{*0} (K^{*0} \rightarrow K_S^0 \pi^0)$

$D^{*+}D^-, D^+D^-$
 $J/\psi \pi^0, D^{*+}D^{*-}$

$\phi K^0, K^+ K^- K_S^0,$
 $K_S^0 K_S^0 K_S^0, \eta' K^0, K_S^0 \pi^0,$
 $\omega K_S^0, f_0(980) K_S^0$

increasing tree diagram amplitude

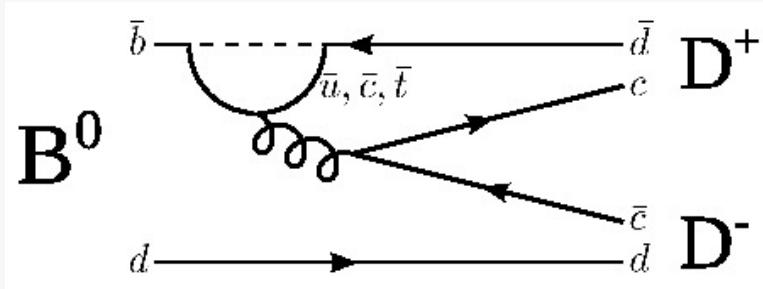
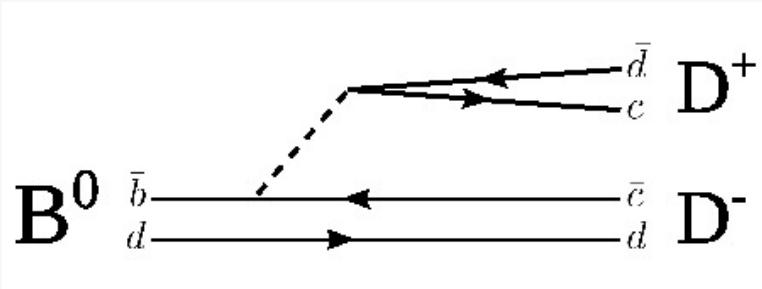
increasing sensitivity to new physics

possible new sources of CPV ?

Recent update of $B^0 \rightarrow D^+ D^-$ mode



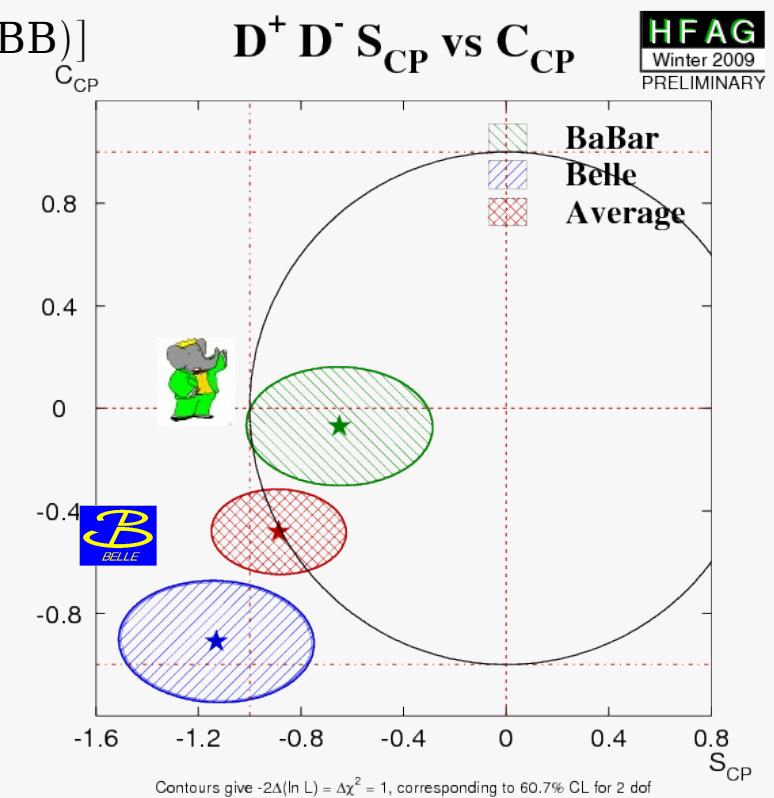
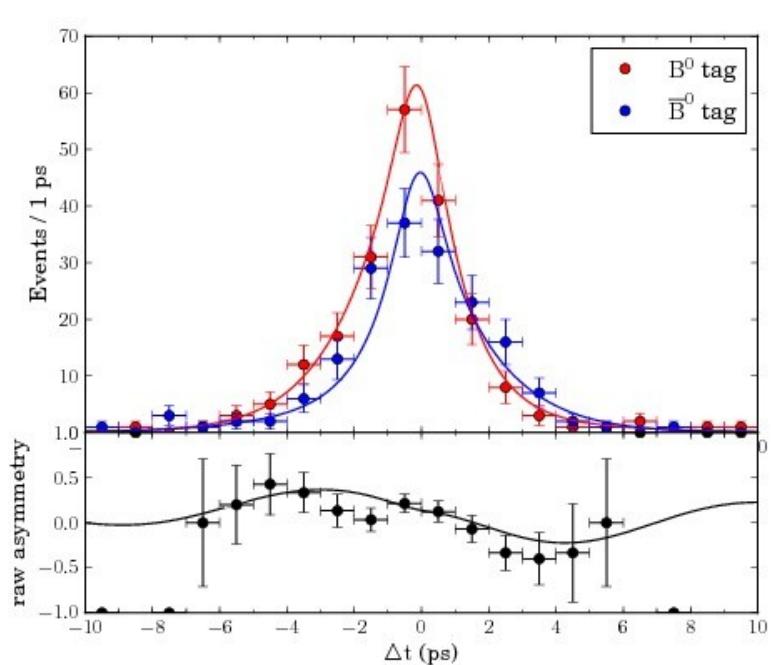
$772 \times 10^6 B\bar{B}$ pairs
preliminary
shown at EPS11



SM prediction: $S = -\sin 2\beta$ and $A=0$ [Z.Z Xing , PRD61 , 014010 (1999)]

$$\begin{aligned} B^0 \rightarrow D^+ D^- &\rightarrow (K^- \pi^+ \pi^+) (K^+ \pi^- \pi^-) \\ &\rightarrow (K^- \pi^+ \pi^+) (K_S^0 \pi^-) \end{aligned}$$

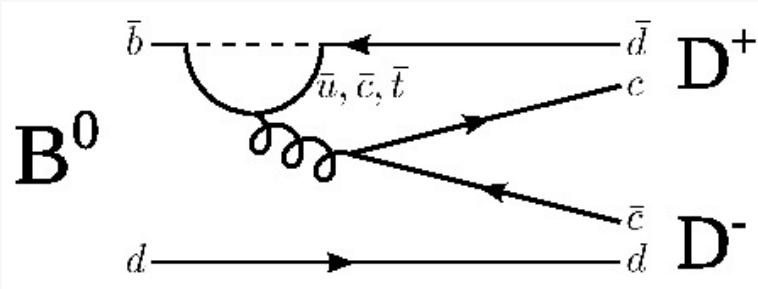
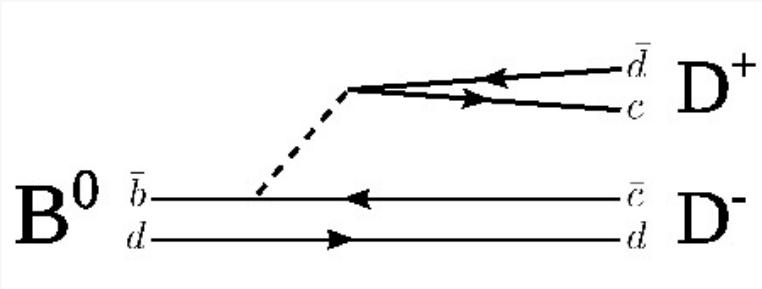
[$> \times 2$ signal yield compared to previous analysis (535 MBB)]



Recent update of $B^0 \rightarrow D^+ D^-$ mode



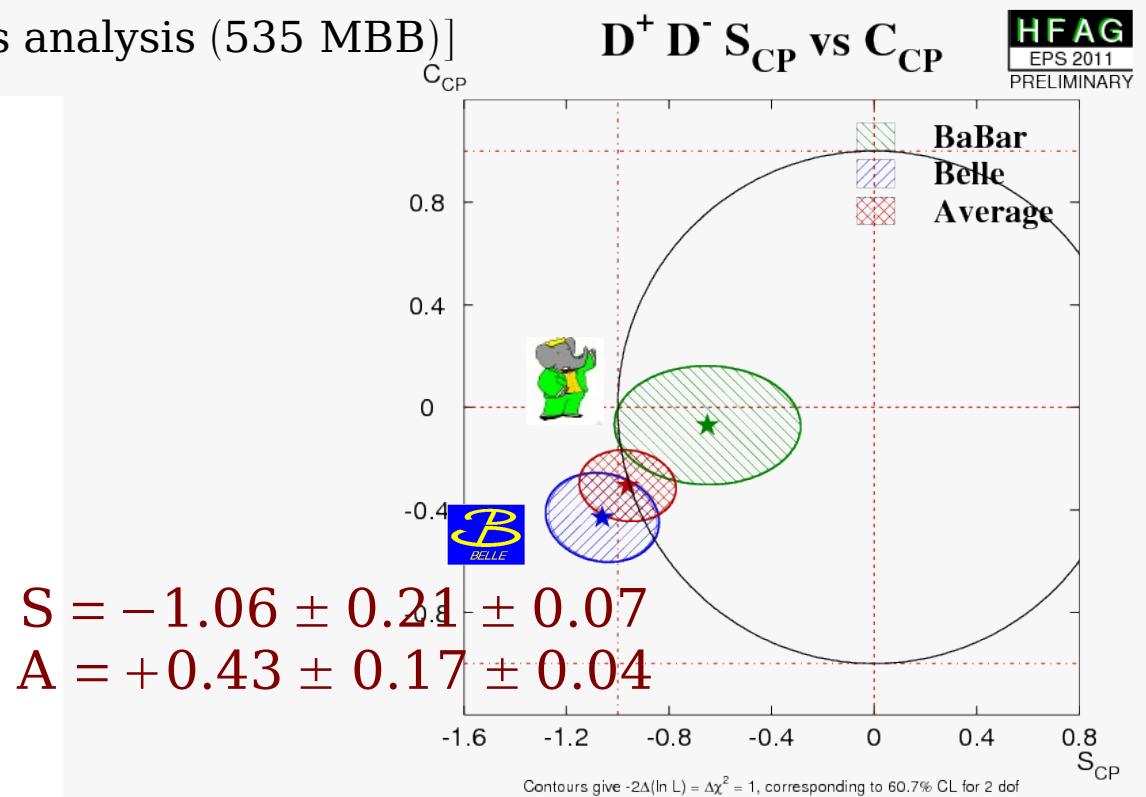
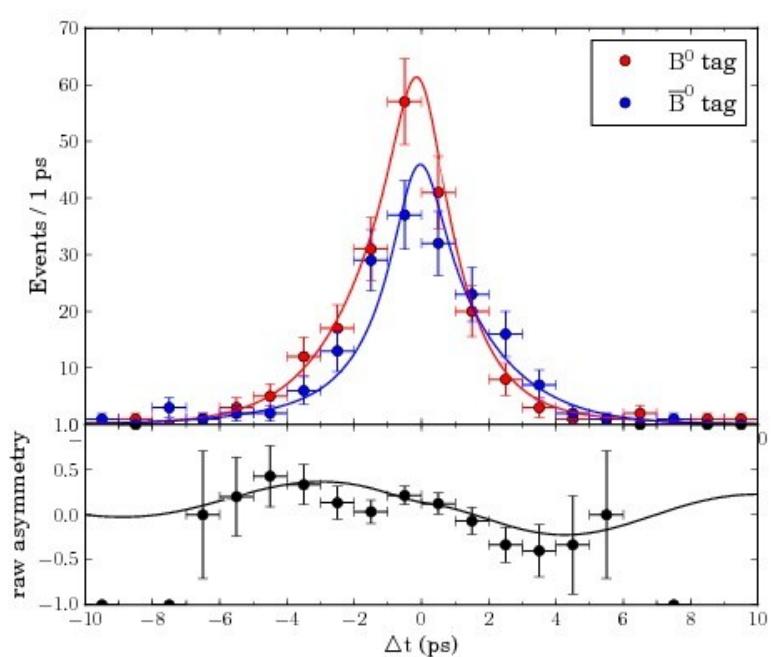
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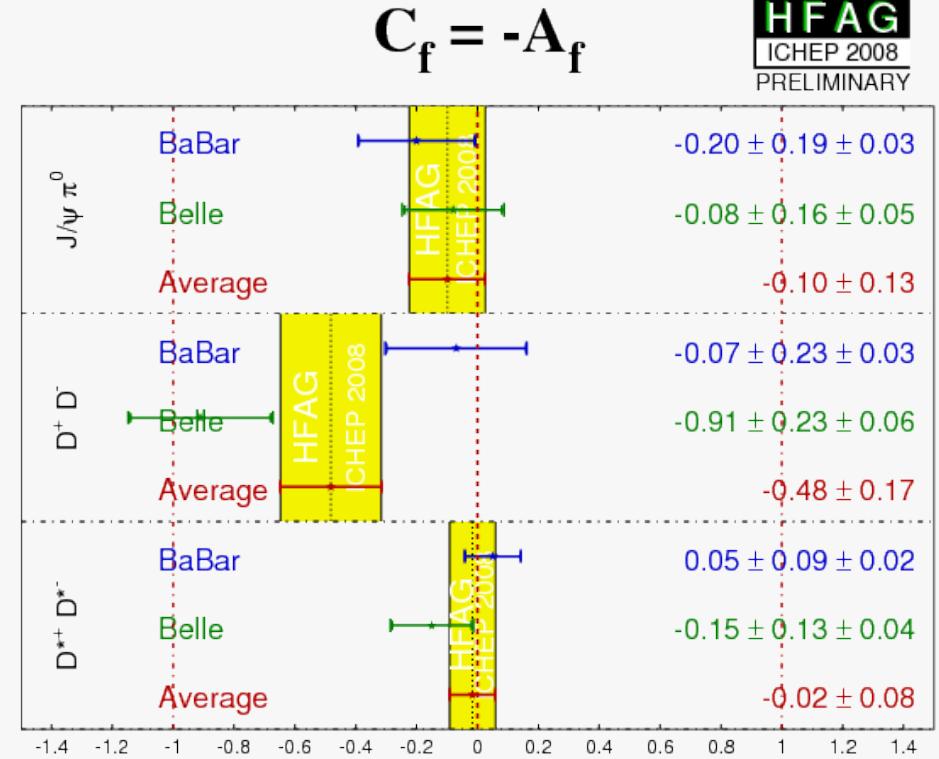
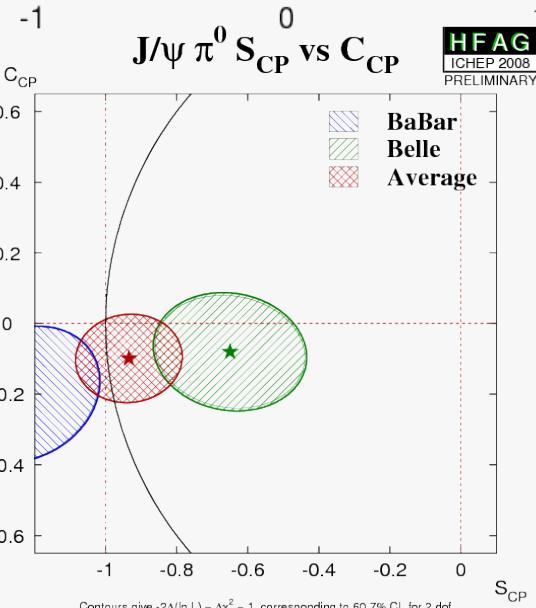
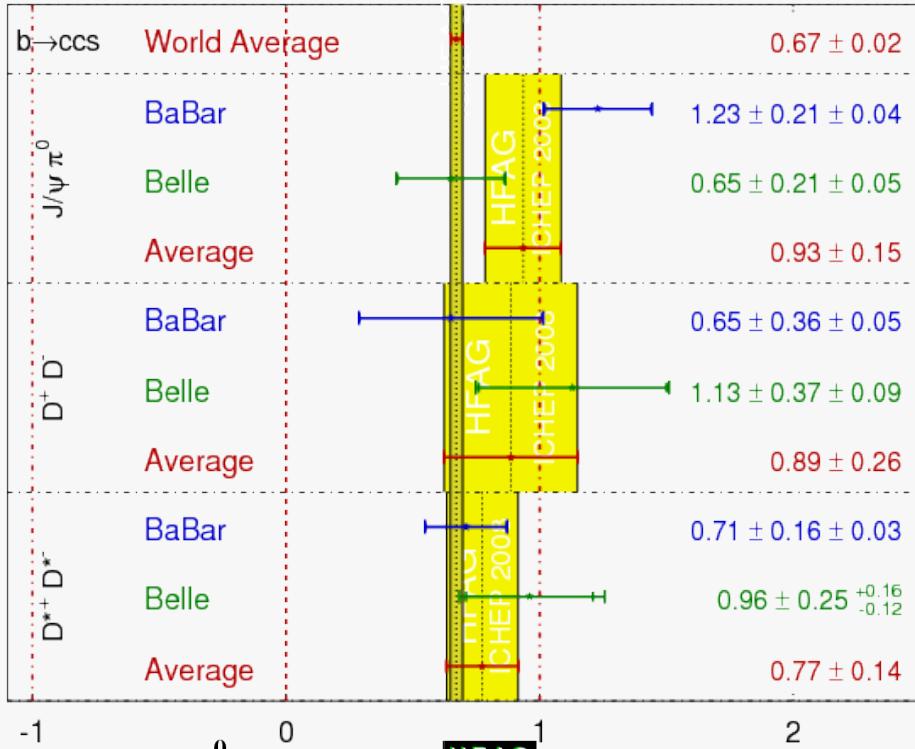
[$> \times 2$ signal yield compared to previous analysis (535 MBB)]



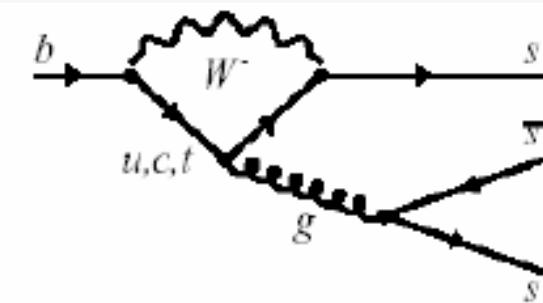
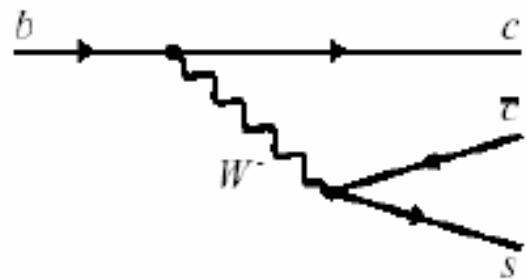
S and C in $b \rightarrow c\bar{c}d$ modes

$$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})$$

HFAG
ICHEP 2008
PRELIMINARY



good agreement with $b \rightarrow c\bar{c}s$ modes result
 $S = -\sin 2\beta, C = 0$
more info needed for C in $D^+ D^-$ mode



$J/\psi K_S^0, \psi(2S)K_S^0, \chi_{c1}K_S^0,$
 $\eta_c K_S^0, J/\psi K_L^0,$
 $J/\psi K^{*0} (K^{*0} \rightarrow K_S^0 \pi^0)$

$D^{*+}D^-, D^+D^-$
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increasing tree diagram amplitude

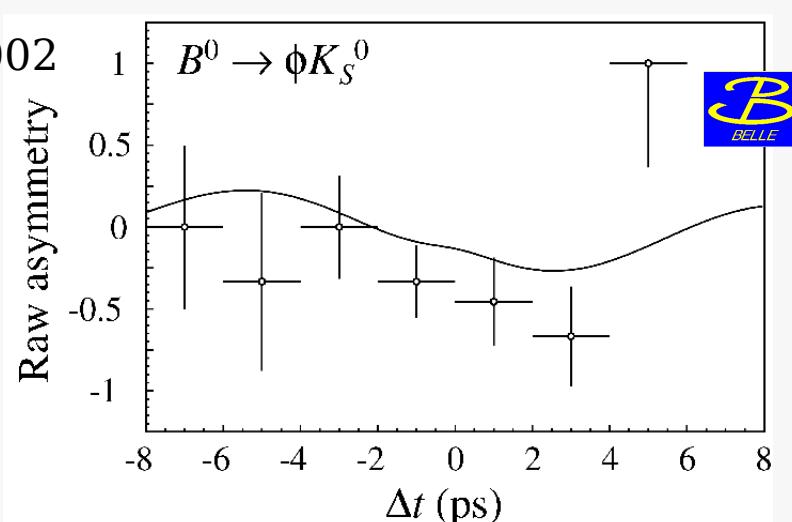


increasing sensitivity to new physics

first reported in Moriond EW 2002

$|\sin 2\beta| = -0.73 \pm 0.64 \pm 0.22$

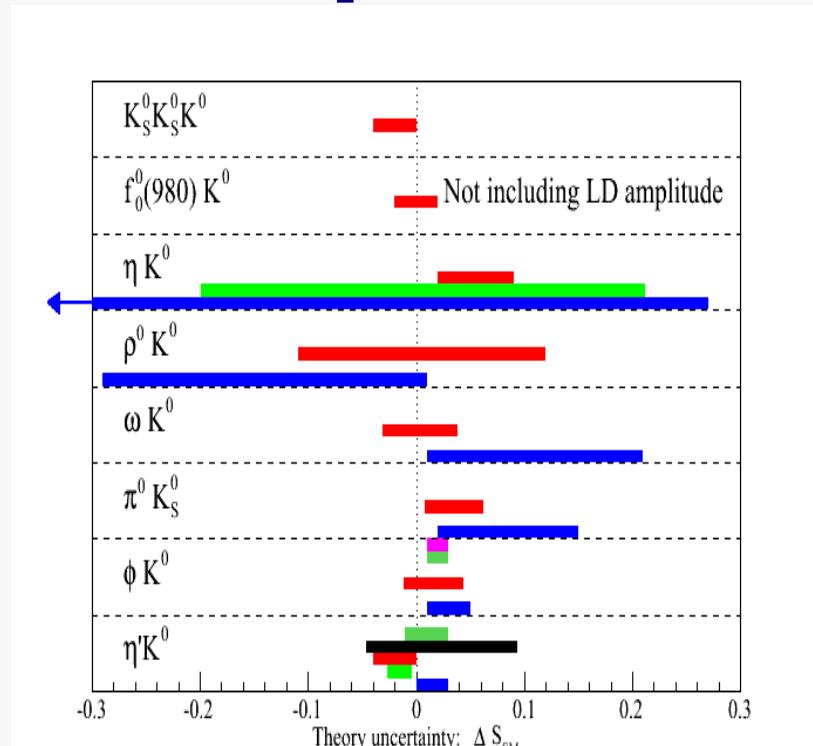
[PRD 67, 031102 (2003)]



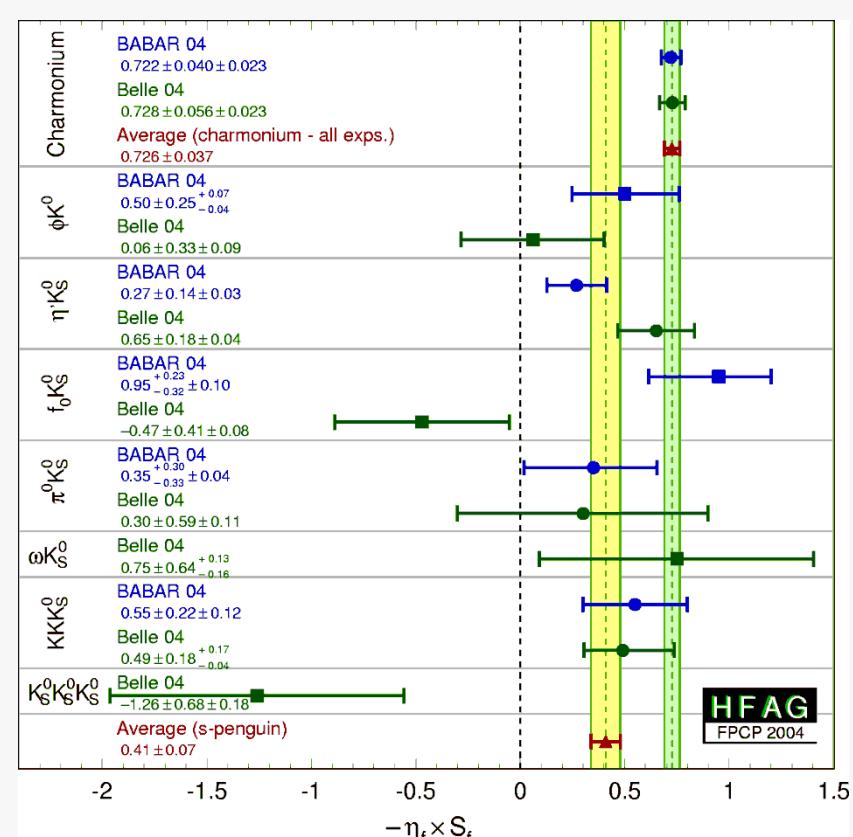
" $\sin 2\phi_1$ " from $b \rightarrow s$ penguins

- dominant phase is the same as in $b \rightarrow c\bar{c}s$
- even in SM, possible deviations (tree pollution)
- New physics in the loop may cause deviation in the values of S and C

Theoretical prediction for ΔS



For most of the modes, theory predicts $\Delta S > 0$



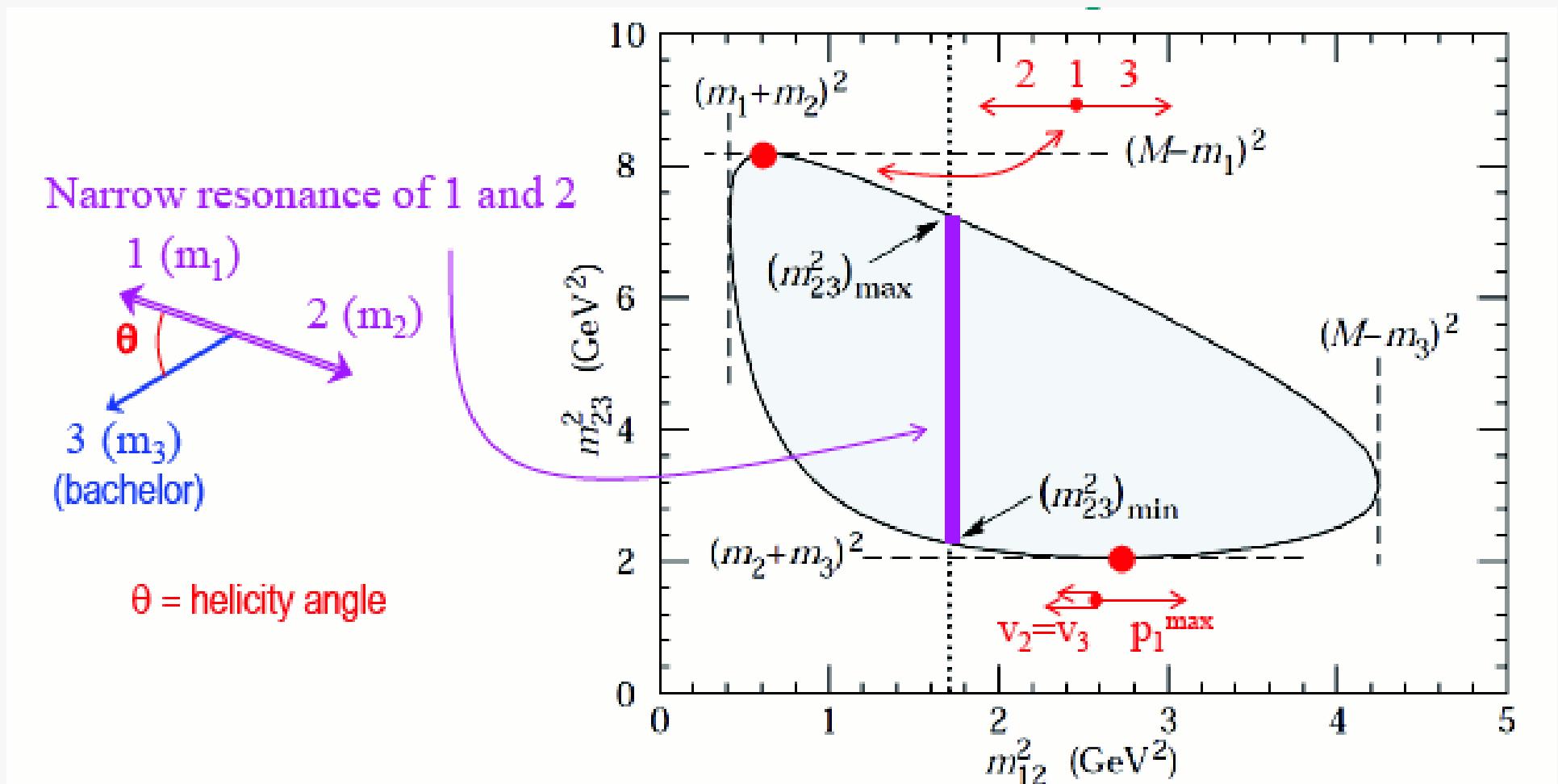
Tension between $\sin 2\phi_1$ from $b \rightarrow c\bar{c}s$ and $b \rightarrow q\bar{q}s$ ($\Delta S < 0$)

Dalitz plot

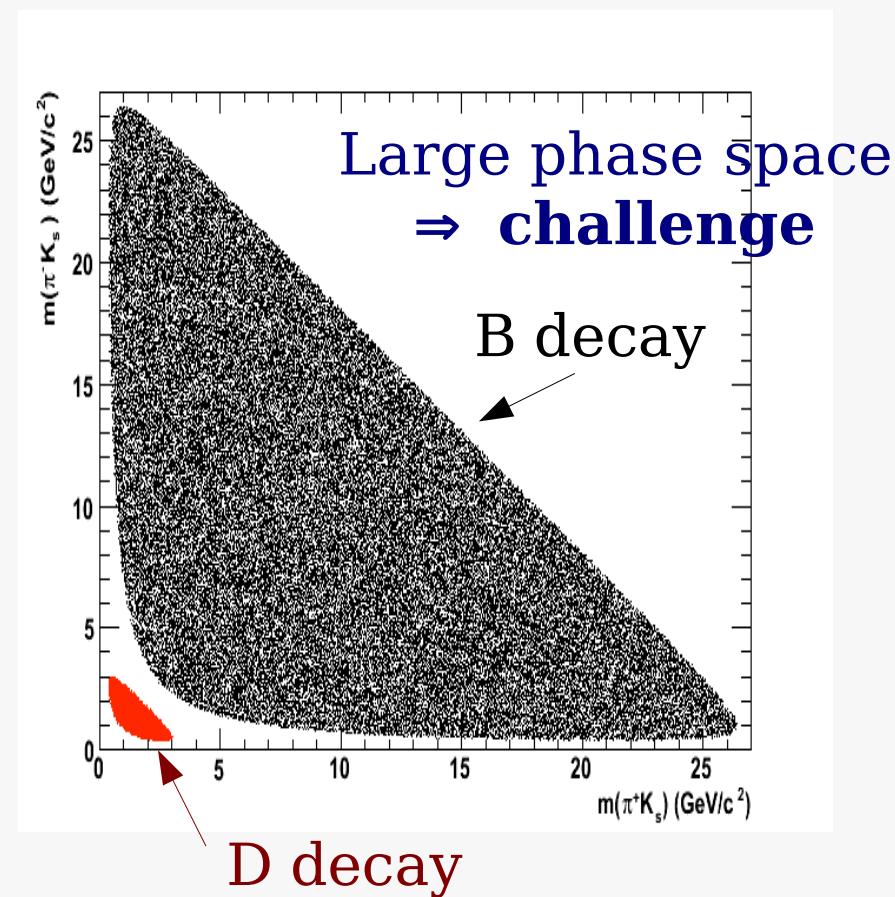
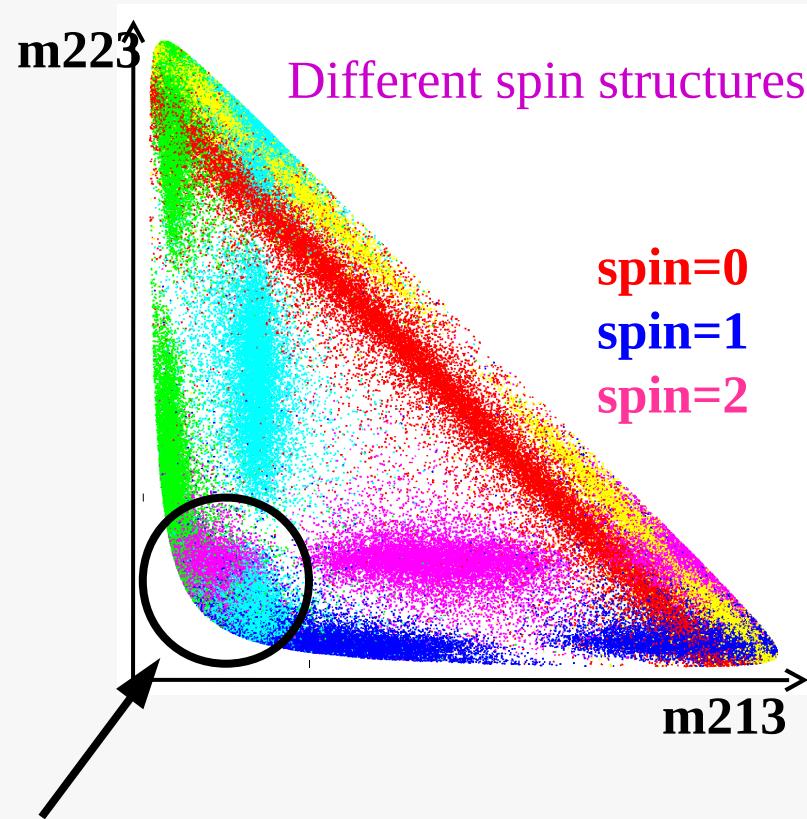
In 3-body decays spin-0 \rightarrow 3 spin-0 (e.g. $B^0 \rightarrow \pi^+ \pi^- K_S^0$), there are only 2 meaningful degrees of freedom: usually taken to be squared invariant masses m_{ij}^2

$$dm_{12} dm_{23} \propto \frac{d^3 p}{E}$$

Intermediate resonance \Rightarrow structure in the DP according to its mass and width



Dalitz plot



Superimposed resonant contributions:

- interference
 - access to phases
- with no ambiguity such as $\sin 2 \phi_1^{\text{eff}} = \sin (\pi - 2 \phi_1^{\text{eff}})$

The isobar model

- Parameterization of intermediate state amplitudes

- In B decays:

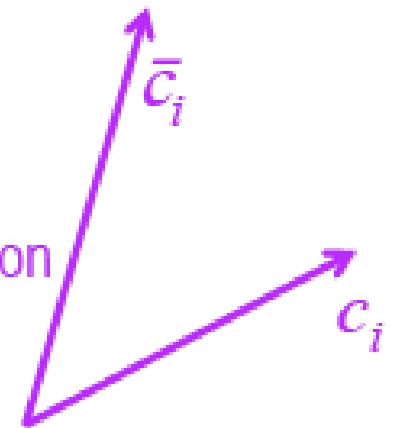
$$A \sim \sum c_i F(m_{13}^2, m_{23}^2)$$

complex

e.g. Breit - Wigner

(similar expression for \bar{B} decays: \bar{A})

$|c_i| \neq |\bar{c}_i|$
 \Rightarrow Direct CP violation



- DP and Δt model:

$$\frac{d\Gamma(m_{13}^2, m_{23}^2, \Delta t, q_{tag})}{d\Delta t dm_{13} dm_{23}} \propto e^{-|\Delta t|/\tau_{B^0}} \times \left[\left(|A|^2 + |\bar{A}|^2 \right) \right.$$

$$\left. - q_{tag} \underbrace{\left(|A|^2 - |\bar{A}|^2 \right)}_{\sim C} \cos(\Delta m_d \Delta t) + q_{tag} \underbrace{2 \operatorname{Im} \left(\bar{A} A^* e^{-i \cdot 2\beta} \right)}_{\sim S} \sin(\Delta m_d \Delta t) \right]$$

Directly extracted parameters: isobar amplitudes c_i

Other parameters (S , C , A_{CP} , phases, BF) are computed from them

Dalitz-Plot Signal Model

(an example: $B \rightarrow K_S \pi \pi$)

- Signal components:

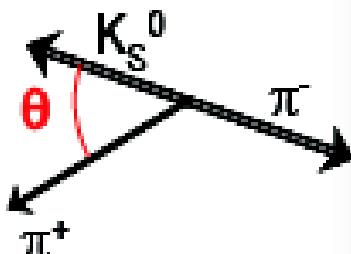
- $B^0 \rightarrow \rho^0(770) K_S^0$
- $B^0 \rightarrow f_0(980) K_S^0$
- $B^0 \rightarrow f_X(1300) K_S^0$
- $B^0 \rightarrow f_2(1270) K_S^0$
- $B^0 \rightarrow \chi_{c0} K_S^0$
- $B^0 \rightarrow K^*(892) \pi$
- $B^0 \rightarrow K\pi$ S-wave
- Non-resonant

} $\pi^+ \pi^-$
 } resonances
 } $K\pi$
 } resonances

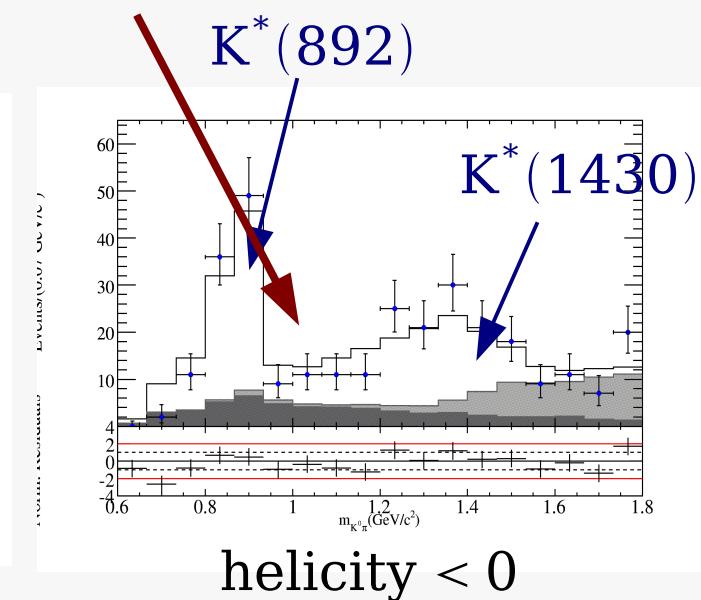
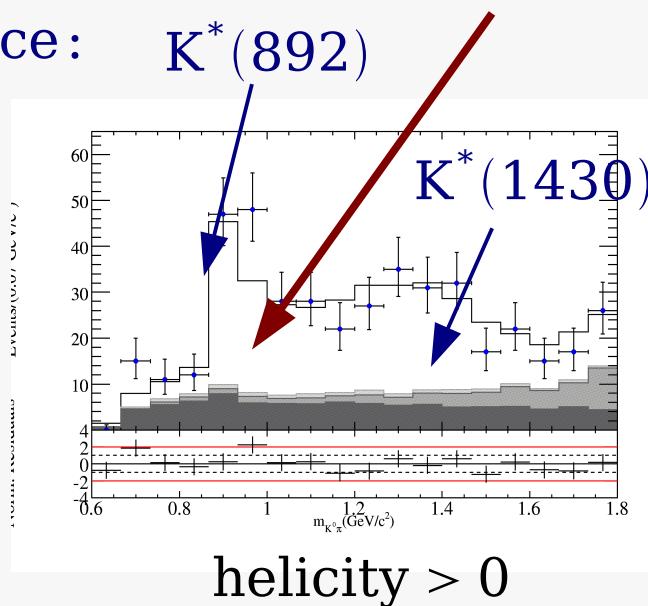
interference
constructive destructive

- Example of interference:

$$\text{helicity} = \cos\theta$$



(bachelor)

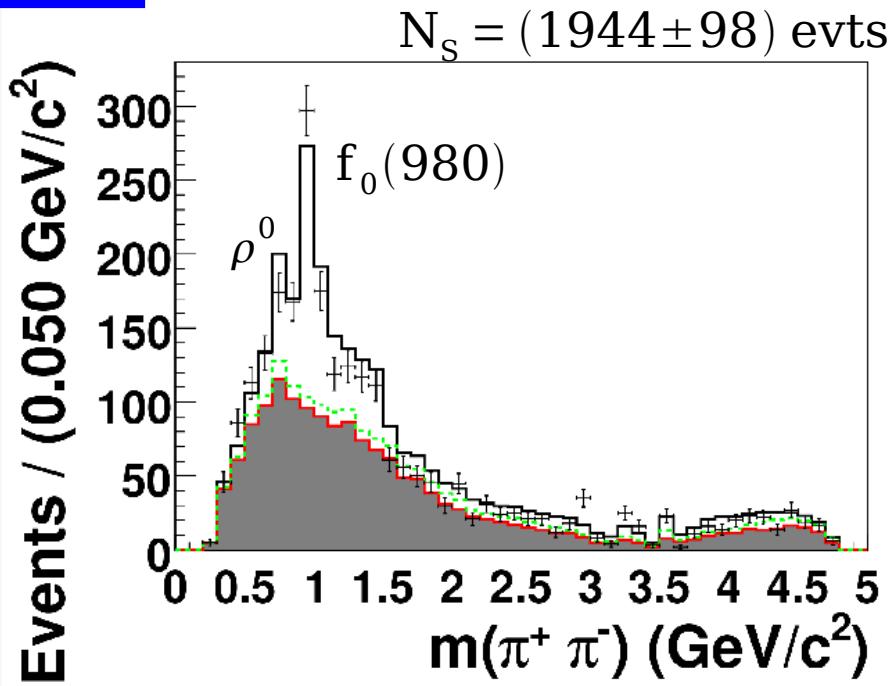


- multiple solutions: major issue in this method...
(choice of the solution: external inputs...)

Time-dependent Dalitz plot analysis $K_S \pi^+ \pi^-$



$657 \times 10^6 B\bar{B}$ pairs
[ArXiv:0811.3665]



$\rho^0 K_S$

$$\beta_{\text{eff}} = (20.0^{+8.6}_{-8.5} \pm 3.2 \pm 3.5)^\circ$$

$$A_{\text{CP}} = +0.03^{+0.23}_{-0.24} \pm 0.11 \pm 0.10$$

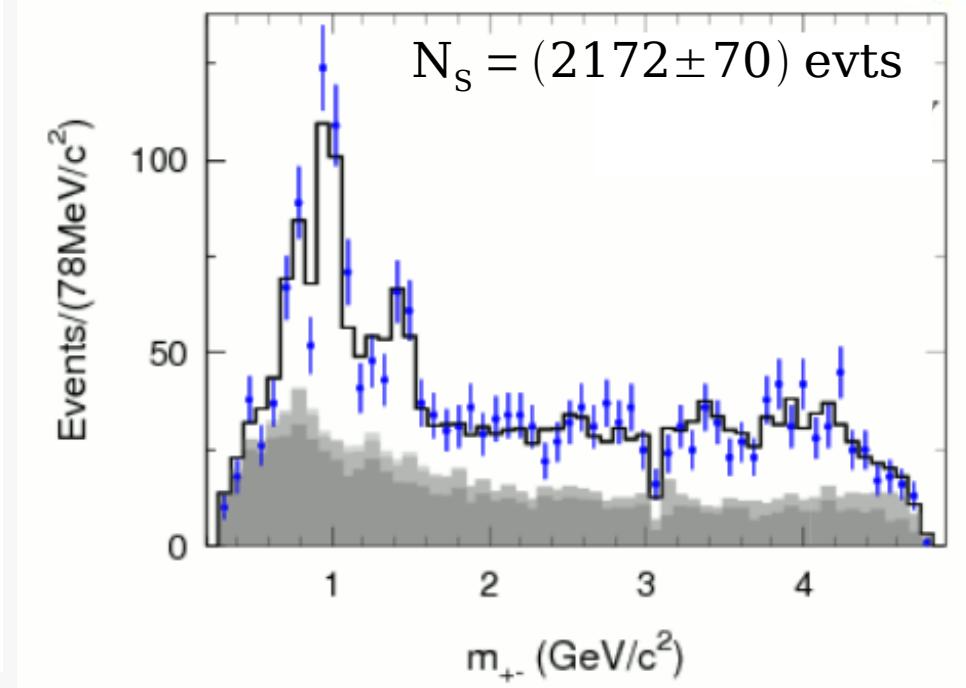
$f_0(980) K_S$

$$\beta_{\text{eff}} = (12.7^{+6.9}_{-6.5} \pm 2.8 \pm 3.3)^\circ$$

$$A_{\text{CP}} = -0.06 \pm 0.17 \pm 0.07 \pm 0.09$$



$383 \times 10^6 B\bar{B}$ pairs
[ArXiv:0708.2097]



$$\beta_{\text{eff}} = (19^{+10}_{-9} \pm 3 \pm 3)^\circ$$

$$A_{\text{CP}} = -0.02 \pm 0.27 \pm 0.08 \pm 0.06$$

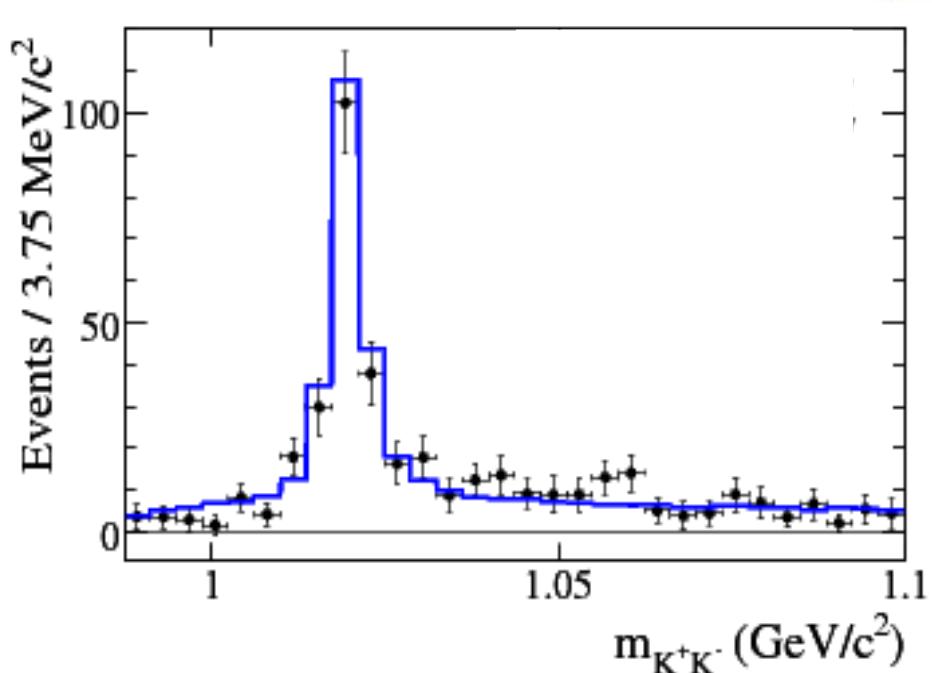
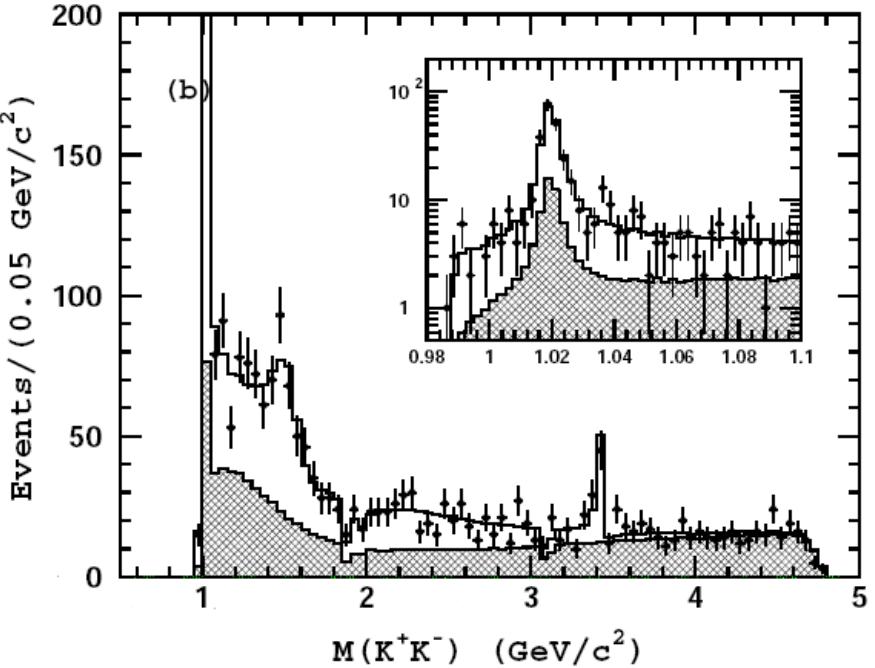
$$\beta_{\text{eff}} = (44^{+11}_{-10} \pm 3 \pm 4)^\circ$$

$$A_{\text{CP}} = -0.35 \pm 0.27 \pm 0.07 \pm 0.04$$

Time-dependent Dalitz plot analysis $K_S K^+ K^-$



$465 \times 10^6 B\bar{B}$ pairs
[ArXiv:0808.0700]



ϕK_S

$$\beta_{\text{eff}} = (21.2^{+9.8}_{-10.4} \pm 2.0 \pm 2.0)^\circ$$

$$A_{\text{CP}} = +0.31^{+0.21}_{-0.23} \pm 0.04 \pm 0.09$$

$f_0(980) K_S$

$$\beta_{\text{eff}} = (28.2^{+9.9}_{-9.8} \pm 2.0 \pm 2.0)^\circ$$

$$A_{\text{CP}} = -0.02 \pm 0.34 \pm 0.08 \pm 0.09$$

$$\beta_{\text{eff}} = (7.7 \pm 7.7 \pm 0.9)^\circ$$

$$A_{\text{CP}} = +0.14 \pm 0.19 \pm 0.02$$

$$\beta_{\text{eff}} = (8.5 \pm 7.5 \pm 1.8)^\circ$$

$$A_{\text{CP}} = +0.01 \pm 0.26 \pm 0.07$$

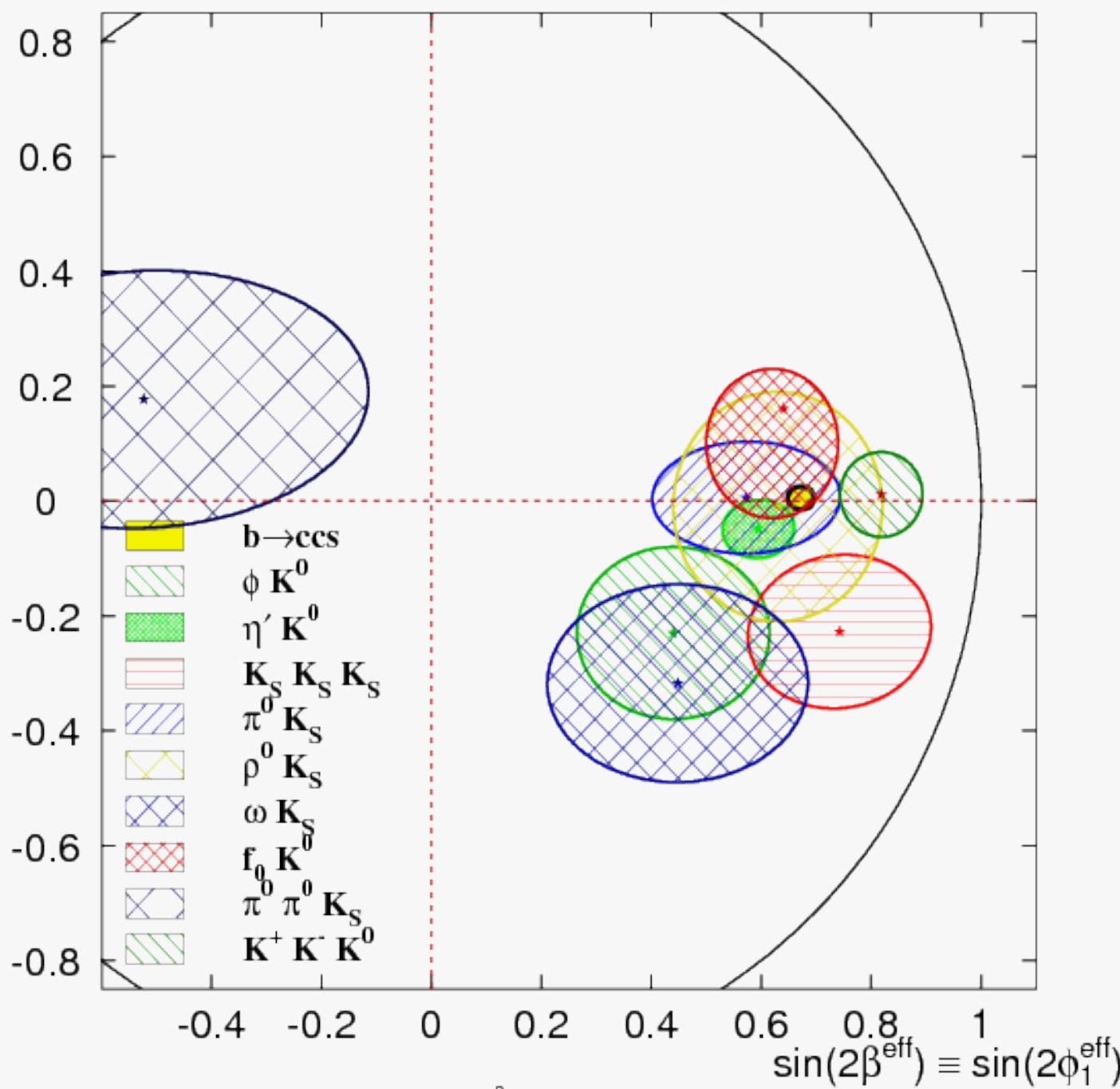
$\sin(2\beta^{\text{eff}}) \equiv \sin(2\phi_1^{\text{eff}})$ vs $C_{\text{CP}} \equiv -A_{\text{CP}}$

HFAG

CKM2008

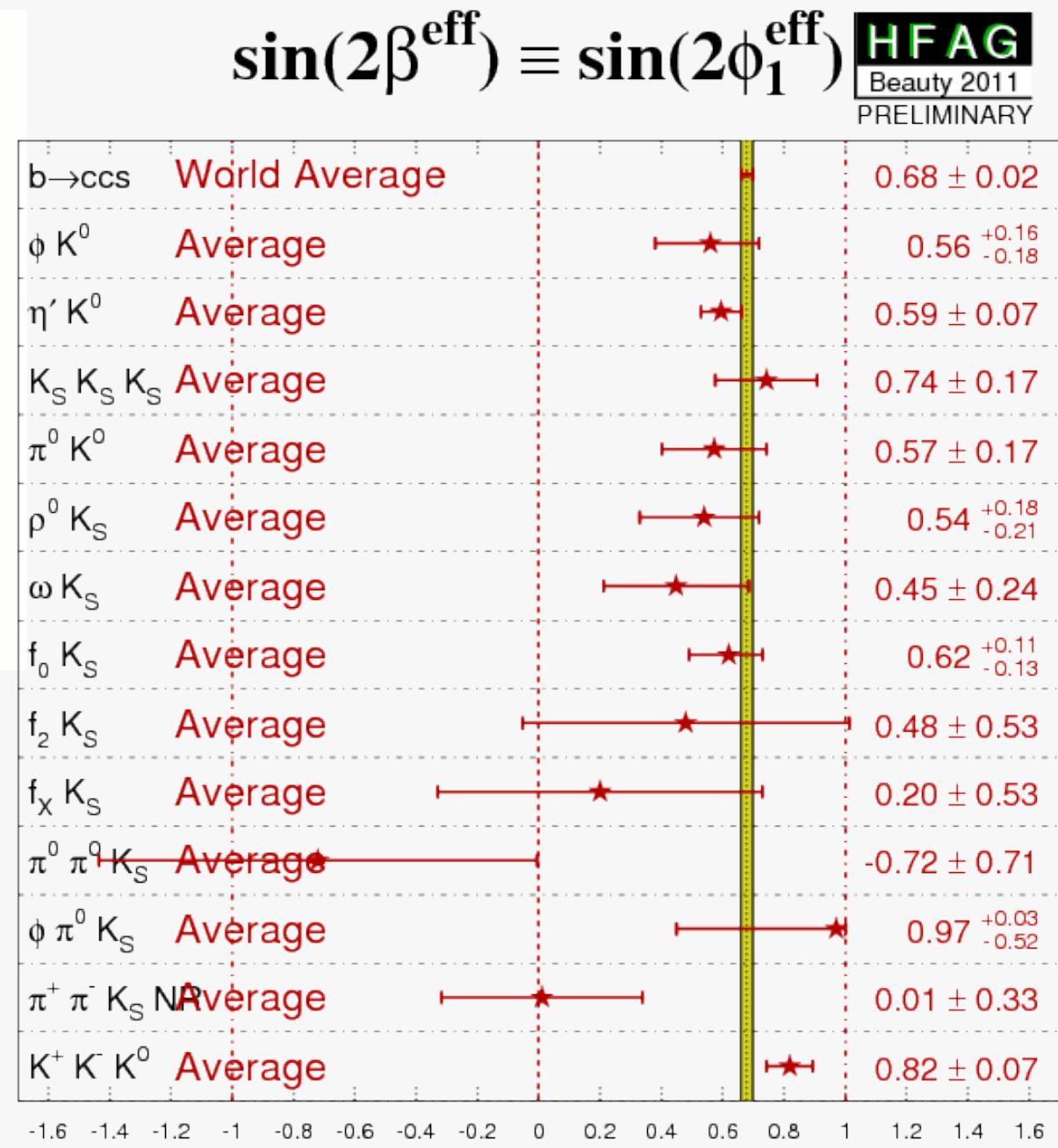
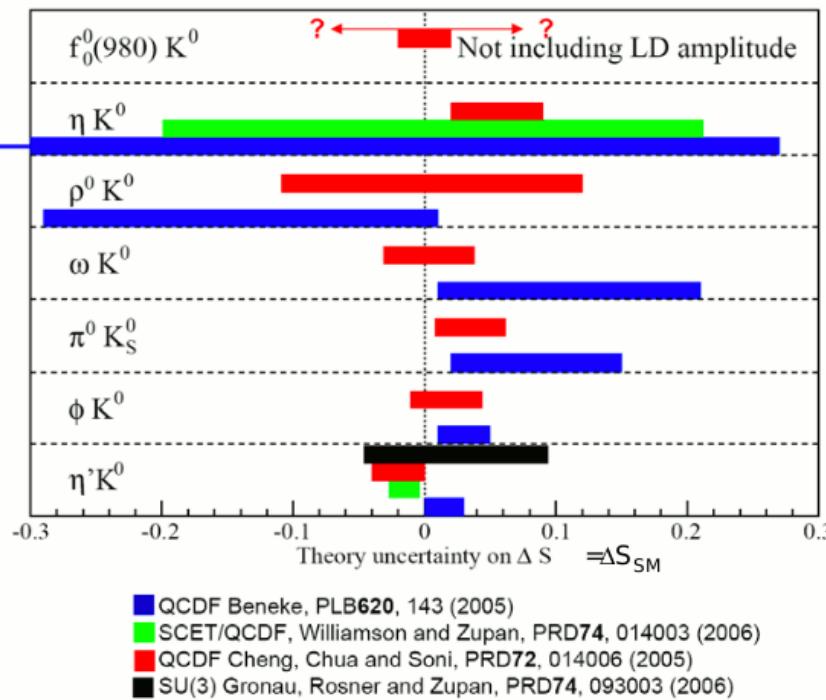
PRELIMINARY

$C_{\text{CP}} \equiv -A_{\text{CP}}$



Contours give $-2\Delta(\ln L) = \Delta\chi^2 = 1$, corresponding to 60.7% CL for 2 dof

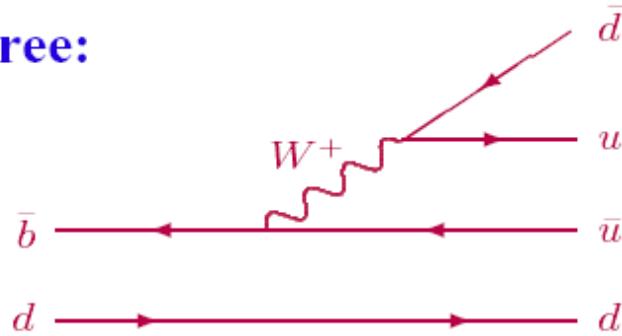
β with $b \rightarrow s$ penguins



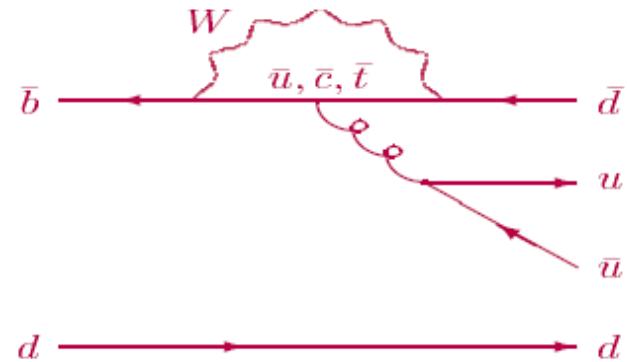
More statistics crucial
for mode-by-mode studies

α determination

Tree:



Penguin:



$$A(B^0 \rightarrow \pi^+ \pi^-) = T e^{i\gamma} + P e^{i\delta}, \quad r = |P|/|T|$$

$$\begin{aligned} A(t) &= S_{\pi^+ \pi^-} \sin(\Delta m t) - C_{\pi^+ \pi^-} \cos(\Delta m t) \\ &= \sqrt{1 - C_{\pi^+ \pi^-}^2} \sin 2\alpha_{\text{eff}} \sin(\Delta m t) - C_{\pi^+ \pi^-} \cos(\Delta m t) \end{aligned}$$

from time dependent CP, we can measure α_{eff} , but we want α !

expanding in r : $S_{\pi^+ \pi^-} = \sin 2\alpha + 2r \cos \delta \sin(\beta + \alpha) \cos 2\alpha + O(r^2)$

time dependent decay width:

$$\Gamma(B^0(t)) \propto \Gamma_{\pi^+ \pi^-} [1 + C_{\pi^+ \pi^-} \cos \Delta m t - S_{\pi^+ \pi^-} \sin \Delta m t]$$

3 measurables vs. 4 unknowns: T, r, δ, γ

→ additional inputs required to determine the penguin pollution to fix r

α determination with isospin analysis

[Gronau-London, PRL65, 3381 (1990)]

Isospin breaking (d and u charges different, $m_u \neq m_d$)

$$A_{+-} = A(B^0 \rightarrow \pi^+ \pi^-) = e^{-i\alpha} T^{+-} + P$$

$$\sqrt{2} A_{00} = \sqrt{2} A(B^0 \rightarrow \pi^0 \pi^0) = e^{-i\alpha} T^{00} + P$$

$$\sqrt{2} A_{+0} = \sqrt{2} A(B^+ \rightarrow \pi^+ \pi^0) = e^{-i\alpha} (T^{00} + T^{+-})$$

$$A_{+-} + \sqrt{2} A_{00} = \sqrt{2} A_{+0}$$

$$\bar{A}_{+-} + \sqrt{2} \bar{A}_{00} = \sqrt{2} \bar{A}_{+0}$$

- neglecting EWP $\Rightarrow A_{+0}$ pure tree
 $|A_{+0}| = |\bar{A}_{+0}|$

$$\Rightarrow \Delta \alpha_{\text{EWP}} = (1.5 \pm 0.3 \pm 0.3)^\circ$$

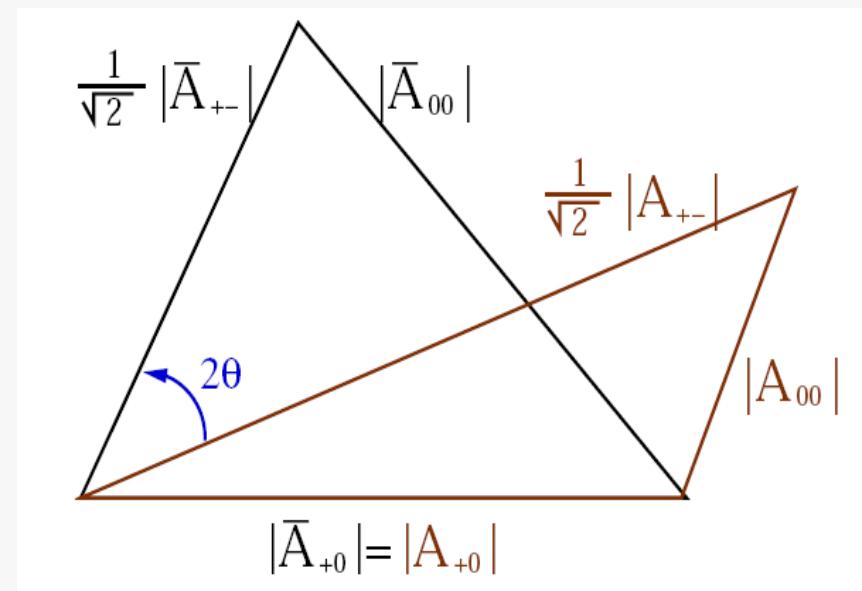
- $\pi^0 - \eta - \eta'$ and $\rho - \omega$ mixing

[J.Zupan, hep-ph/0701004]

(mass eigenstates do not coincide with isospin eigenstates)

$$\Rightarrow |\Delta \alpha_{\pi\pi}^{\pi-\eta-\eta'}| < 1.6^\circ$$

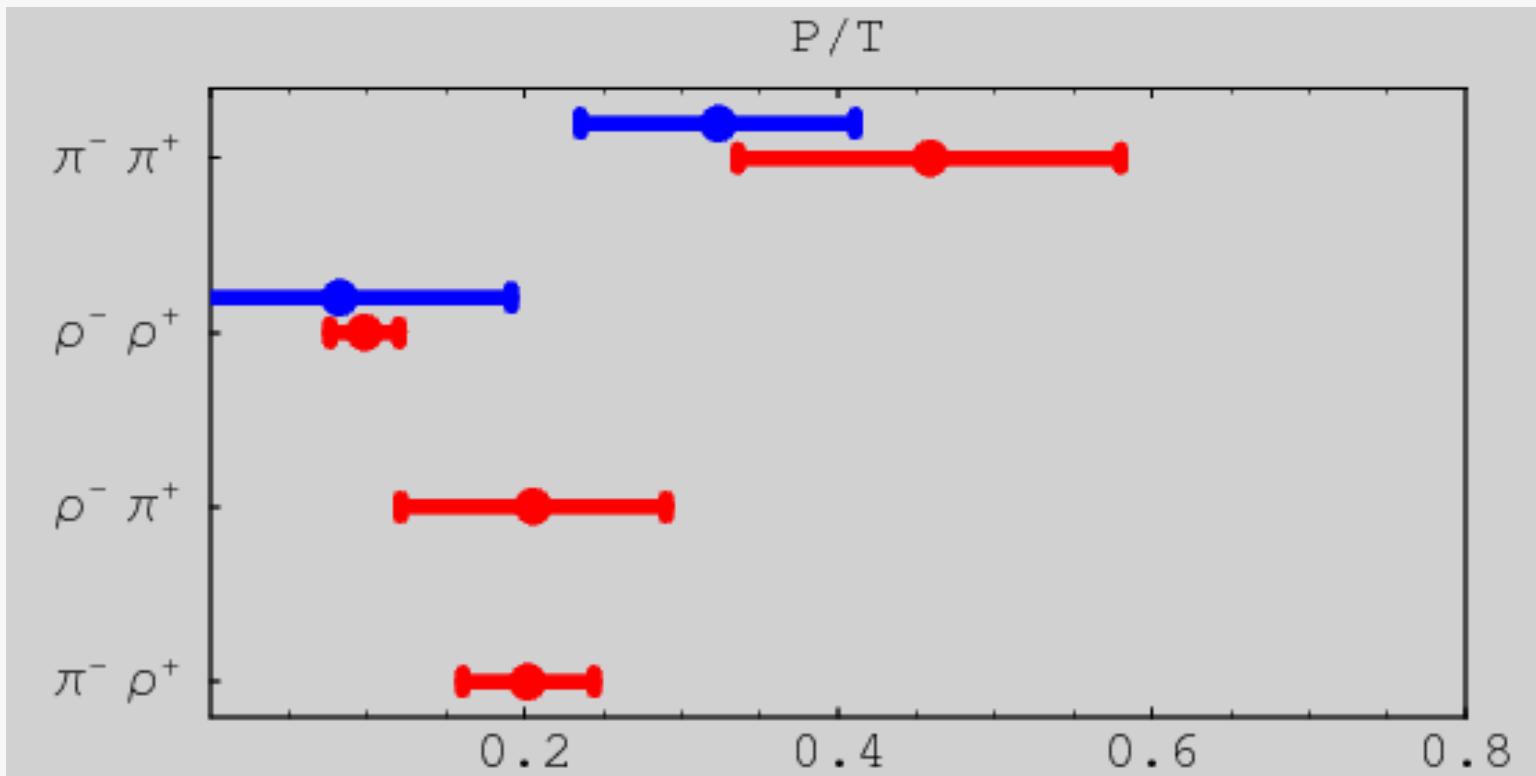
α can be resolved up to an 8-fold ambiguity ($\alpha \in [0, \pi]$)



Sizes of penguin-to-tree ratios r

hierarchy: $r(\pi^+ \pi^-) > r(\rho^+ \pi^-) \sim r(\rho^+ \pi^-) > r(\rho^+ \rho^-)$

larger $r \Rightarrow$ larger difference $\sin 2\alpha - \sin 2\alpha_{\text{eff}}$



(blue) from isospin decomposition , (red) using SU(3)

Concrete example: $B \rightarrow \pi\pi$ at Belle

Gronau and London,
PRL 65, 3381 (1990)

$$|A_{\text{th}}^{+-}| = \sqrt{a^{+-}(1 - \mathcal{A}_{\pi\pi})}$$

$$|\bar{A}_{\text{th}}^{+-}| = \sqrt{a^{+-}(1 + \mathcal{A}_{\pi\pi})}$$

$$|A_{\text{th}}^{0+}| = |A_{\text{th}}^{0-}| = \sqrt{a^{0+}}$$

$$|A_{\text{th}}^{00}|^2 = \frac{|A_{\text{th}}^{+-}|^2}{2} + |A_{\text{th}}^{0+}|^2 - \sqrt{2}|A_{\text{th}}^{+-}||A_{\text{th}}^{+0}|\cos(\omega - \kappa/2)$$

$$|\bar{A}_{\text{th}}^{00}|^2 = \frac{|\bar{A}_{\text{th}}^{+-}|^2}{2} + |A_{\text{th}}^{0+}|^2 - \sqrt{2}|\bar{A}_{\text{th}}^{+-}||A_{\text{th}}^{+0}|\cos(\omega + \kappa/2)$$

$$B_{\text{th}}^{\pi^+\pi^-} = \left(|A_{\text{th}}^{+-}|^2 + |\bar{A}_{\text{th}}^{+-}|^2 \right) / 2 = a^{+-}$$

$$B_{\text{th}}^{\pi^0\pi^0} = \left(|A_{\text{th}}^{00}|^2 + |\bar{A}_{\text{th}}^{00}|^2 \right) / 2$$

$$B_{\text{th}}^{\pi^0\pi^+} = |A_{\text{th}}^{0+}|^2 (\tau_{B^\pm}/\tau_{B^0}) = a^{+0} \cdot (\tau_{B^\pm}/\tau_{B^0})$$

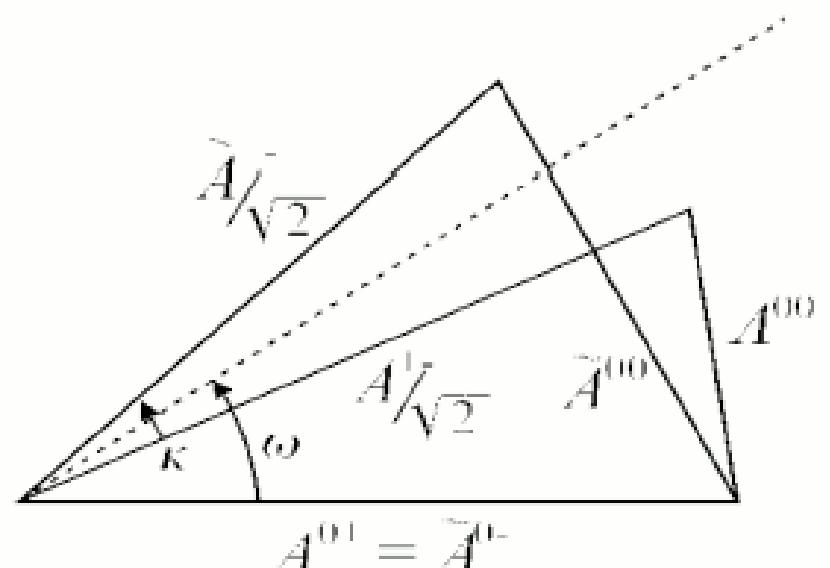
$$\mathcal{A}_{\text{th}}^{\pi^0\pi^0} = \frac{|\bar{A}_{\text{th}}^{00}|^2 - |A_{\text{th}}^{00}|^2}{|\bar{A}_{\text{th}}^{00}|^2 + |A_{\text{th}}^{00}|^2}$$

$$\mathcal{A}_{\text{th}}^{\pi^+\pi^-} = \mathcal{A}_{\pi\pi}$$

$$S_{\text{th}}^{\pi^+\pi^-} = \sqrt{1 - \mathcal{A}_{\pi\pi}^2} \sin(2\phi_2 + \kappa)$$

$$\frac{A(B^0 \rightarrow \pi^+\pi^-)}{\sqrt{2}} + A(B^0 \rightarrow \pi^0\pi^0) = A(B^+ \rightarrow \pi^+\pi^0)$$

$$\frac{A(\bar{B}^0 \rightarrow \pi^+\pi^-)}{\sqrt{2}} + A(\bar{B}^0 \rightarrow \pi^0\pi^0) = A(B^- \rightarrow \pi^-\pi^0)$$



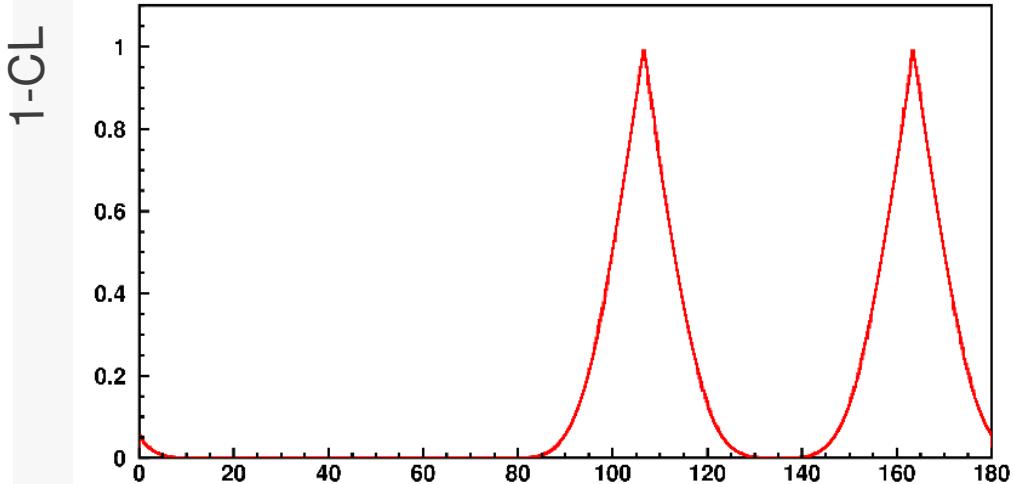
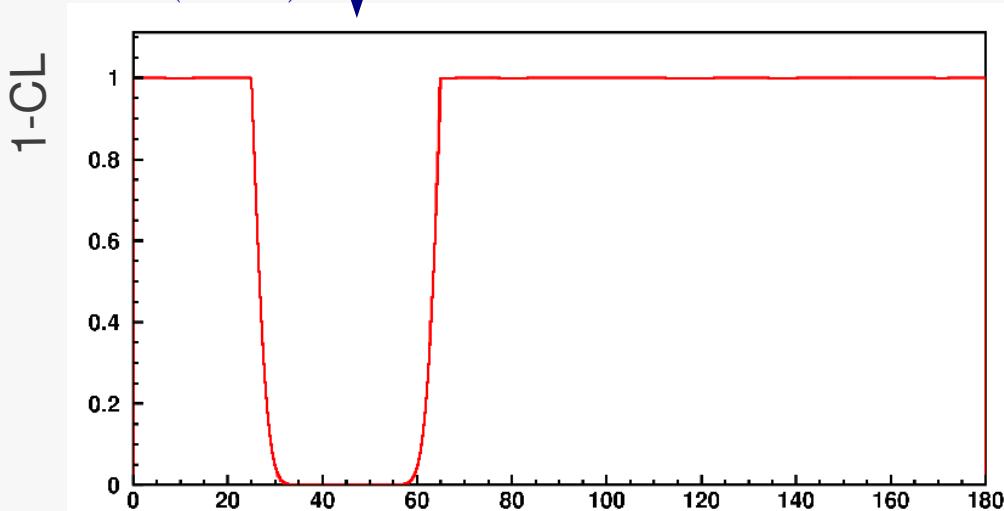
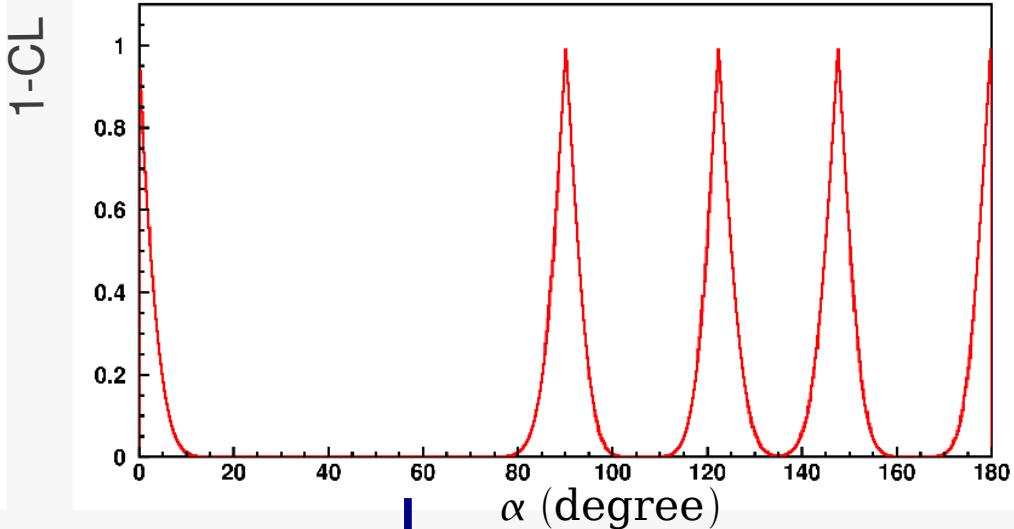
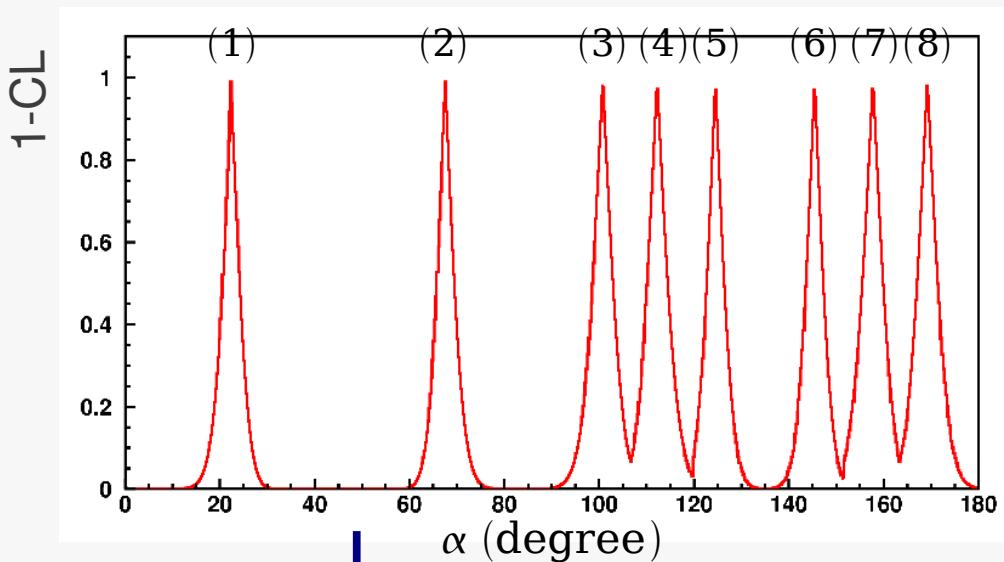
**6 parameters + 6 observables
⇒ all determined**

Some examples to illustrate α extraction

$$\begin{aligned}\text{Br}(\pi^+ \pi^-) &= (5.0 \pm 0.2) \times 10^{-6} \\ \text{Br}(\pi^0 \pi^0) &= (1.5 \pm 0.1) \times 10^{-6} \\ \text{Br}(\pi^+ \pi^0) &= (3.5 \pm 0.2) \times 10^{-6} \\ C(\pi^+ \pi^-) &= -0.40 \pm 0.03 \\ S(\pi^+ \pi^-) &= -0.50 \pm 0.04 \\ C(\pi^0 \pi^0) &= -0.30 \pm 0.10\end{aligned}$$



$$\boxed{\begin{aligned}\text{Br}(\pi^+ \pi^-) &= (2.0 \pm 0.2) \times 10^{-6} \\ \text{Br}(\pi^0 \pi^0) &= (0.5 \pm 0.1) \times 10^{-6} \\ \text{Br}(\pi^+ \pi^0) &= (3.5 \pm 0.2) \times 10^{-6} \\ C(\pi^+ \pi^-) &= -0.40 \pm 0.03 \\ S(\pi^+ \pi^-) &= -0.50 \pm 0.04 \\ C(\pi^0 \pi^0) &= -0.30 \pm 0.10\end{aligned}}$$



α : $\pi\pi$ system

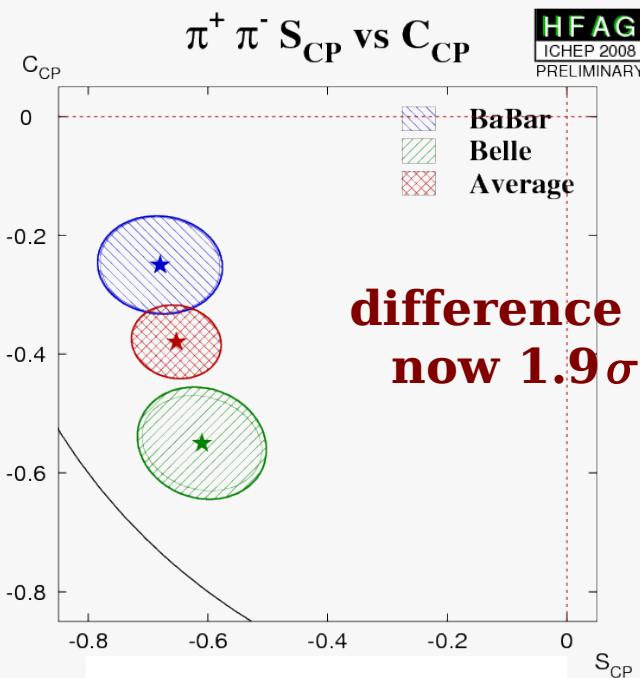


$535 \times 10^6 B\bar{B}$ pairs
PRL 98, 221801(2007)

$$C = -0.55 \pm 0.08 \pm 0.05$$

$$S = -0.61 \pm 0.10 \pm 0.04$$

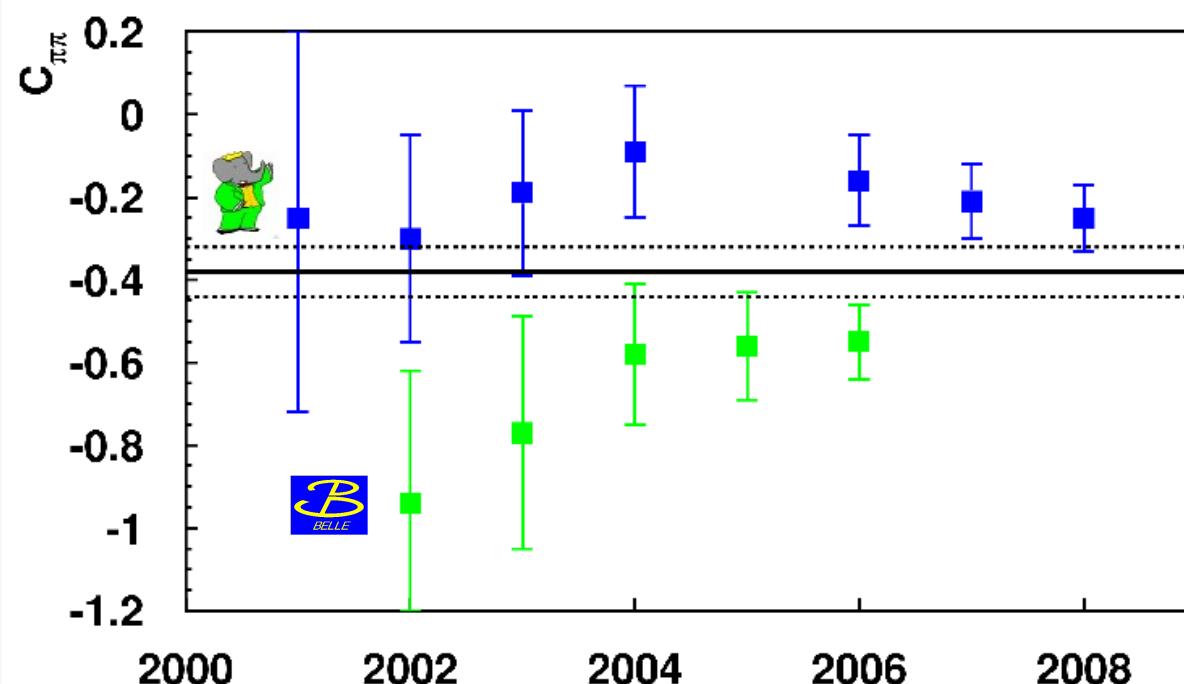
direct CPV @ 5.5σ



$467 \times 10^6 B\bar{B}$ pairs
ArXiv: 0807.4226

$$C = -0.25 \pm 0.08 \pm 0.02$$

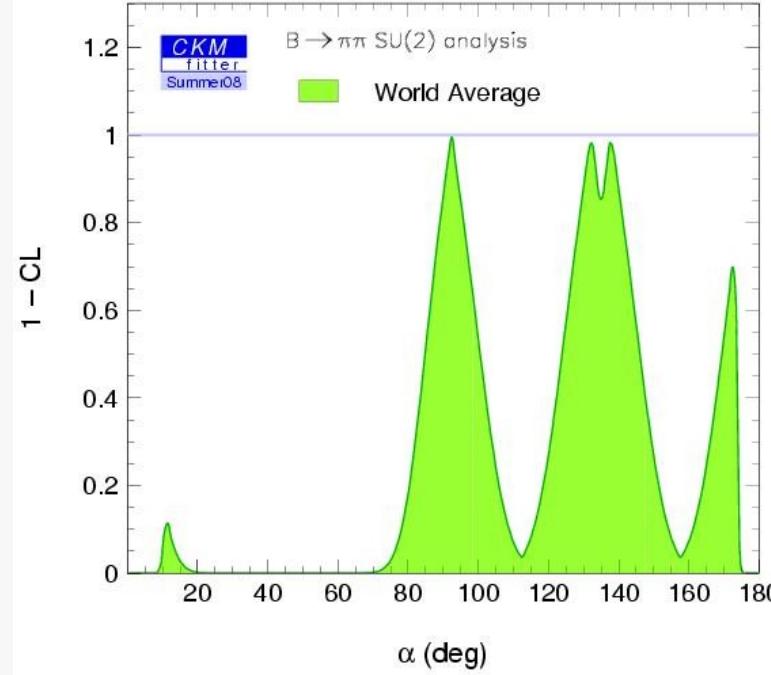
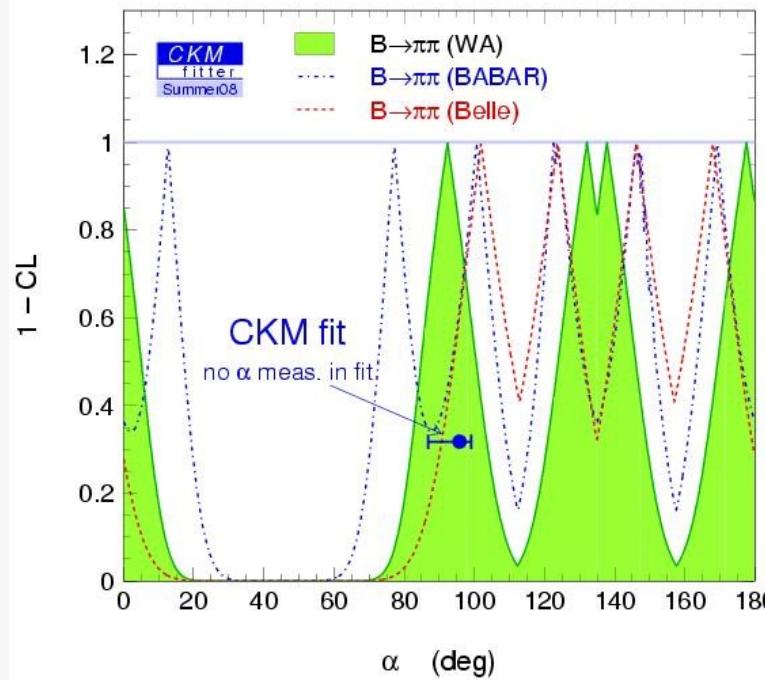
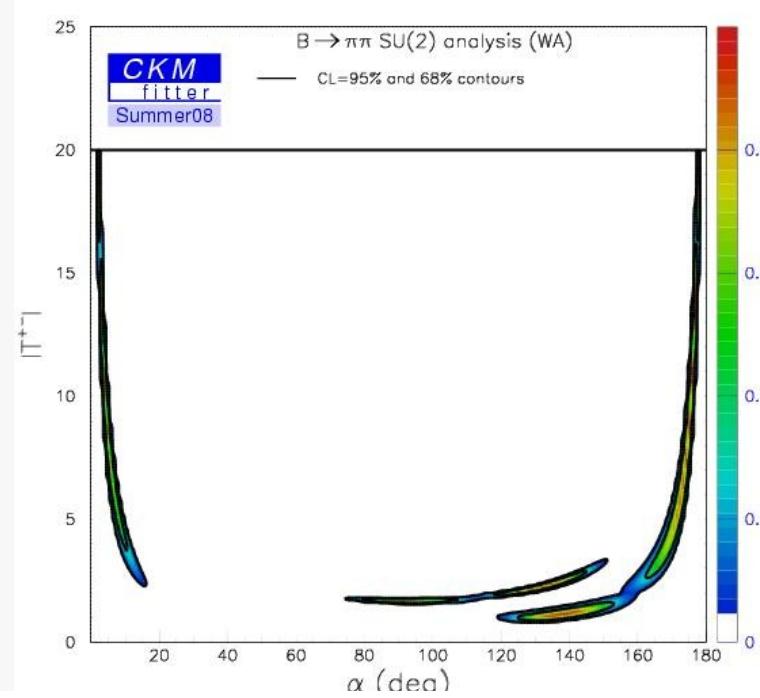
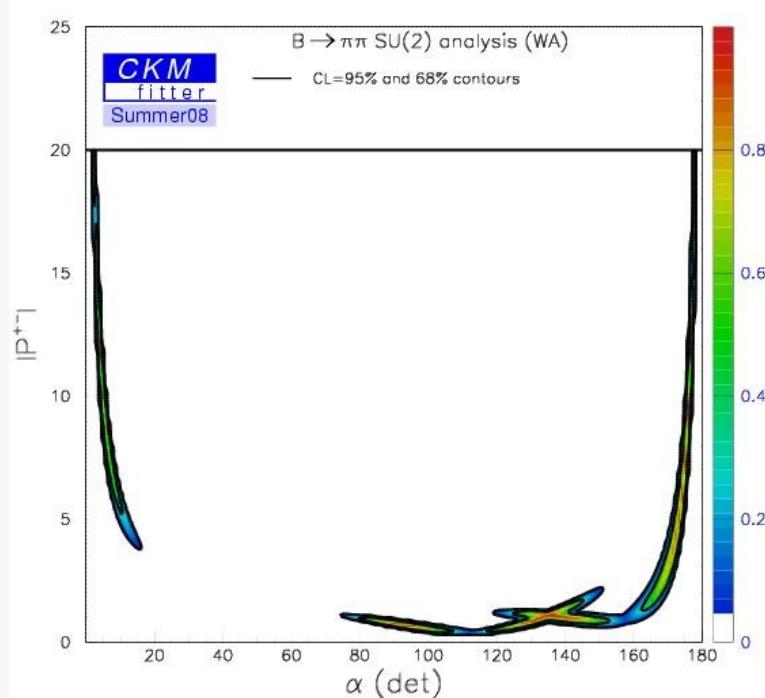
$$S = -0.68 \pm 0.10 \pm 0.03$$



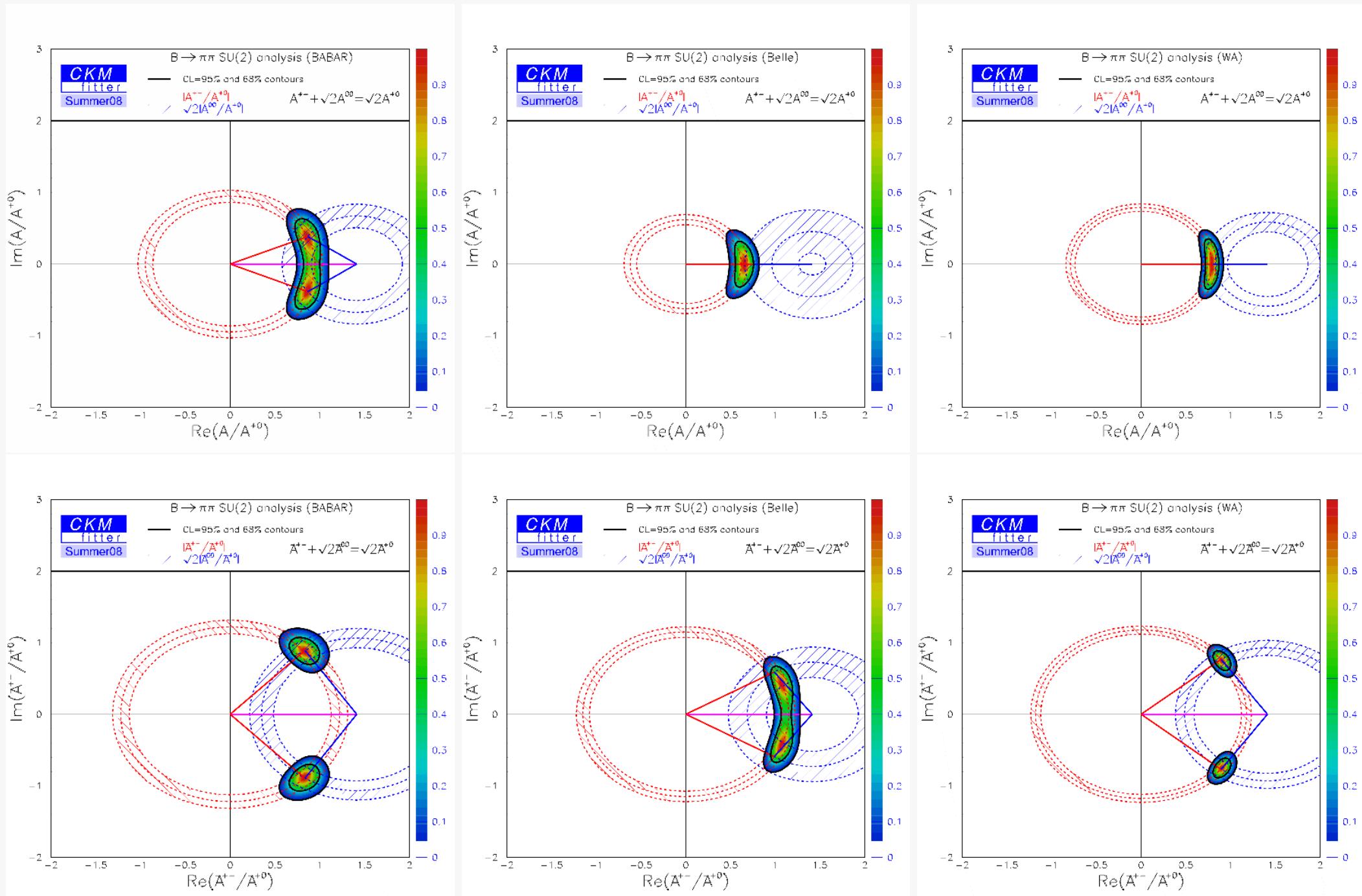
Summary of $C_{\pi\pi}$

Year	BaBar	Belle	Difference
2001	$-0.25 \pm 0.45 \pm 0.14$ PRD 65, 051502 (33M)		
2002	$-0.30 \pm 0.25 \pm 0.04$ PRL 89, 281802 (88M)	$-0.94^{+0.25}_{-0.31} \pm 0.09$ PRL 89, 071801 (45M)	
2003	$-0.19 \pm 0.19 \pm 0.05$ preliminary LP2003 (123M)	$-0.77 \pm 0.27 \pm 0.08$ PRD 68, 012001 (85M)	2.0σ
2004	$-0.09 \pm 0.15 \pm 0.04$ PRL 95, 151803 (227M)	$-0.58 \pm 0.15 \pm 0.07$ PRL 93, 021601 (152M)	3.2σ
2005		$-0.56 \pm 0.12 \pm 0.06$ PRL 95, 101801 (275M)	2.3σ
2006	$-0.16 \pm 0.11 \pm 0.03$ ArXiv:0607106 (347M)	$-0.55 \pm 0.08 \pm 0.05$ PRL 98, 211801 (535M)	2.3σ
2007	$-0.21 \pm 0.09 \pm 0.02$ PRL 99, 021603 (383M)		2.1σ
2008	$-0.25 \pm 0.08 \pm 0.02$ ArXiv:0807.4226 (467M)		1.9σ

P and T



Isospin triangles $B \rightarrow \pi\pi$ case



α : $\pi\pi$ system (6 observables for 6 parameters)

$\text{Br}(\text{B} \rightarrow \pi^+ \pi^-)$, $S_{\pi^+ \pi^-}$, $C_{\pi^+ \pi^-}$, $\text{Br}(\text{B} \rightarrow \pi^+ \pi^0)$, $\text{Br}(\text{B} \rightarrow \pi^0 \pi^0)$, $C_{\pi^0 \pi^0}$

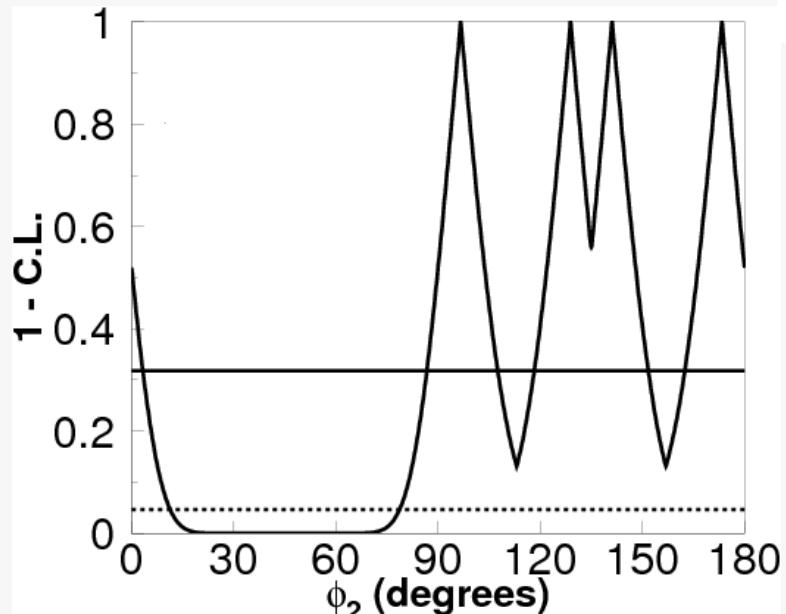


$535 \times 10^6 \text{ B}\bar{\text{B}}$ pairs
PRL 98, 221801(2007)

$$C = -0.55 \pm 0.08 \pm 0.05$$

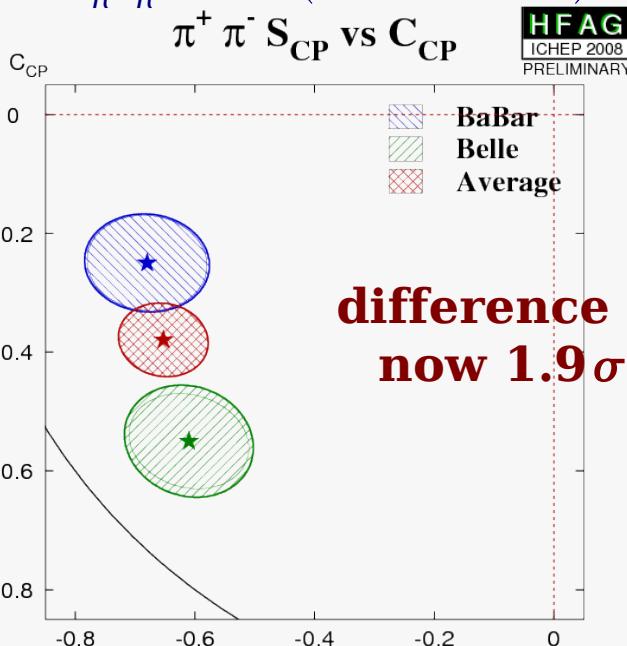
$$S = -0.61 \pm 0.10 \pm 0.04$$

direct CPV @ 5.5σ



standard peak

$[86^\circ, 108^\circ]$ @ 68% C.L.

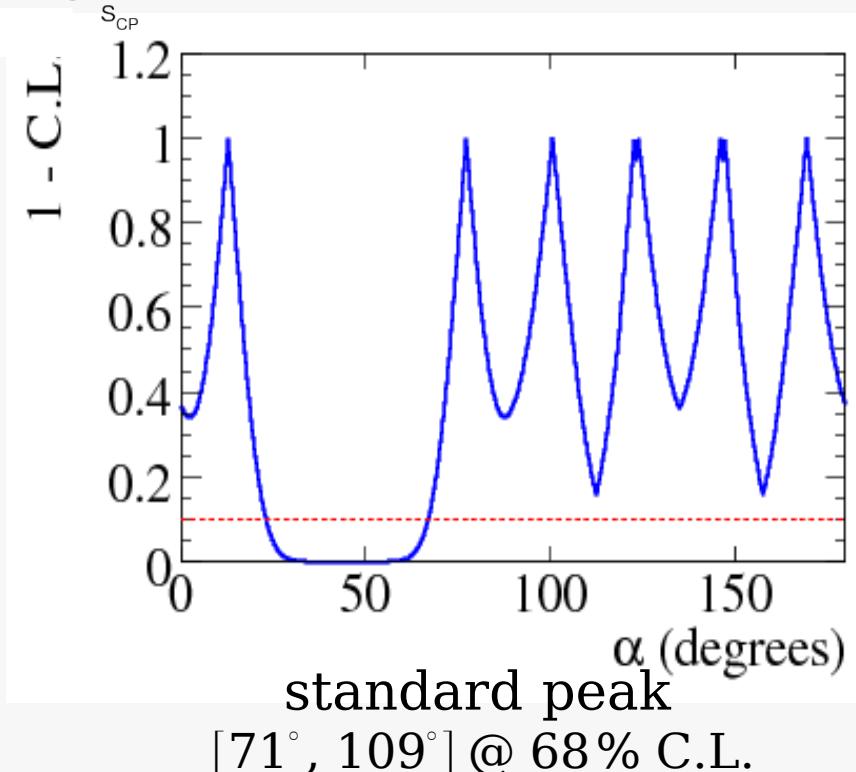


difference is
now 1.9σ

$467 \times 10^6 \text{ B}\bar{\text{B}}$ pairs
ArXiv: 0807.4226

$$C = -0.25 \pm 0.08 \pm 0.02$$

$$S = -0.68 \pm 0.10 \pm 0.03$$



standard peak

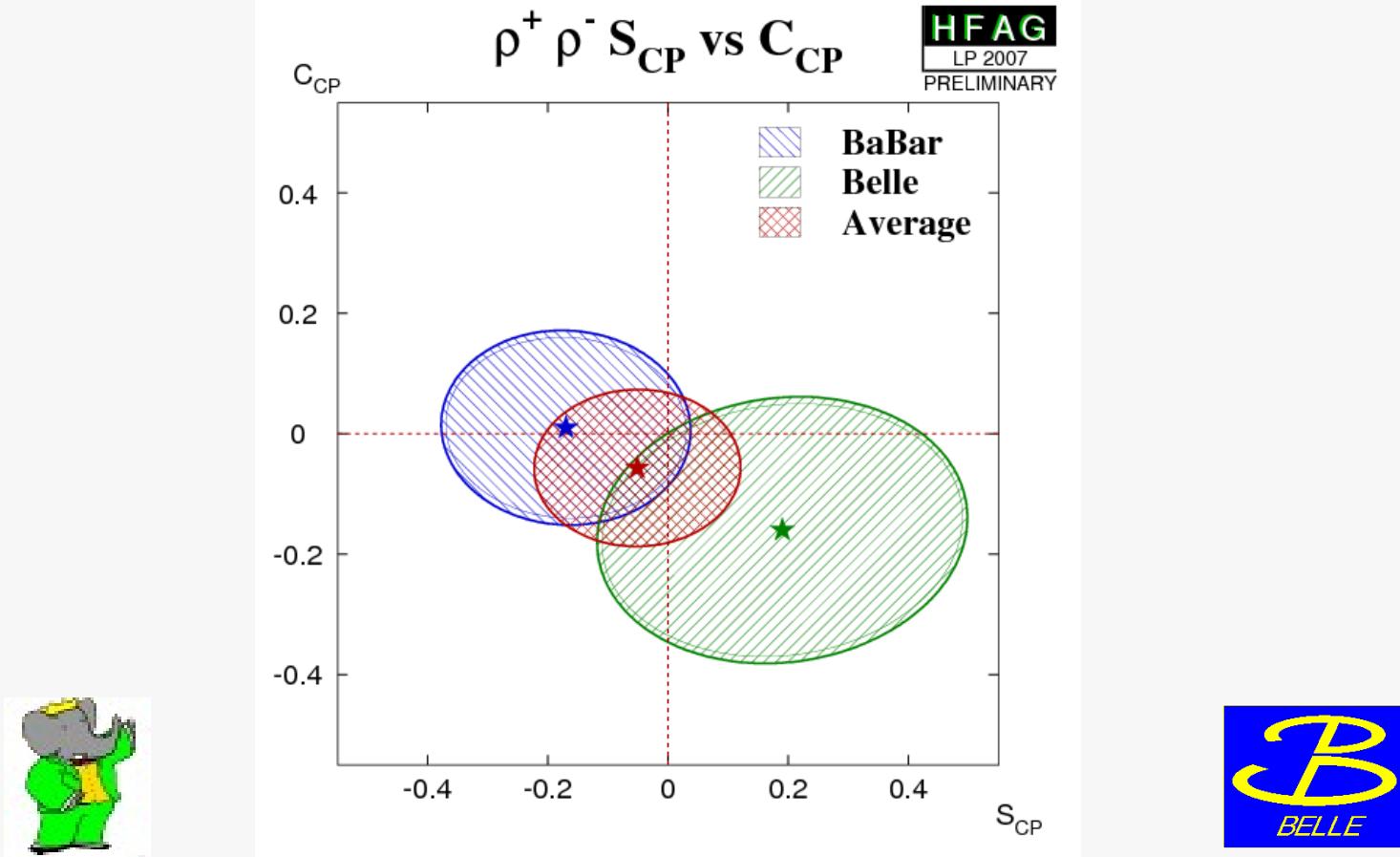
$[71^\circ, 109^\circ]$ @ 68% C.L.



$\rho\rho$ system (5 observables for 6 parameters)

$(\text{Br}(\text{B} \rightarrow \rho^+ \rho^-), S_{\rho^+ \rho^-}, C_{\rho^+ \rho^-}, \text{Br}(\text{B} \rightarrow \rho^+ \rho^0), \text{Br}(\text{B} \rightarrow \rho^0 \rho^0)) + f_L$

$\rho^+ \rho^-$: $\sim 100\%$ longitudinally polarized (similar isospin analysis)



$387 \times 10^6 B\bar{B}$ pairs
PRD 76, 052007(R)(2007)

$$C = +0.01 \pm 0.15 \pm 0.06$$

$$S = -0.17 \pm 0.20^{+0.05}_{-0.06}$$

$535 \times 10^6 B\bar{B}$ pairs
PRD 76, 011104(R)(2007)

$$C = -0.16 \pm 0.21 \pm 0.07$$

$$S = +0.19 \pm 0.30 \pm 0.07$$

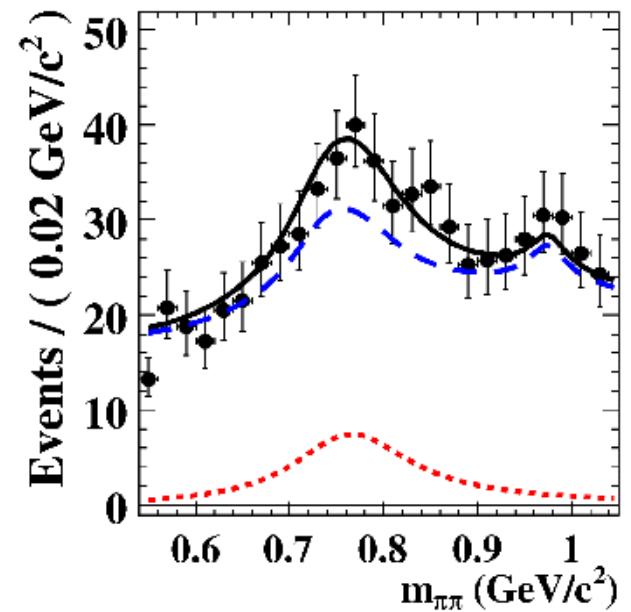
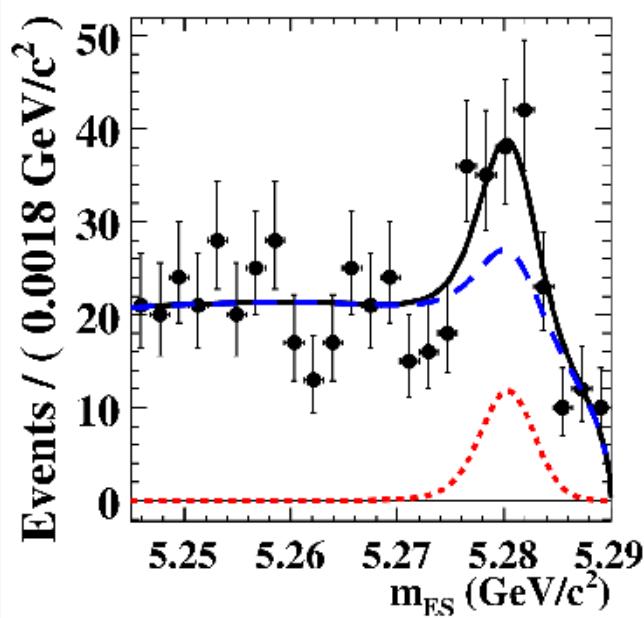
$\rho^0 \rho^0$ mode

**465 × 10⁶ B̄B pairs
[PRD78, 071104(R)]**



10-dim fit to extract yields, f_L , C_L^{00} , S_L^{00} :

$$N_S(\rho^0 \rho^0) = 99^{+35}_{-34} \pm 15 \quad (\Sigma = 3.1 \sigma)$$



$$\text{Br}(B^0 \rightarrow \rho^0 \rho^0) = (0.92 \pm 0.32 \pm 0.14) \times 10^{-6}$$

$$N_S(\rho^0 \pi^+ \pi^-) = -12^{+39}_{-35} \pm 52$$

$$\text{Br}(B^0 \rightarrow \rho^0 \pi^+ \pi^-) < 8.7 \times 10^{-6} \text{ @ 90% C.L.}$$

$$N_S(4\pi^\pm) = 8^{+30}_{-25} \pm 6$$

$$\text{Br}(B^0 \rightarrow 4\pi^\pm) < 21.1 \times 10^{-6} \text{ @ 90% C.L.}$$

$\rho^0 \rho^0$ mode

465 × 10⁶ B̄B pairs
[PRD78, 071104(R)]



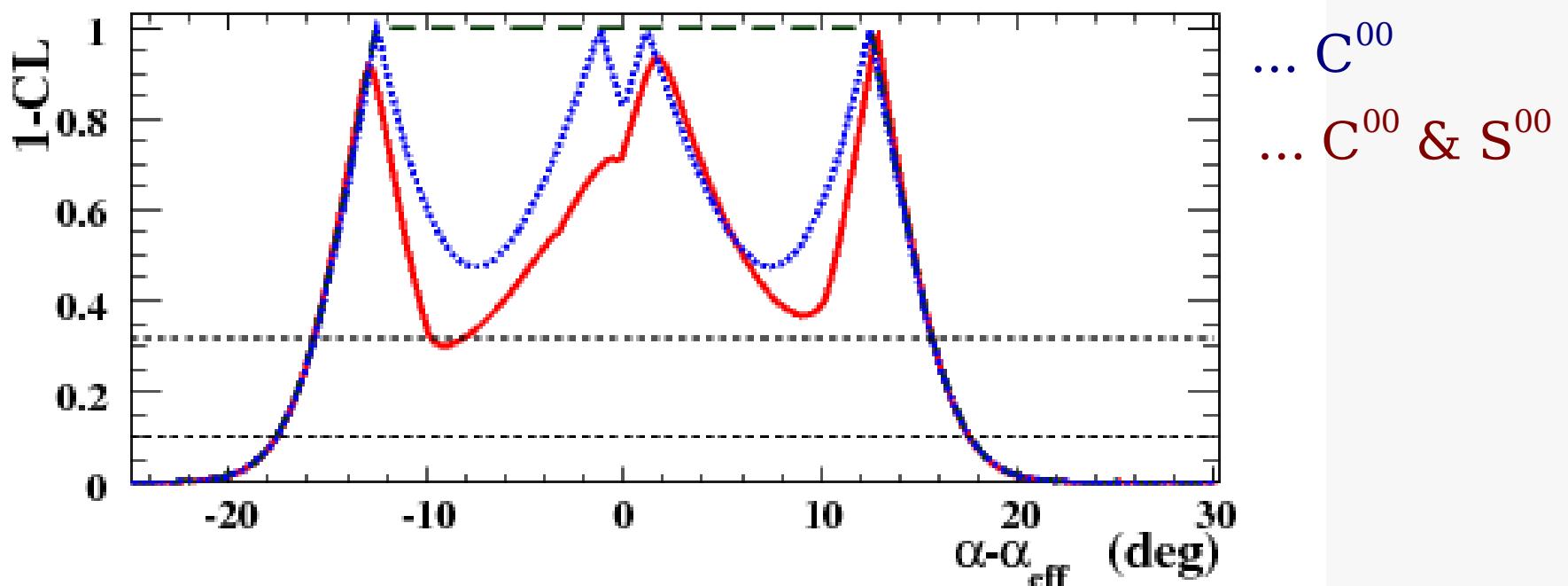
$$f_L = 0.75^{+0.11}_{-0.14} \pm 0.04$$

$$S_L^{00} = 0.3 \pm 0.7 \pm 0.2 \quad C_L^{00} = 0.2 \pm 0.8 \pm 0.3$$

using $\rho^+ \rho^-$: $f_L(\rho^+ \rho^-)$, $\text{Br}(\rho^+ \rho^-)$, S^{+-} , C^{+-} PRD76, 052007 (2007)

$\rho^+ \rho^0$: $f_L(\rho^+ \rho^0)$, $\text{Br}(\rho^+ \rho^0)$ PRL97, 261801 (2006)

$\rho^0 \rho^0$: $f_L(\rho^0 \rho^0)$, $\text{Br}(\rho^0 \rho^0)$ and ...

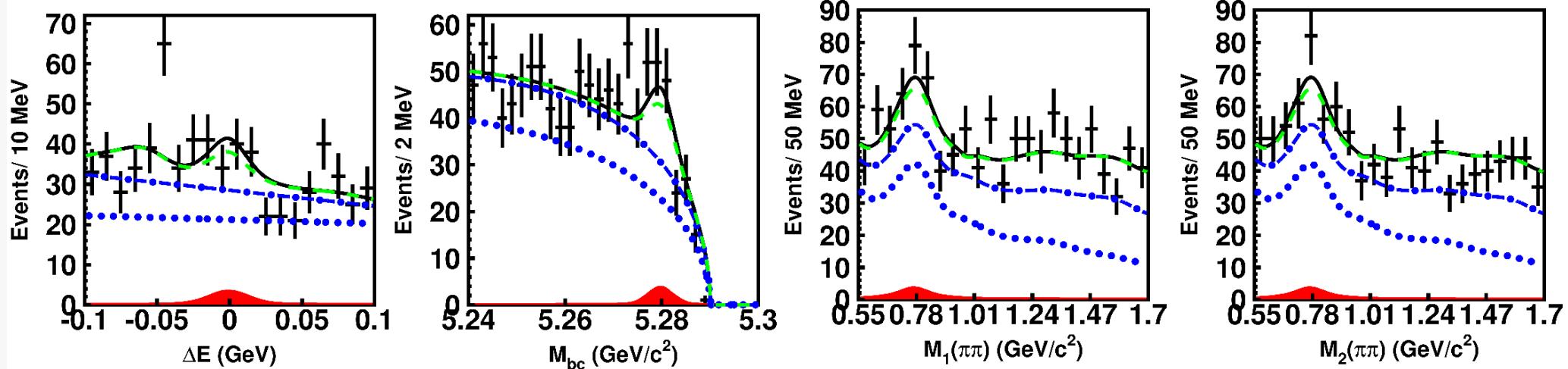


$\rho^0 \rho^0$ mode

$657 \times 10^6 B\bar{B}$ pairs
[PRD78, 111102(R)]



4-dim (ΔE , M_{bc} , $M_{\pi\pi}$, $M_{\pi\pi}$) fit:



$$N_S(\rho^0 \rho^0) = 24.5^{+23.6+10.1}_{-22.1-16.2} \ (\Sigma = 1.0 \ \sigma)$$

$$\text{Br}(B^0 \rightarrow \rho^0 \rho^0) < 1.0 \times 10^{-6} @ 90\% \text{ C.L.}$$

$$= (0.4 \pm 0.4^{+0.2}_{-0.3}) \times 10^{-6}$$

$$N_S(\rho^0 \pi^+ \pi^-) = 113^{+67}_{-66} \pm 52 \ (\Sigma = 1.3 \ \sigma)$$

$$\begin{aligned} \text{Br}(B^0 \rightarrow \rho^0 \pi^+ \pi^-) &< 12.0 \times 10^{-6} @ 90\% \text{ C.L.} \\ &= (5.9^{+3.5}_{-3.4} \pm 2.7) \times 10^{-6} \end{aligned}$$

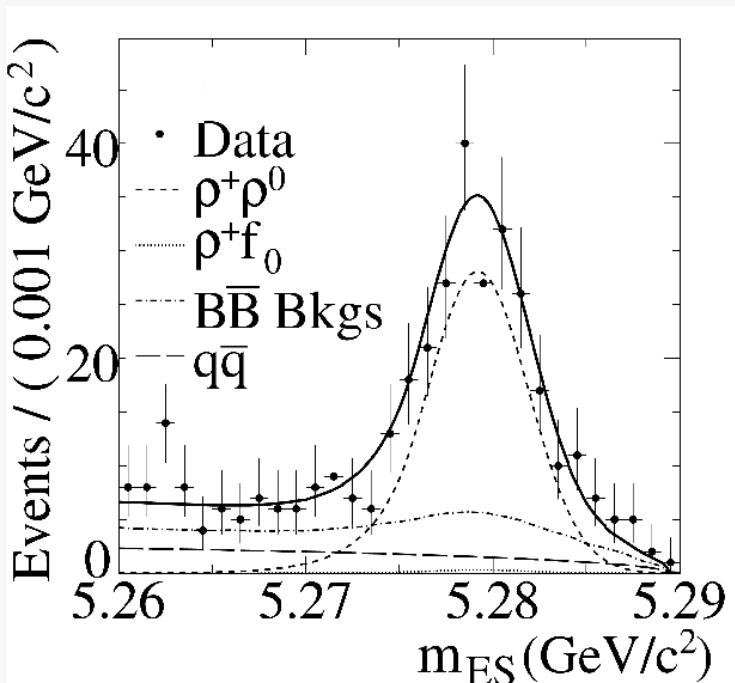
$$N_S(4\pi^\pm) = 161^{+61+28}_{-59-25} \ (\Sigma = 2.5 \ \sigma)$$

$$\begin{aligned} \text{Br}(B^0 \rightarrow 4\pi^\pm) &< 19.3 \times 10^{-6} @ 90\% \text{ C.L.} \\ &= (12.4^{+4.7+2.1}_{-4.6-1.9}) \times 10^{-6} \end{aligned}$$

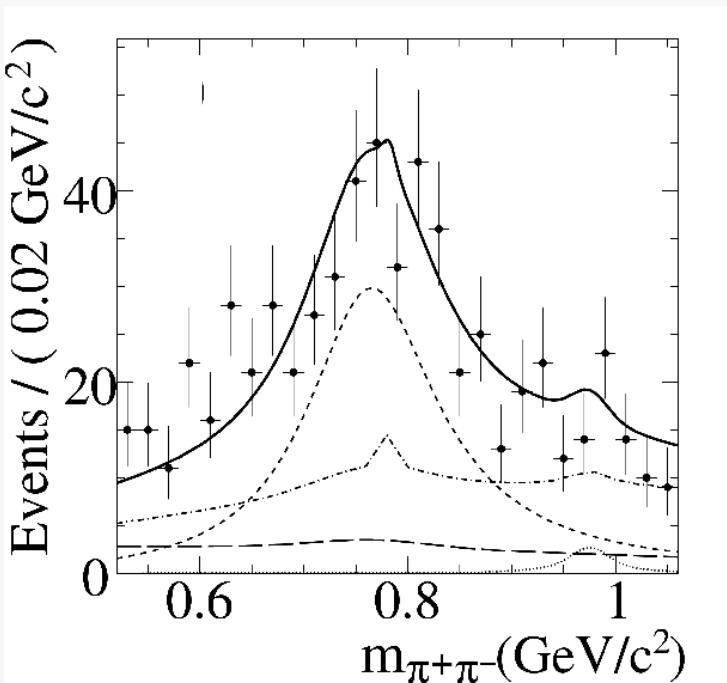
Br($\rho^0 \rho^0$) is small ! SU(2) triangle even more squashed...

$\rho^+ \rho^0$ mode

**465 $\times 10^6$ B \bar{B} pairs
[ArXiv:0901.3522]**



$$N_s(\rho^+ \rho^0) = 1122 \pm 63(\text{stat})$$



$$\text{Br}(B^+ \rightarrow \rho^+ \rho^0) = (23.7 \pm 1.4 \pm 1.4) \times 10^{-6}$$

$$f_L = 0.950 \pm 0.015 \pm 0.006$$

Previous results:

232 $\times 10^6$ B \bar{B} pairs [PRL97, 261801 (2006)]

$$\text{Br}(B^+ \rightarrow \rho^+ \rho^0) = (16.8 \pm 2.2 \pm 2.3) \times 10^{-6}$$

$$f_L = 0.905 \pm 0.042 \quad {}^{+0.023}_{-0.027}$$

85 $\times 10^6$ B \bar{B} pairs [PRL91, 221801 (2003)]

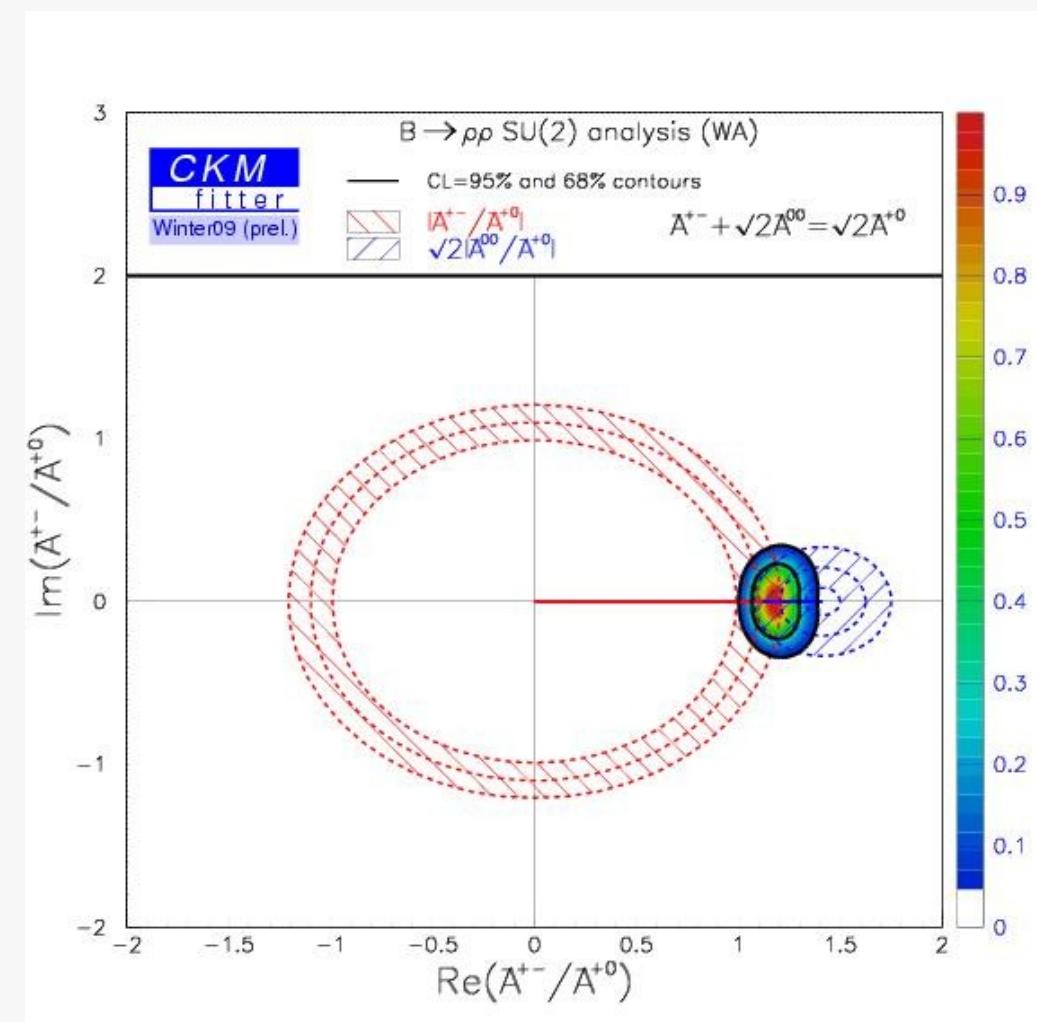
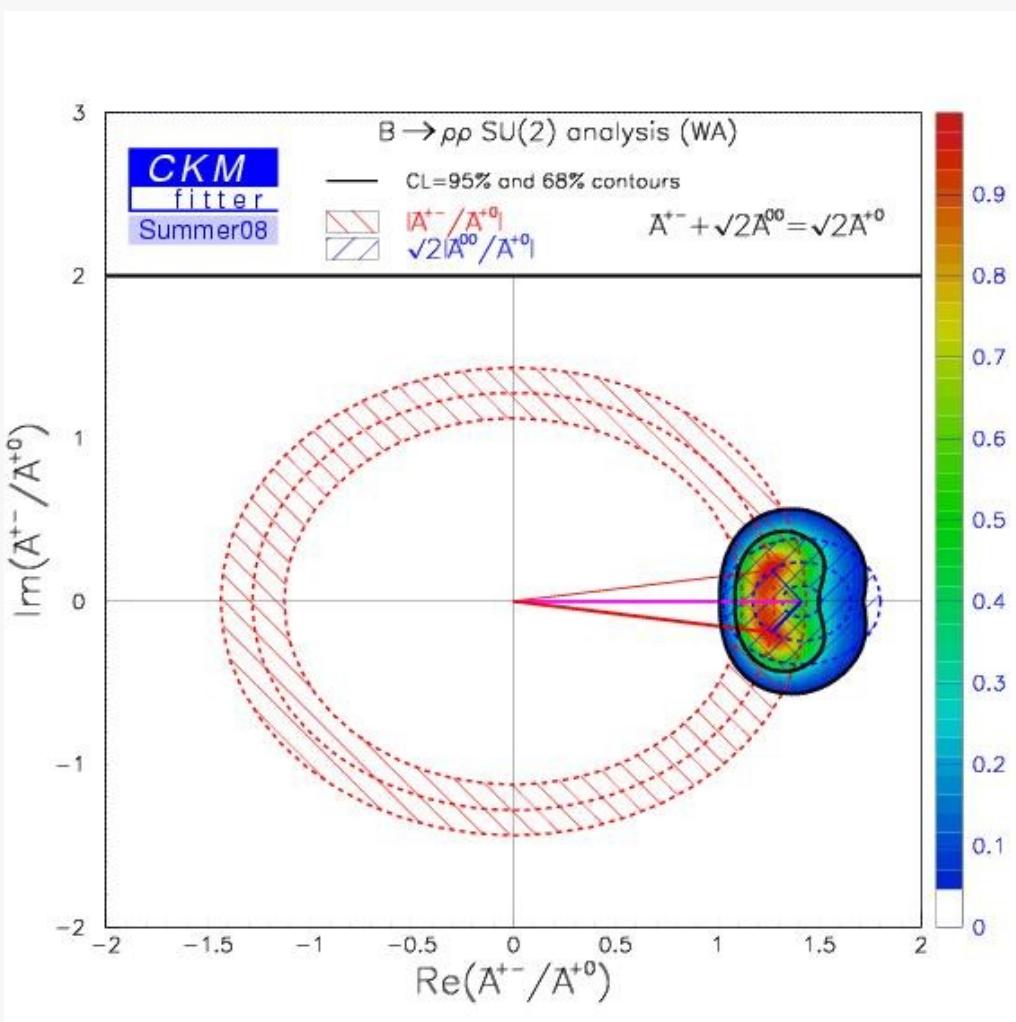
$$\text{Br}(B^+ \rightarrow \rho^+ \rho^0) = (31.7 \pm 7.1 {}^{+3.8}_{-6.7}) \times 10^{-6}$$

$$f_L = 0.948 \pm 0.106 \pm 0.021$$

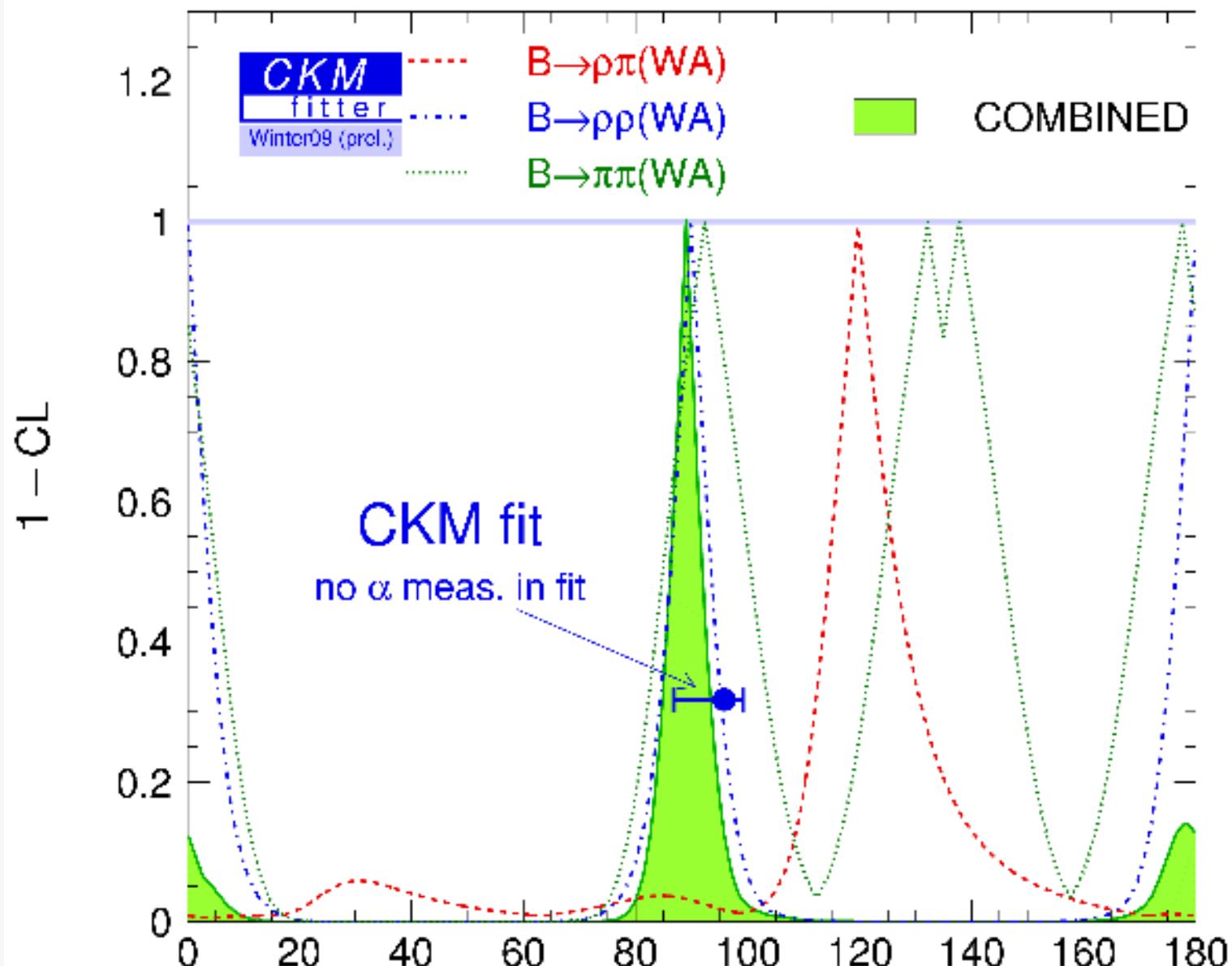


Isospin triangles (Summer 08 to Winter 09)

$B \rightarrow \rho \rho$ case



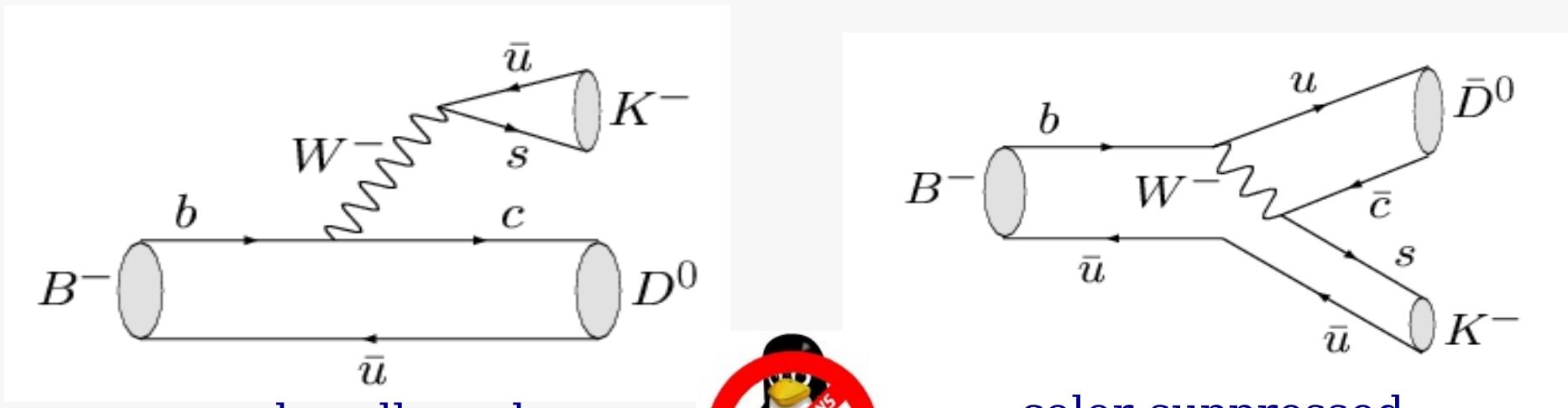
α determination



68% C.L. interval: $\alpha = (89.0^{+4.4}_{-4.2})^\circ$

γ measurements from $B^\pm \rightarrow D\bar{K}^\pm$

- Theoretically pristine $B \rightarrow D\bar{K}$ approach
- Access γ via interference between $B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$



color allowed
 $B^- \rightarrow D^0 K^- \sim V_{cb} V_{us}^*$
 $\sim \mathbf{A} \lambda^3$



color suppressed
 $B^- \rightarrow \bar{D}^0 K^- \sim V_{ub} V_{cs}^*$
 $\sim \mathbf{A} \lambda^3 (\rho + i\eta)$

relative magnitude of suppressed amplitude is r_B

$$r_B = \frac{|A_{\text{suppressed}}|}{|A_{\text{favoured}}|} \sim \frac{|V_{ub} V_{cs}^*|}{|V_{cb} V_{us}^*|} \times [\text{color supp}] = 0.1 - 0.2$$

relative weak phase is γ , relative strong phase is δ_B

γ measurements from $B^\pm \rightarrow D K^\pm$

- Reconstruct D in final states accessible to both D^0 and \bar{D}^0
 - $D = D_{CP}$, CP eigenstates as $K^+ K^-$, $\pi^+ \pi^-$, $K_S \pi^0$
GLW method (Gronau-London-Wyler)
 - $D = D_{sup}$, Doubly-Cabbibo suppressed decays as $K\pi$
ADS method (Atwood-Dunietz-Soni)
 - Three-body decays as $D \rightarrow K_S \pi^+ \pi^-$, $K_S K^+ K^-$
GGSZ (Dalitz) method (Giri-Grossman-Soffer-Zupan)
- Largest effects due to
 - charm mixing
 - charm CP violation

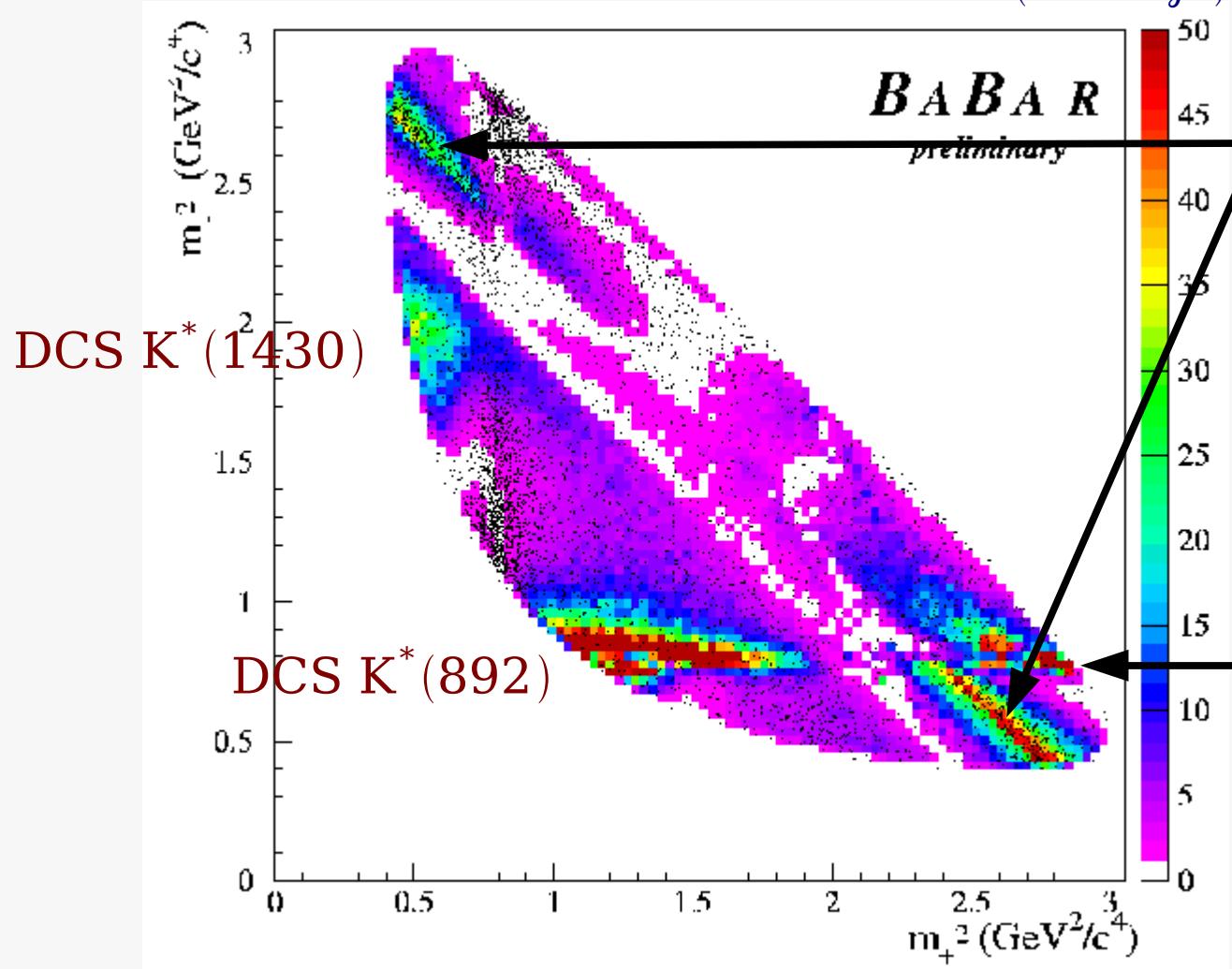
negligible
Y.Grossman, A.Soffer, J.Zupan
[PRD 72, 031501 (2005)]
- Different B decays ($D K$, $D^* K$, $D K^*$)
 - different hadronic factors (r_B , δ_B) for each

Sensitivity to γ

sensitivity to γ/ϕ_3 varies across the Dalitz plot

$$\gamma=75^\circ, \delta=180^\circ, r_B=0.125$$

$$w = 1/(d^2 L/d\gamma^2)$$



GLW like
Interference of
 $B^- \rightarrow D^0 K^-$, $D^0 \rightarrow K_S^0 \rho^0$
with
 $B^- \rightarrow \bar{D}^0 K^-$, $\bar{D}^0 \rightarrow K_S^0 \rho^0$

ADS like
Interference of
 $B^- \rightarrow D^0 K^-$, $D^0 \rightarrow K^{*+} \pi^-$
with
 $B^- \rightarrow \bar{D}^0 K^-$, $\bar{D}^0 \rightarrow K^{*+} \pi^-$

$B \rightarrow D^{(*)} K^{(*)}$ Dalitz analysis

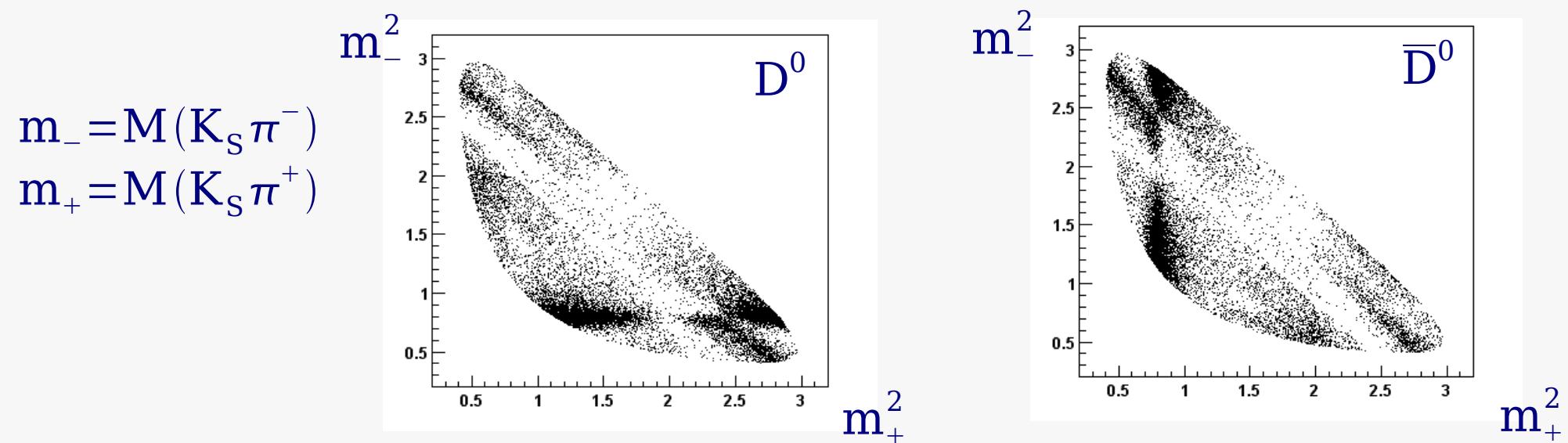
Reconstruction of three-body final states D^0 , $\bar{D}^0 \rightarrow K_S \pi^+ \pi^-$

Amplitude for each Dalitz point is described as:

$$\bar{D}^0 \rightarrow K_S \pi^+ \pi^- \sim f(m_+^2, m_-^2)$$

$$D^0 \rightarrow K_S \pi^+ \pi^- \sim f(m_-^2, m_+^2)$$

$$B^+ \rightarrow (K_S \pi^+ \pi^-)_D K^+ : f(m_+^2, m_-^2) + r e^{i(\delta_B + \gamma)} f(m_-^2, m_+^2)$$



$$B^- \rightarrow (K_S \pi^+ \pi^-)_D K^- : f(m_-^2, m_+^2) + r e^{i(\delta_B - \gamma)} f(m_+^2, m_-^2)$$

Simultaneous fit of B^+ and B^- to extract parameters r_B , ϕ_3 and δ_B

Note: 2 fold ambiguity on γ : $(\gamma, \delta_B) \rightarrow (\gamma + \pi, \delta_B + \pi)$

γ measurements from $B^\pm \rightarrow D K^\pm$ r_B dependence

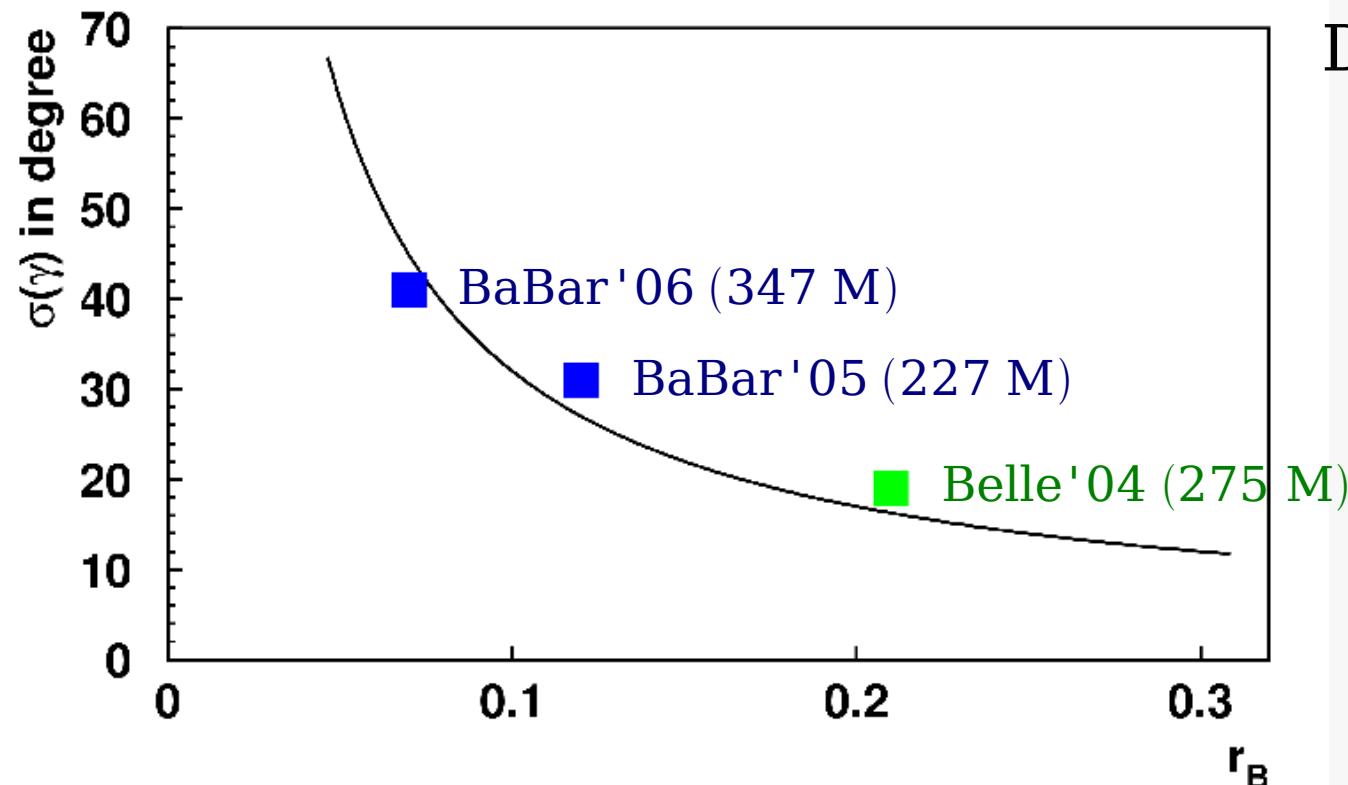
Dalitz $B \rightarrow D(K_S \pi \pi)K$

experimental inputs:

$$x_\pm = r_B \cos(\delta_B \pm \gamma)$$

$$y_\pm = r_B \sin(\delta_B \pm \gamma)$$

uncertainty on γ scales as $1/r_B$!

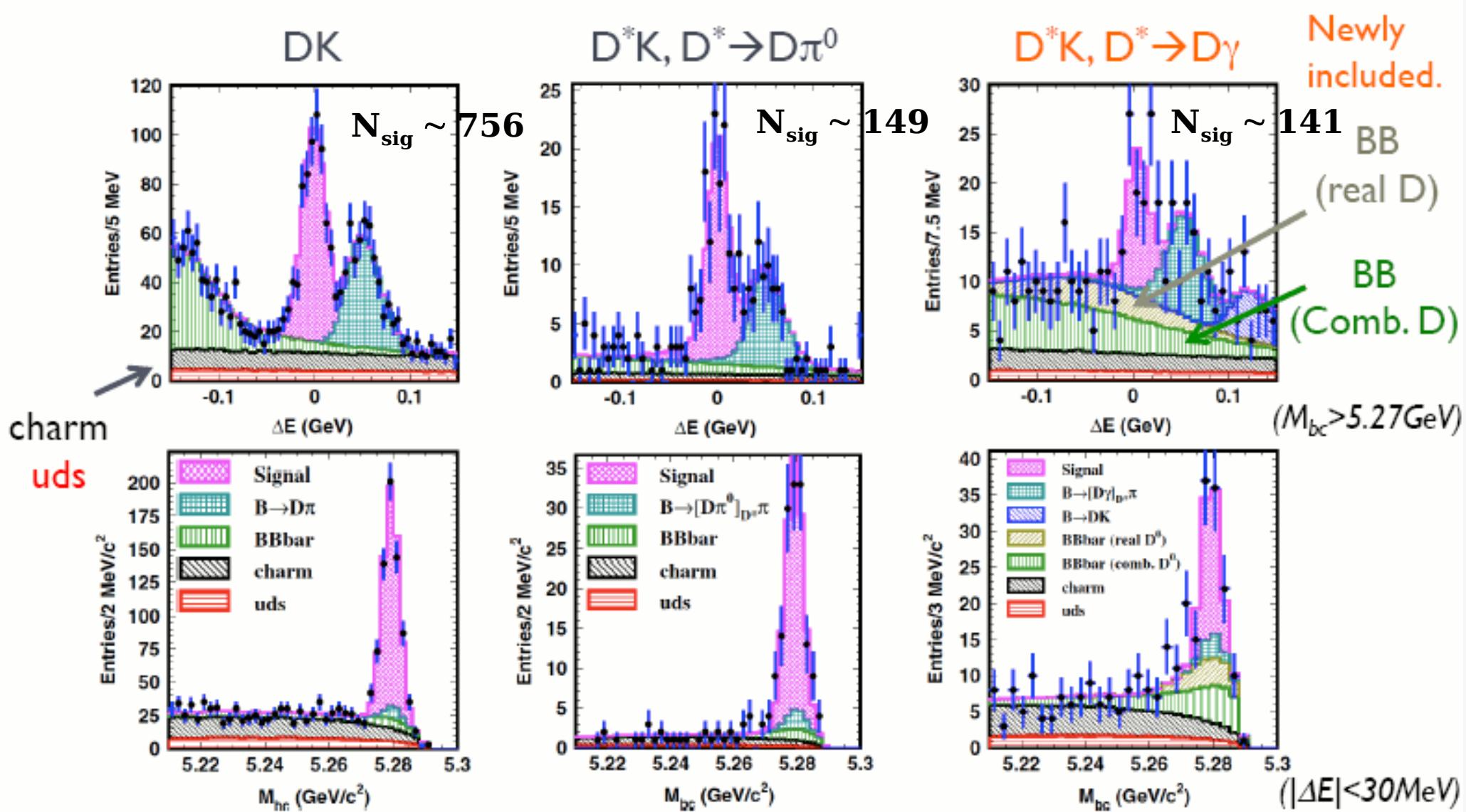


DK (stat only)
 $\sigma(x_\pm) \sim 0.07$
 $\sigma(y_\pm) \sim 0.08$

$B^- \rightarrow D^{(*)}(K_S \pi \pi) K^-$ Dalitz, ΔE and M_{bc} projections

$|\cos\theta_{\text{thr}}| < 0.8$ and $F > -0.7$

PRD 81, 112002 (2010)
 $657 \times 10^6 B\bar{B}$ pairs

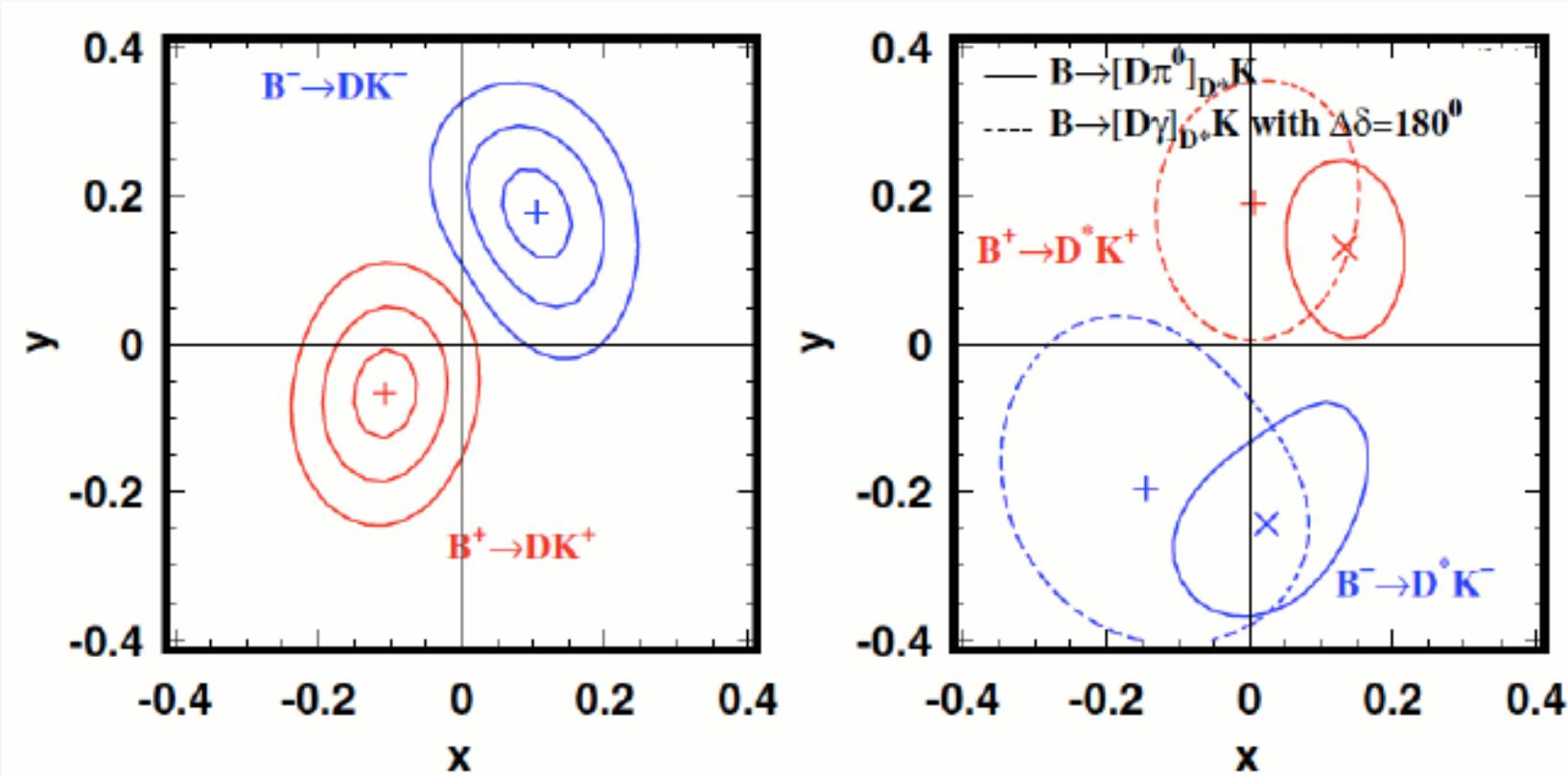


γ measurement with $B \rightarrow D(K_S \pi \pi)K$

PRD 81, 112002 (2010)

$657 \times 10^6 B\bar{B}$ pairs

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma), \quad y_{\pm} = r_B \sin(\delta_B \pm \gamma)$$



$$\gamma = (80.8^{+13.1}_{-14.8} \pm 5.0 \pm 8.9)^\circ$$

$$r_B = 0.161^{+0.040}_{-0.038} \pm 0.011^{+0.050}_{-0.010}$$

$$\delta_B = (137.4^{+13.0}_{-15.7} \pm 4.0 \pm 22.9)^\circ$$

$$\gamma = (73.9^{+18.9}_{-20.2} \pm 4.2 \pm 8.9)^\circ$$

$$r_B = 0.196^{+0.073}_{-0.072} \pm 0.013^{+0.062}_{-0.012}$$

$$\delta_B = (341.7^{+18.6}_{-20.9} \pm 3.2 \pm 22.9)^\circ$$

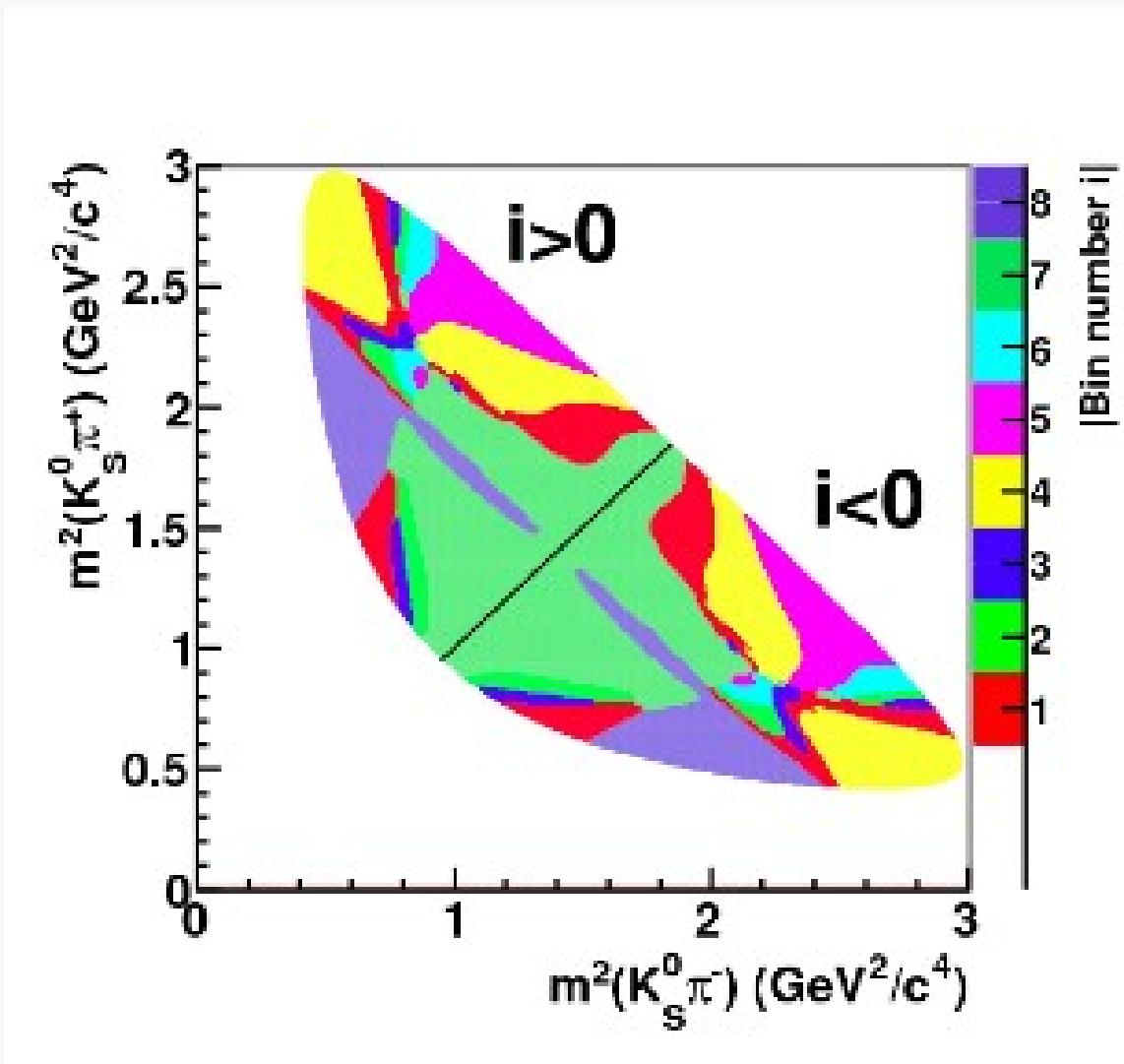
combining both B modes (Dalitz): $\gamma = (78.4^{+10.8}_{-11.6} \pm 3.6 \pm 8.9)^\circ$

CPV significance is 3.5 standard deviations

(model-dependent error will limit viability of this approach)



Binned Dalitz method: avoid the modeling error by
"optimal" binning of the Dalitz plot



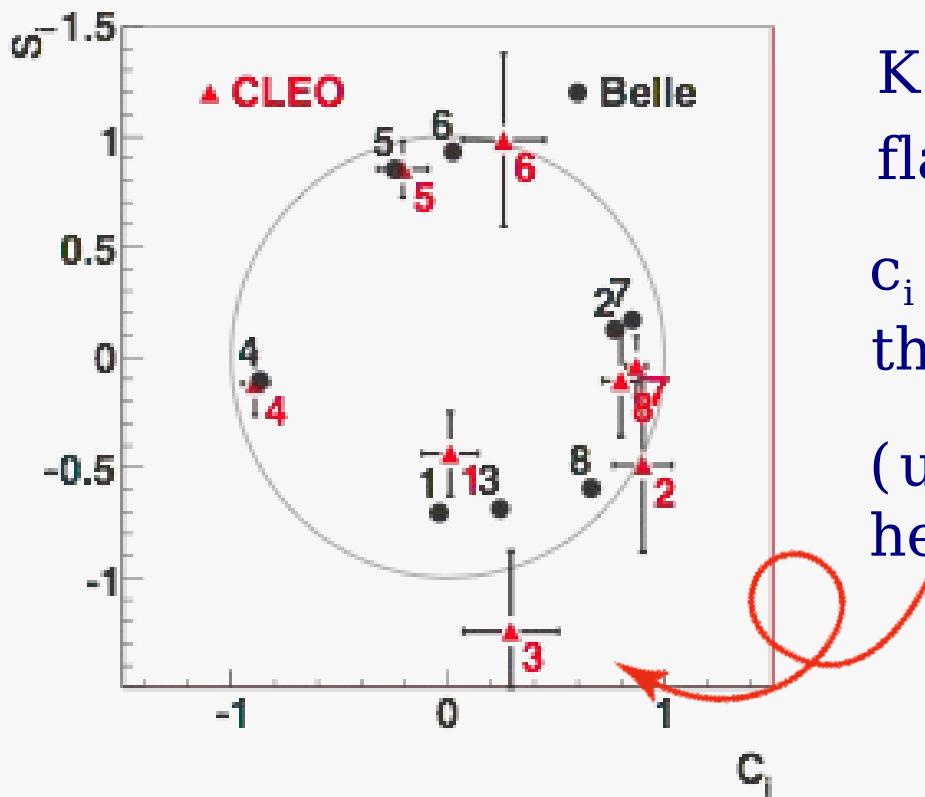
Bondar and
Poluektov
EPJ C55, 51 (2008)

choice of bins guided by model, but extracted γ is
not biased by this choice

Binned Dalitz method: minimize χ^2
in fit to all bins for each mode

Expected number of $B^\pm \rightarrow D K^\pm$ events in bin i is:

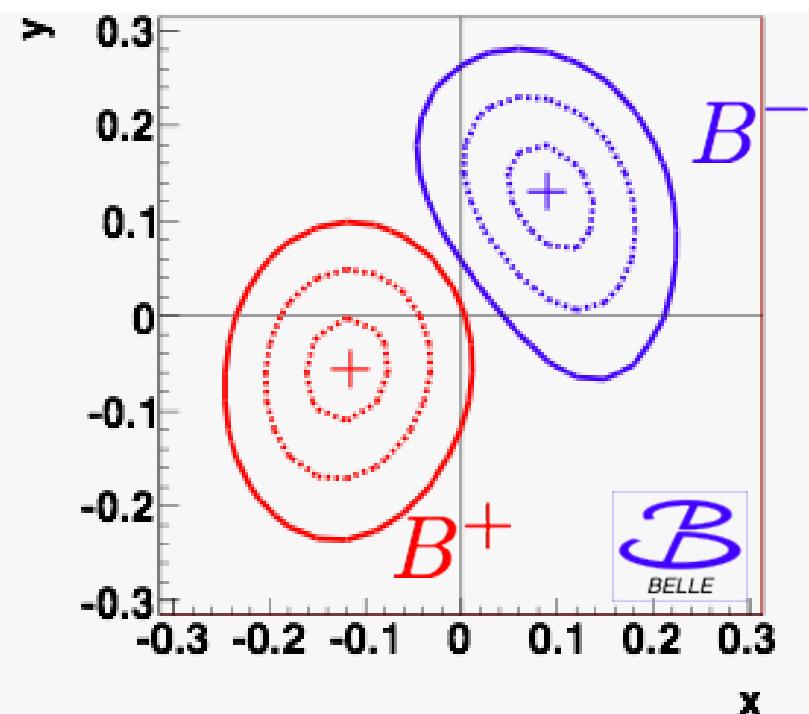
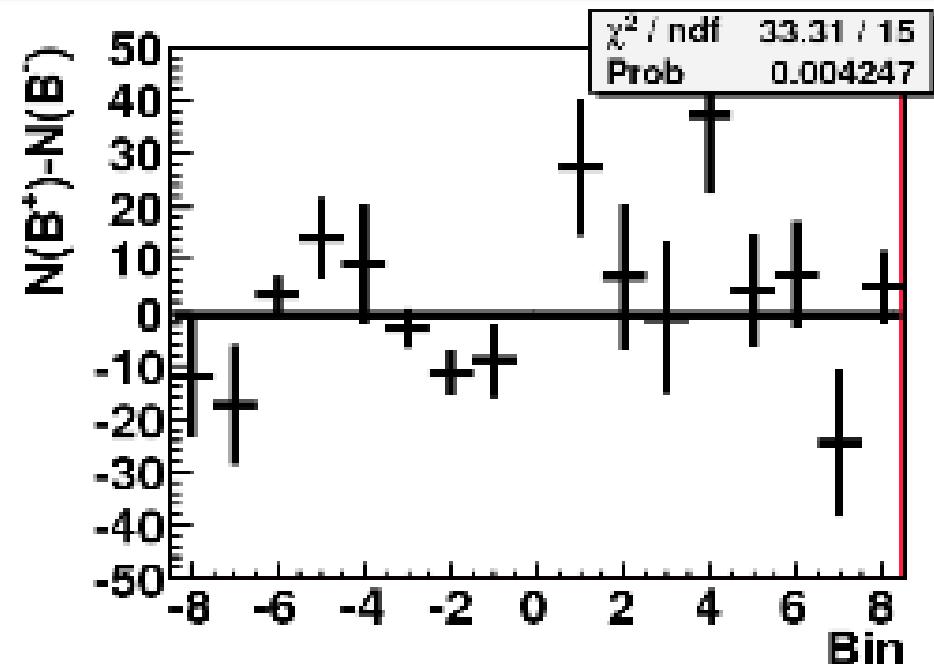
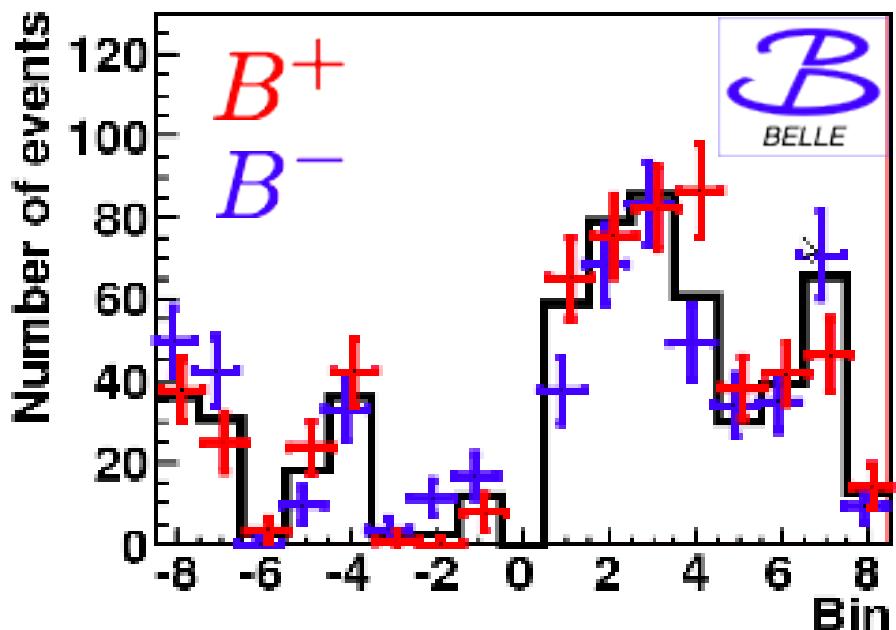
$$N_i^\pm = h \left\{ K_i + r_B^2 K_{-i} + 2\sqrt{K_i K_{-i}}(x_\pm c_i + y_\pm s_i) \right\}^{1 - \frac{m^2(K_s^0 \pi^-)}{m^2(K_s^0 \pi^+)} \frac{2}{(GeV^2/c^4)} \frac{3}{}$$



- K_i is the # of events in bin i from a flavour-tagged sample ($D^{*\pm} \rightarrow D\pi^\pm$)
- c_i and s_i contain information about the strong-phase difference in bin i
(use CLEO data for $\psi(3770) \rightarrow D^0 \bar{D}^0$ here; can be measured by BES-III too)

Binned Dalitz method result in $B \rightarrow DK$ from 772 million events

arXiv:1106.4046



$$\gamma = (77.3 {}^{+15.1}_{-14.9} \pm 4.2 \pm 4.3)^\circ$$

$$r_B = 0.145 \pm 0.030 \pm 0.011 \pm 0.011$$

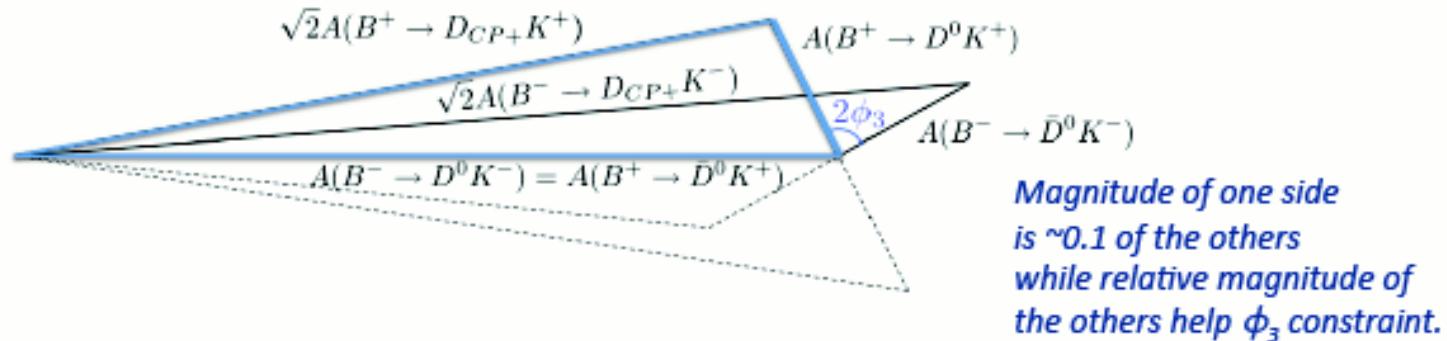
$$\delta_B = (129.9 \pm 15.0 \pm 3.9 \pm 4.7)^\circ$$

uncertainty in c_i, s_i
from CLEO data
(can reduce using
future BES-III data)

GLW with $D_{CP}K$

D decays to CP eigenstates

➤ Amplitude triangle:



Usually measured observables:

$$\mathcal{R}_{CP\pm} \equiv \frac{\mathcal{B}(B^- \rightarrow D_{CP\pm} K^-) + \mathcal{B}(B^+ \rightarrow D_{CP\pm} K^+)}{\mathcal{B}(B^- \rightarrow D^0 K^-) + \mathcal{B}(B^+ \rightarrow \bar{D}^0 K^+)}$$

$$\mathcal{A}_{CP\pm} \equiv \frac{\mathcal{B}(B^- \rightarrow D_{CP\pm} K^-) - \mathcal{B}(B^+ \rightarrow D_{CP\pm} K^+)}{\mathcal{B}(B^- \rightarrow D_{CP\pm} K^-) + \mathcal{B}(B^+ \rightarrow D_{CP\pm} K^+)}$$

Relation between $(A_{CP+}, A_{CP-}, R_{CP+}, R_{CP-})$ and (γ, r_B, δ_B)

$$A_{CP+} = \frac{2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma}$$

$$A_{CP-} = \frac{-2r_B \sin \delta_B \sin \gamma}{1 + r_B^2 - 2r_B \cos \delta_B \cos \gamma}$$

$$R_{CP+} = 1 + r_B^2 + 2r_B \cos \delta_B \cos \gamma$$

$$R_{CP-} = 1 + r_B^2 - 2r_B \cos \delta_B \cos \gamma$$

⇒ look for $R_{CP\pm} \neq 1$ and $A_{CP\pm} \neq 0$

$\mathbf{B} \rightarrow \mathbf{Dh}$, $\mathbf{D} \rightarrow \mathbf{K}\pi \rightarrow \mathbf{R}_{\mathbf{D}_{\text{fav}}}$

MC (case B)

$B \rightarrow D\pi$

$B \rightarrow DK$

$B\bar{B}$

continuum

$$\begin{aligned} N_{\eta, KID>0.6}^{DK} &= \frac{1}{2} (1 - \eta A^{DK}) N_{tot}^{D\pi} R_{K/\pi} \epsilon \\ N_{\eta, KID<0.6}^{DK} &= \frac{1}{2} (1 - \eta A^{DK}) N_{tot}^{D\pi} R_{K/\pi} (1 - \epsilon) \\ N_{\eta, KID>0.6}^{D\pi} &= \frac{1}{2} (1 - \eta A^{D\pi}) N_{tot}^{D\pi} \kappa \\ N_{\eta, KID<0.6}^{D\pi} &= \frac{1}{2} (1 - \eta A^{D\pi}) N_{tot}^{D\pi} (1 - \kappa) \end{aligned}$$

	kaon fake (1- ϵ)	kaon eff ϵ	pion eff (1- κ)	pion fake κ	
MC	14.70 ± 0.06	85.41 ± 0.06	95.42 ± 0.03	4.47 ± 0.03	←
data	15.86 ± 0.40	84.32 ± 0.39	92.13 ± 0.46	7.94 ± 0.31	

Table 5: Efficiency and fake rate (in %) for kaon and pion, for data and MC. ϵ will be fixed in the fit but κ will be floated (see text for further explanations). These numbers are obtained after properly weighting the values provided by PID group for SVD1 and SVD2.

$\Rightarrow \kappa = (4.58 \pm 0.10)\%$ in the MC fit

$B \rightarrow D h$, $D \rightarrow K \pi \rightarrow R_{D_{\text{fav}}}$

MC

$B \rightarrow D \pi$

$B \rightarrow D K$

$B \bar{B}$

continuum

$$N_{\eta, KID>0.6}^{DK} = \frac{1}{2} (1 - \eta A^{DK}) N_{tot}^{D\pi} R_{K/\pi} \epsilon$$

$$N_{\eta, KID<0.6}^{DK} = \frac{1}{2} (1 - \eta A^{DK}) N_{tot}^{D\pi} R_{K/\pi} (1 - \epsilon)$$

$$N_{\eta, KID>0.6}^{D\pi} = \frac{1}{2} (1 - \eta A^{D\pi}) N_{tot}^{D\pi} \kappa$$

$$N_{\eta, KID<0.6}^{D\pi} = \frac{1}{2} (1 - \eta A^{D\pi}) N_{tot}^{D\pi} (1 - \kappa)$$

KID<0.6

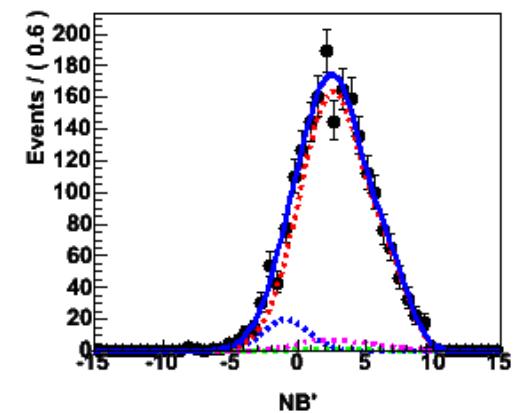
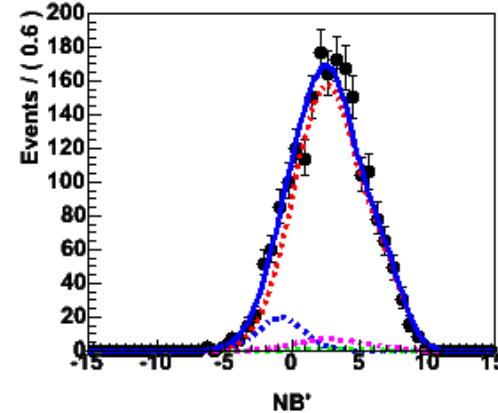
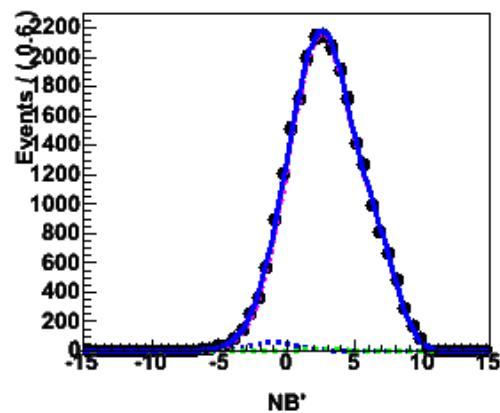
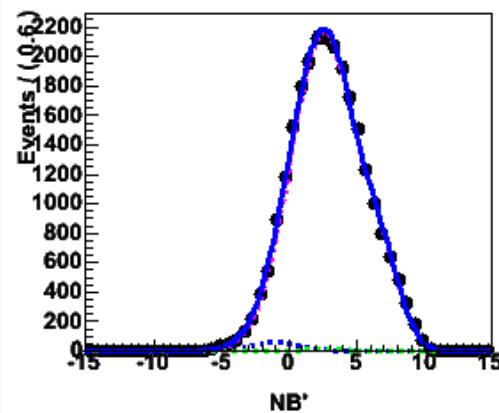
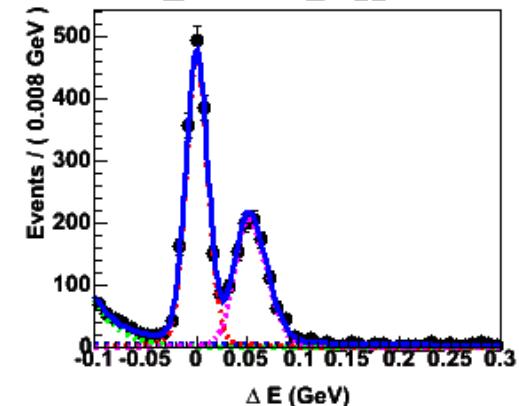
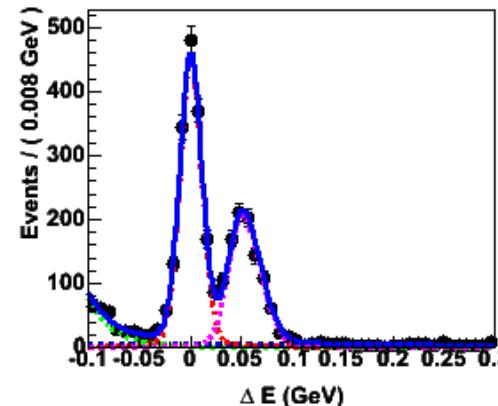
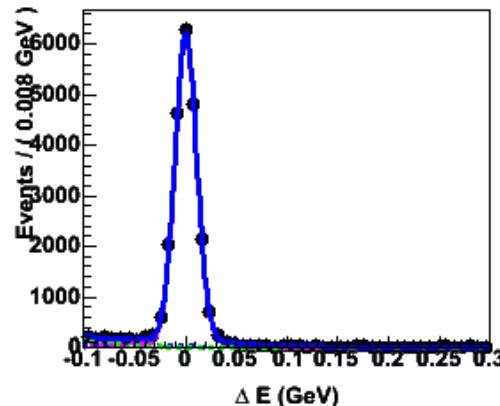
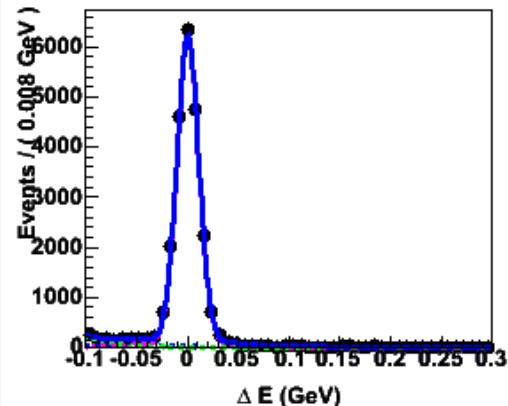
KID>0.6

$B^+ \rightarrow D h^+$

$B^- \rightarrow D h^-$

$B^+ \rightarrow D h^+$

$B^- \rightarrow D h^-$



$B \rightarrow D h$, $D \rightarrow K \pi \rightarrow R_{D_{\text{fav}}}$ data (772 MB \bar{B})

$B \rightarrow D\pi$

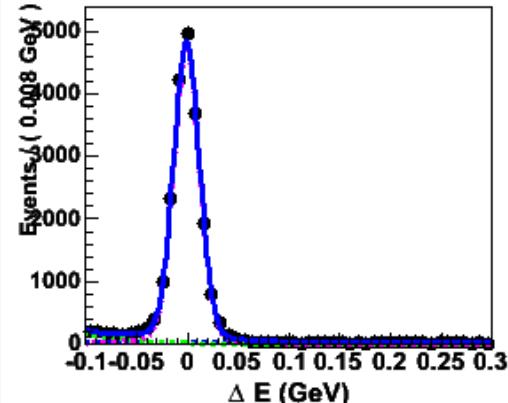
$B \rightarrow DK$

$B\bar{B}$

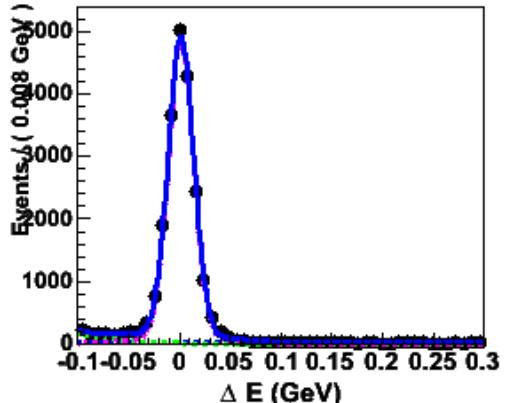
continuum

h is a pion candidate ($KID < 0.6$)

$B^- \rightarrow Dh^-$

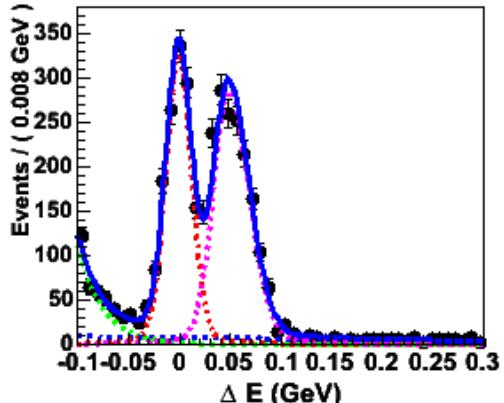


$B^+ \rightarrow Dh^+$

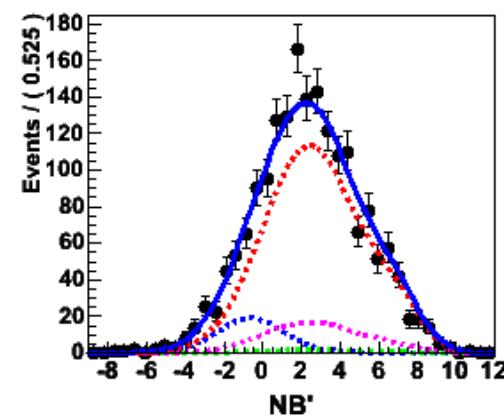
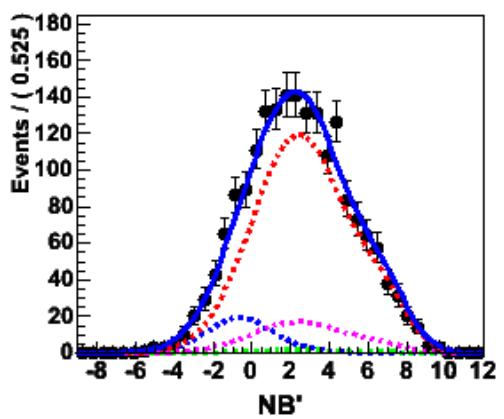
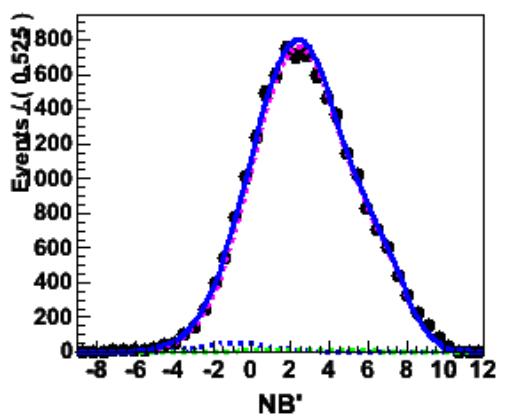
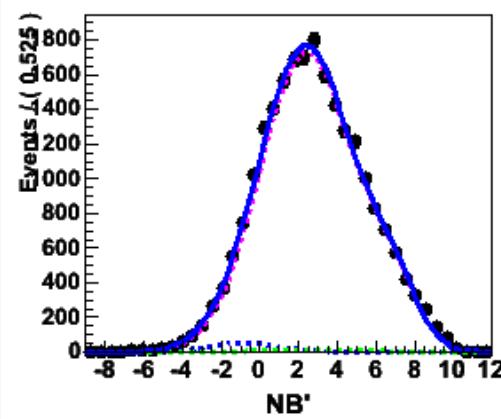
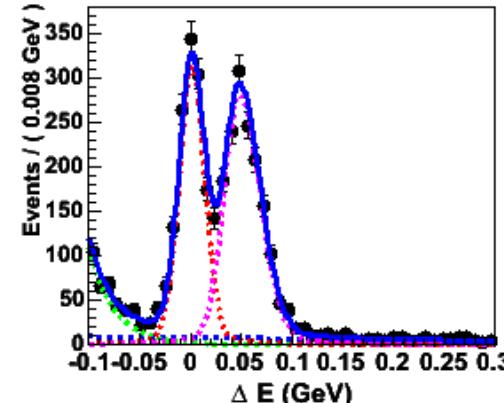


h is a kaon candidate ($KID > 0.6$)

$B^- \rightarrow Dh^-$



$B^+ \rightarrow Dh^+$



$$\Rightarrow R_{D_{\text{fav}}} = (7.32 \pm 0.16)\%, \quad A(DK) = (1.4 \pm 2.0)\%$$

$B \rightarrow D h$, $D \rightarrow K\pi \rightarrow R_+$ data (772 MB \bar{B})

$D \rightarrow K^+ K^-$, $\pi^+ \pi^-$

Preliminary
LP 2011

$B \rightarrow D\pi$

$B \rightarrow DK$

$B\bar{B}$

continuum

h is a pion candidate ($KID < 0.6$)

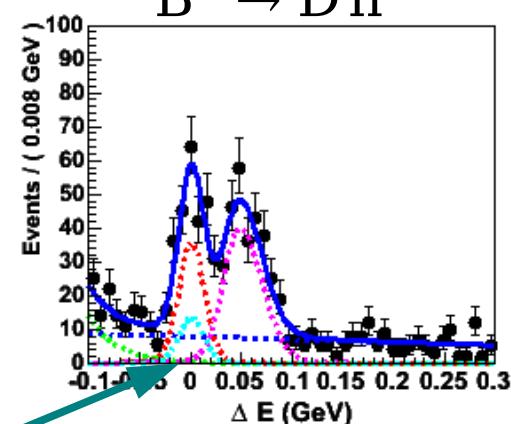
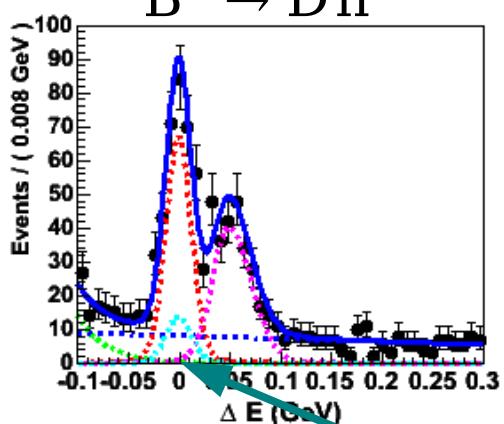
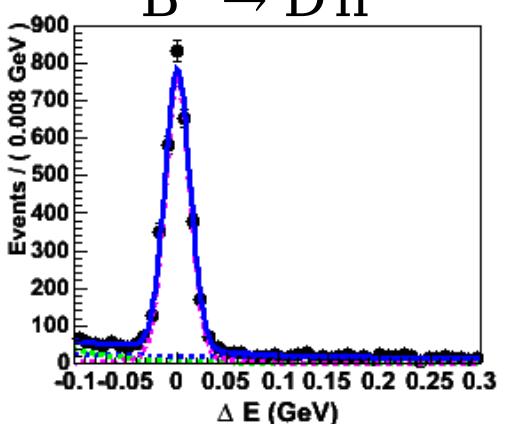
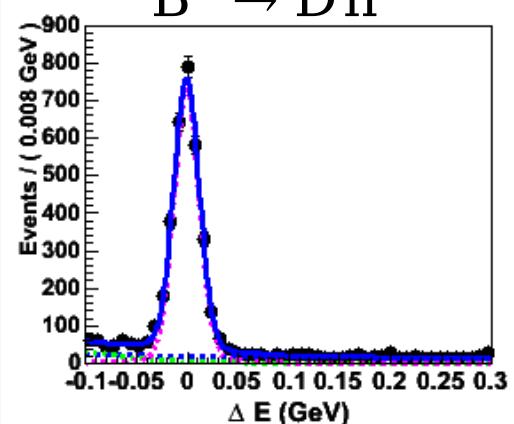
h is a kaon candidate ($KID > 0.6$)

$B^- \rightarrow Dh^-$

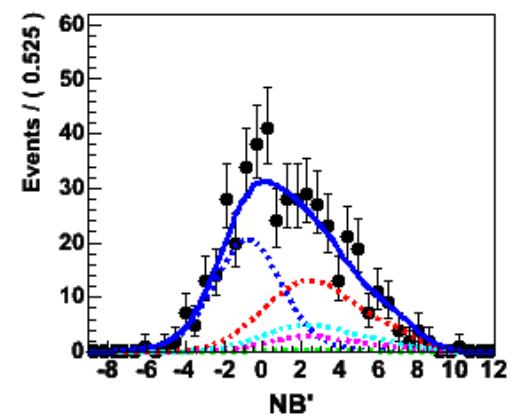
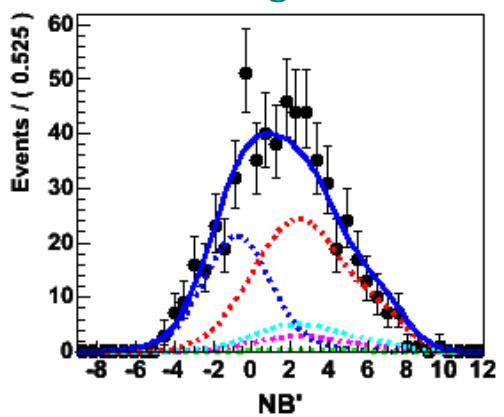
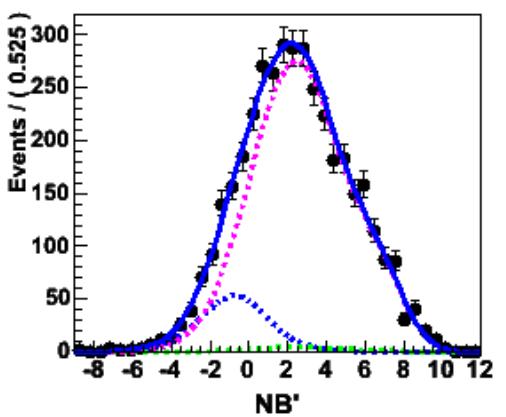
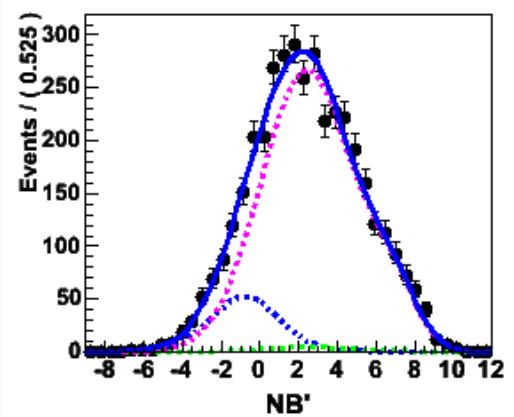
$B^+ \rightarrow Dh^+$

$B^- \rightarrow Dh^-$

$B^+ \rightarrow Dh^+$



large KKK contribution !!



$$\Rightarrow R_{D_{CP+}} = (7.56 \pm 0.51)\%, \quad A_{D_{CP+}} = (28.7 \pm 6.0)\%$$

large asymmetry !!

$B \rightarrow D h$, $D \rightarrow K\pi \rightarrow R_-$ data (772 MB \bar{B})

$D \rightarrow K_S \pi^0, K_S \eta (\gamma\gamma)$

Preliminary
LP 2011

$B \rightarrow D\pi$

$B \rightarrow DK$

$B\bar{B}$

continuum

h is a pion candidate ($KID < 0.6$)

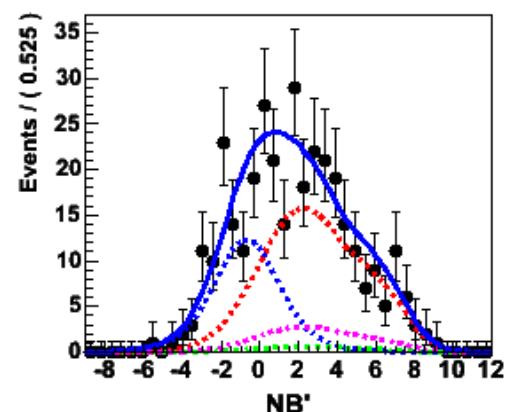
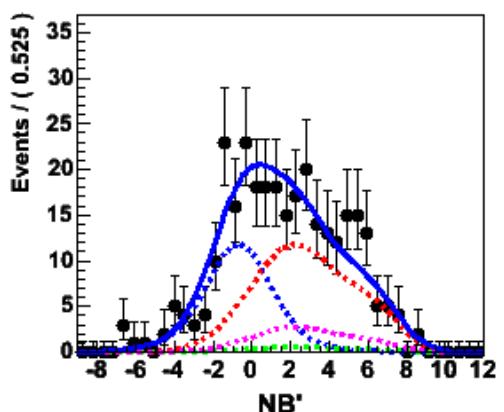
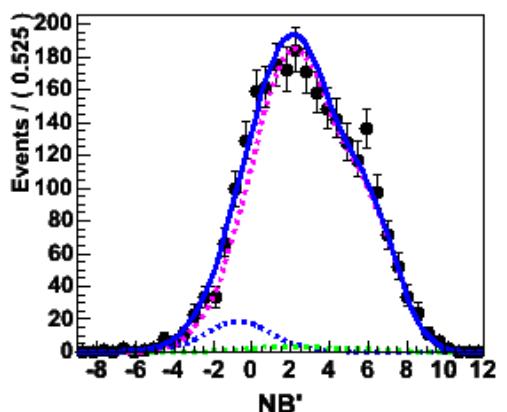
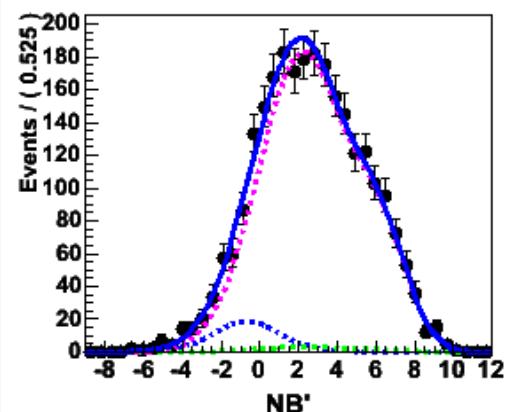
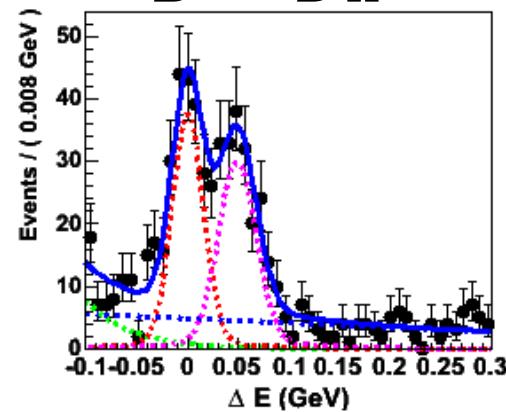
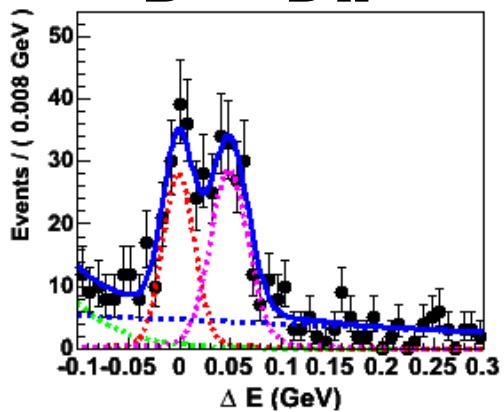
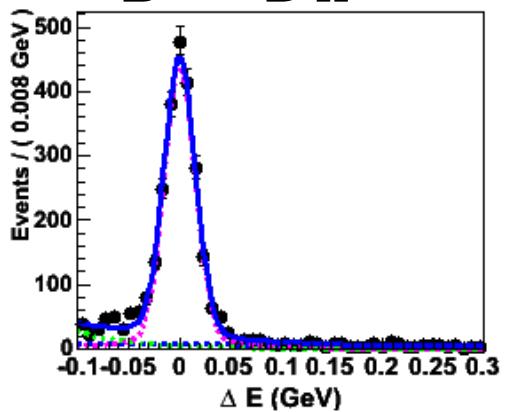
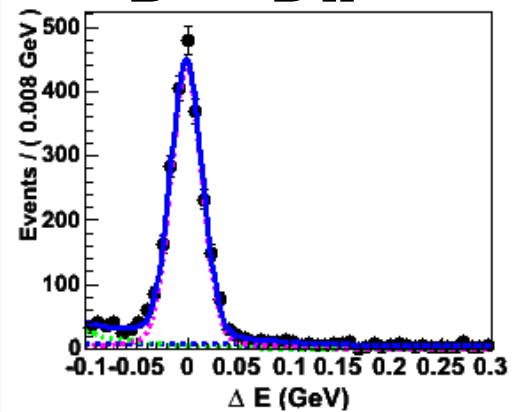
h is a kaon candidate ($KID > 0.6$)

$B^- \rightarrow Dh^-$

$B^+ \rightarrow Dh^+$

$B^- \rightarrow Dh^-$

$B^+ \rightarrow Dh^+$



$$\Rightarrow R_{D_{CP-}} = (8.29 \pm 0.63)\%, \quad A_{D_{CP-}} = (-12.4 \pm 6.4)\%$$

opposite asymmetry !!

GLW Results

Preliminary
LP 2011

Yields	$B \rightarrow D\pi$	$B \rightarrow DK$
$D \rightarrow K\pi$	50432 ± 243	3692 ± 83
$D \rightarrow KK, \pi\pi$	7696 ± 106	582 ± 40
$D \rightarrow K_S\pi^0, K_S\eta$	5745 ± 91	476 ± 37

$$R_{CP+} = 1.03 \pm 0.07 \pm 0.03$$

$$R_{CP-} = 1.13 \pm 0.09 \pm 0.05$$

$$A_{CP+} = +0.29 \pm 0.06 \pm 0.02$$

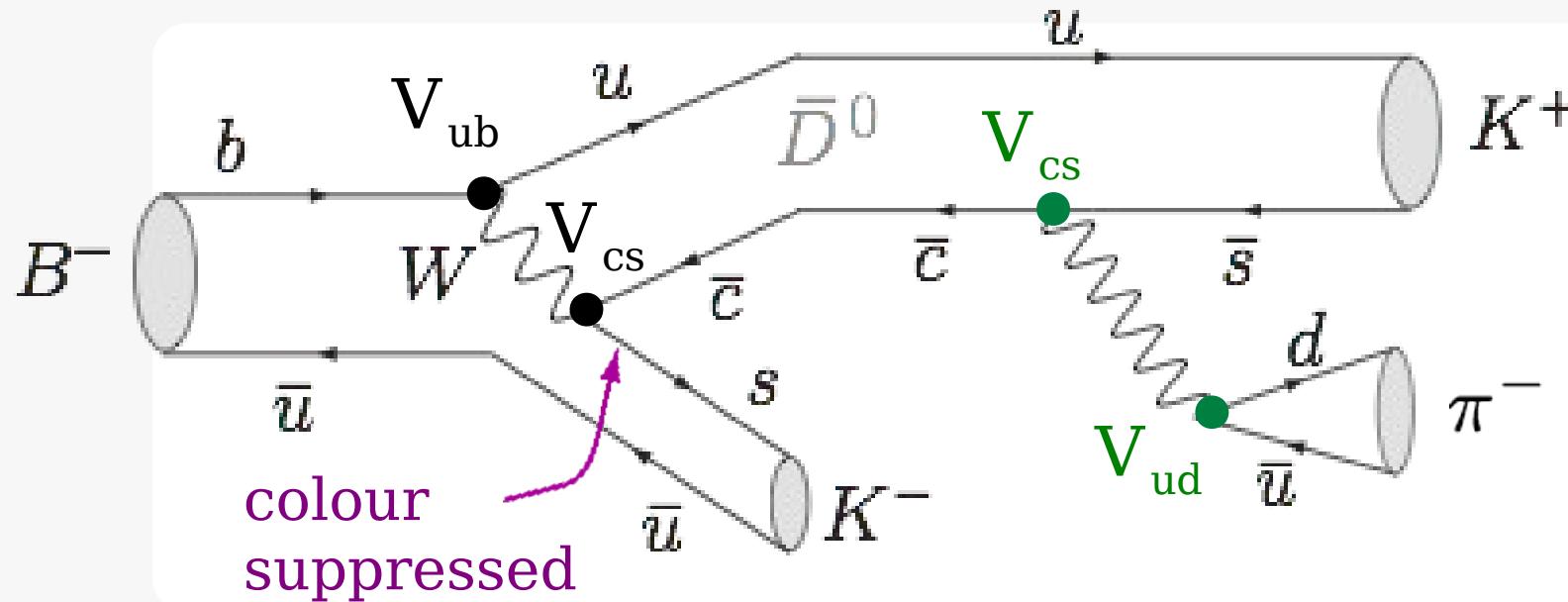
$$A_{CP-} = -0.12 \pm 0.06 \pm 0.01$$

systematics dominated by peaking background,
double ratio approximation

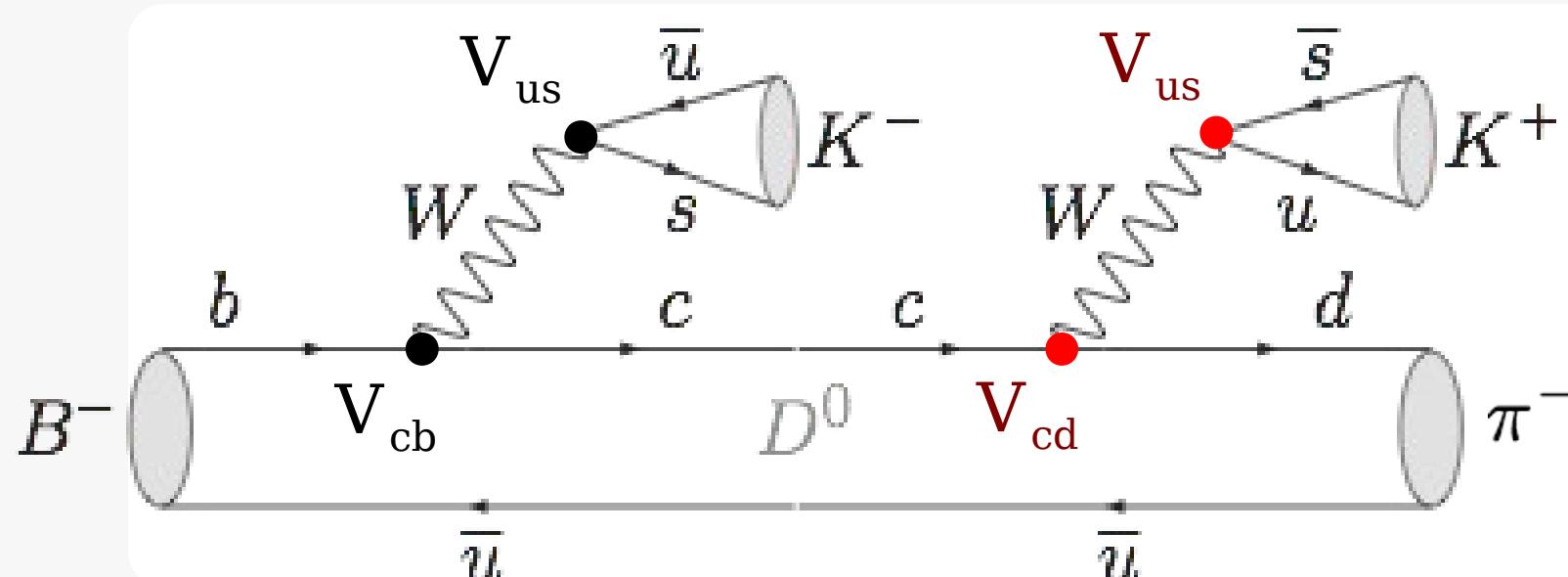
coming improvement: adding $K_S\omega, K_S\eta'$ for CP-odd modes

coming update: D^*K modes

ADS method measures ϕ_3 via the interference in rare $B^- \rightarrow [K^+ \pi^-]_D K^-$ decays



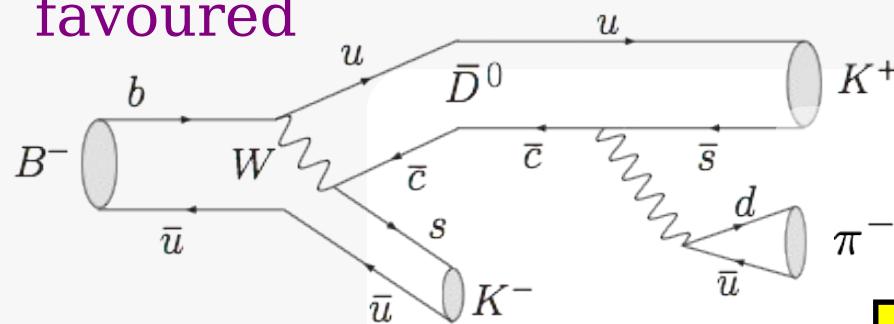
Cabibbo
favoured
D decay



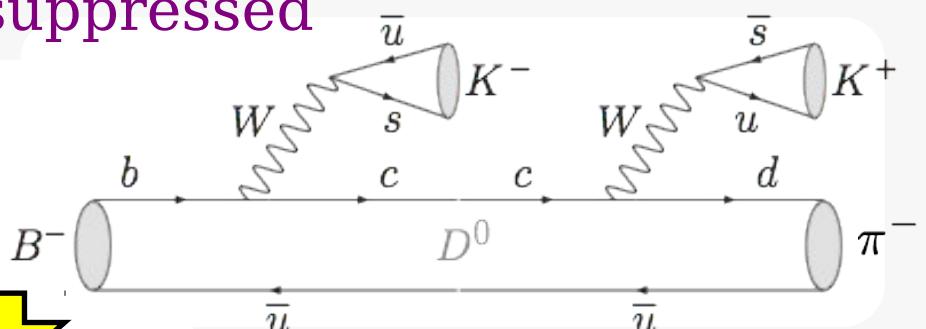
doubly
Cabibbo
suppressed
D decay

ADS rate and asymmetry (relative to the common decay):

favoured

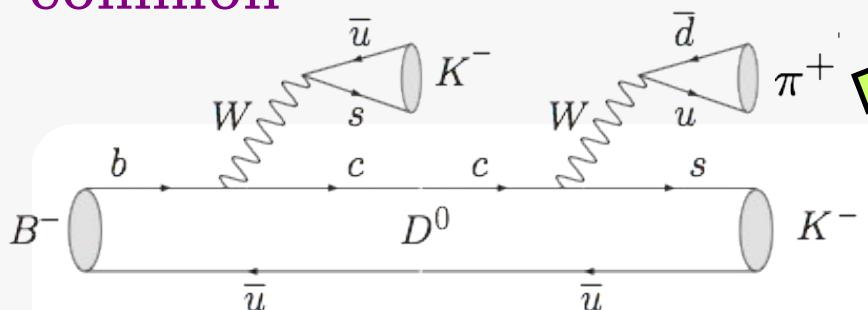


suppressed



$$\mathcal{R}_{DK} = \frac{\Gamma([K^+\pi^-] K^-) + \Gamma([K^-\pi^+] K^+)}{\Gamma([K^-\pi^+] K^-) + \Gamma([K^+\pi^-] K^+)}$$

common



$$= r_B^2 + r_D^2 + 2r_B r_D \cos(\delta_B + \delta_D) \cos \phi_3$$

$$\mathcal{A}_{DK} = \frac{\Gamma([K^+\pi^-] K^-) - \Gamma([K^-\pi^+] K^+)}{\Gamma([K^-\pi^+] K^-) + \Gamma([K^+\pi^-] K^+)}$$

$$= 2r_B r_D \sin(\delta_B + \delta_D) \sin \phi_3 / \mathcal{R}_{DK}$$

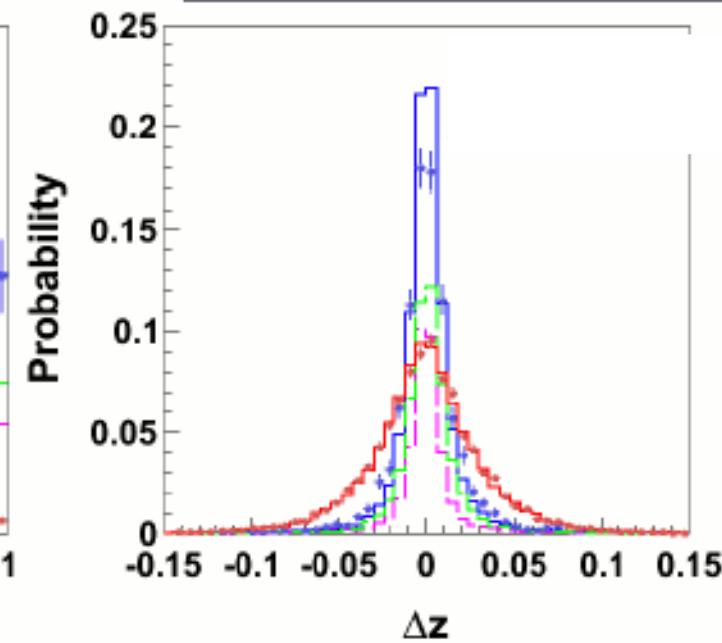
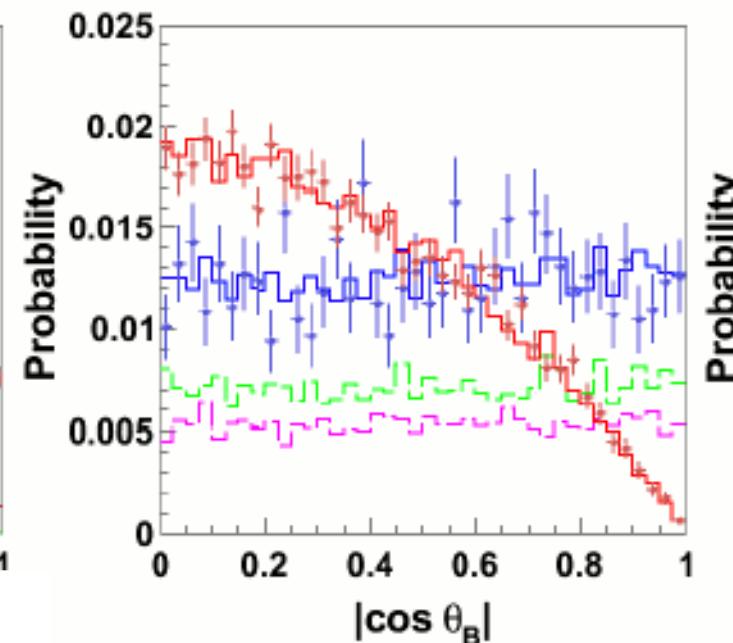
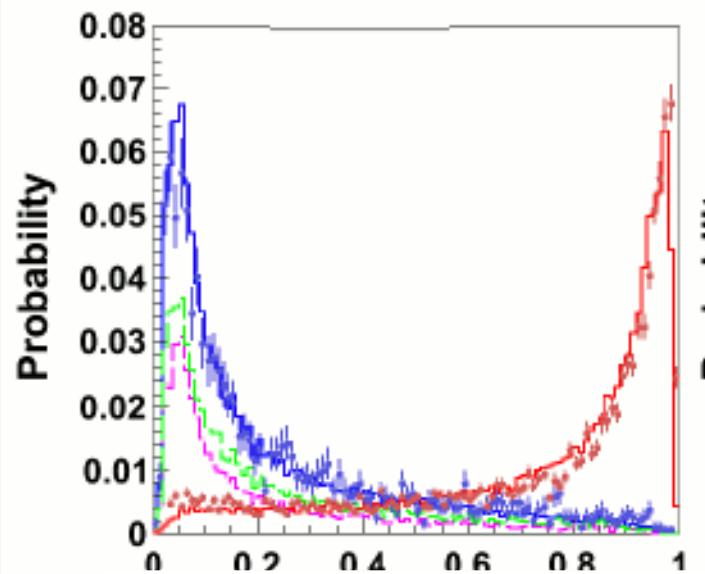
where $r_D = \left| \frac{\mathcal{A}(D^0 \rightarrow K^+\pi^-)}{\mathcal{A}(\bar{D}^0 \rightarrow K^+\pi^-)} \right| = 0.0613 \pm 0.0010$

$B^- \rightarrow D K^-$, $D \rightarrow K^+ \pi^-$ ADS

Main background is $e^+ e^- \rightarrow q\bar{q}$ ($q=u, d, s, c$) continuum
combine 10 variables with neural network:

$B^- \rightarrow D \pi^-$
 $\rightarrow K^- \pi^+$
 M_{bc} -sideband

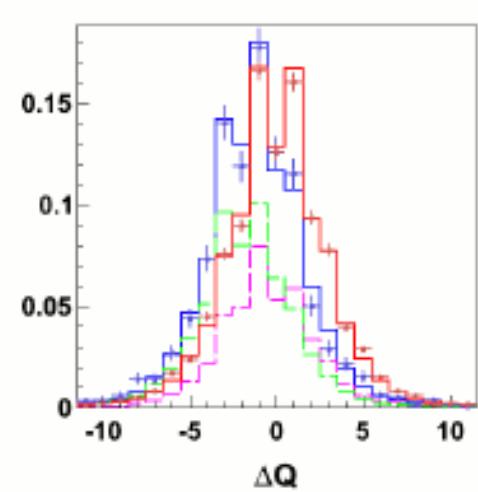
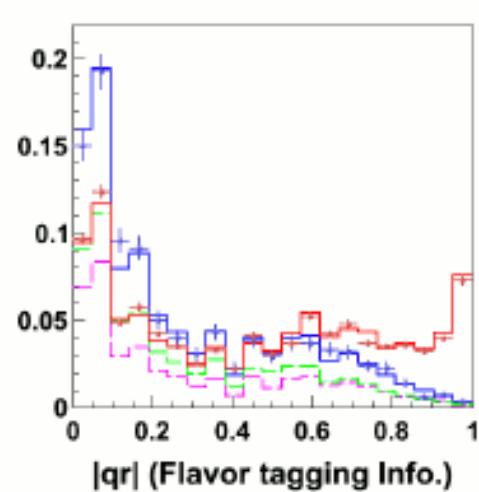
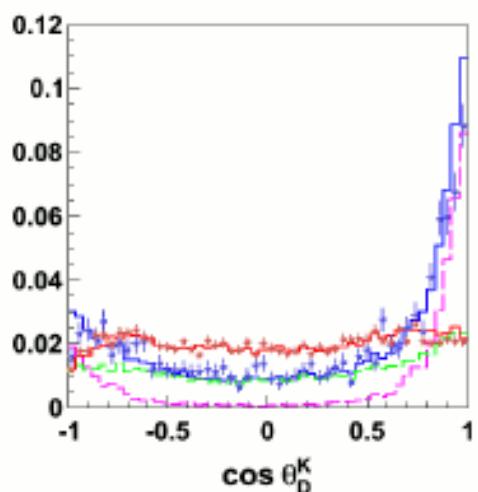
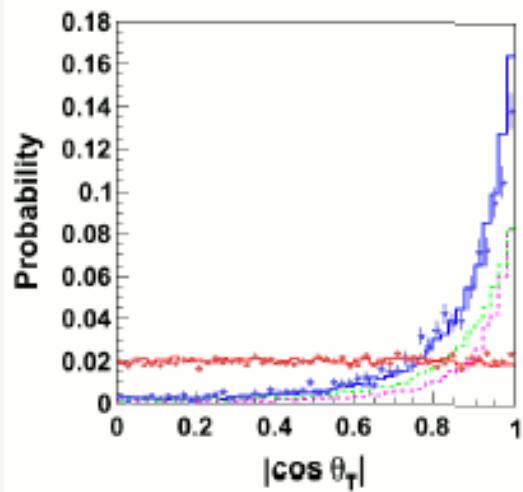
- Variables which have different distributions for signal and $q\bar{q}$ background are used.



(Here, we use “old” package.)
(RooKSFW is also possible.)
Most powerful.

θ_B is angle between B -flight direction and beam axis.

If tag is not possible, $\Delta z = -999$.
NB deals with as a δ -function.

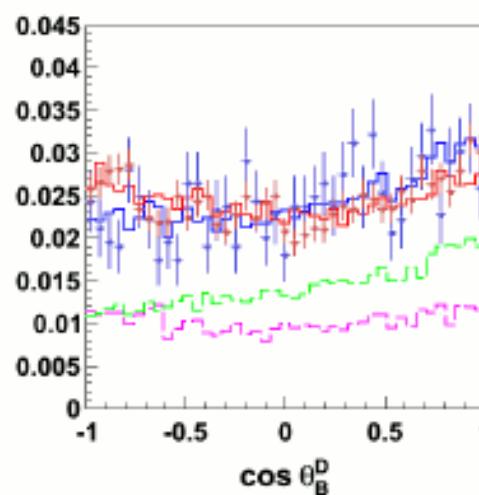
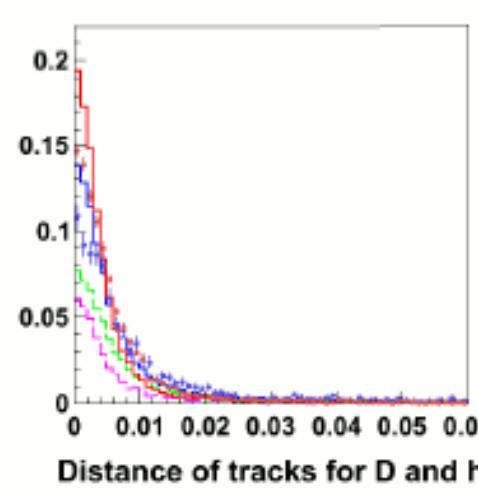
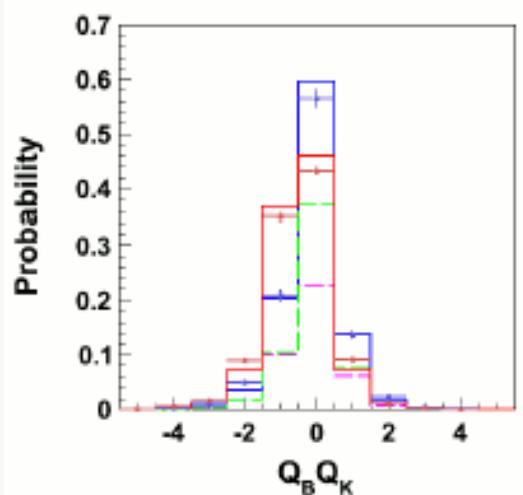


*Angle between thrust axes
of B decay and remainder.
No full correlation to LR(KSFW).*

Decay angle of $D \rightarrow K\pi$.

*Flavor tagging Info. by
MDLH. (NB possible.)*

*Difference of charges in
 D hemisphere and
opposite hemisphere.*



*Product of charge of B and
sum of charges for K not
used in B reconstruction.*

*Distance of tracks
for D and K .*

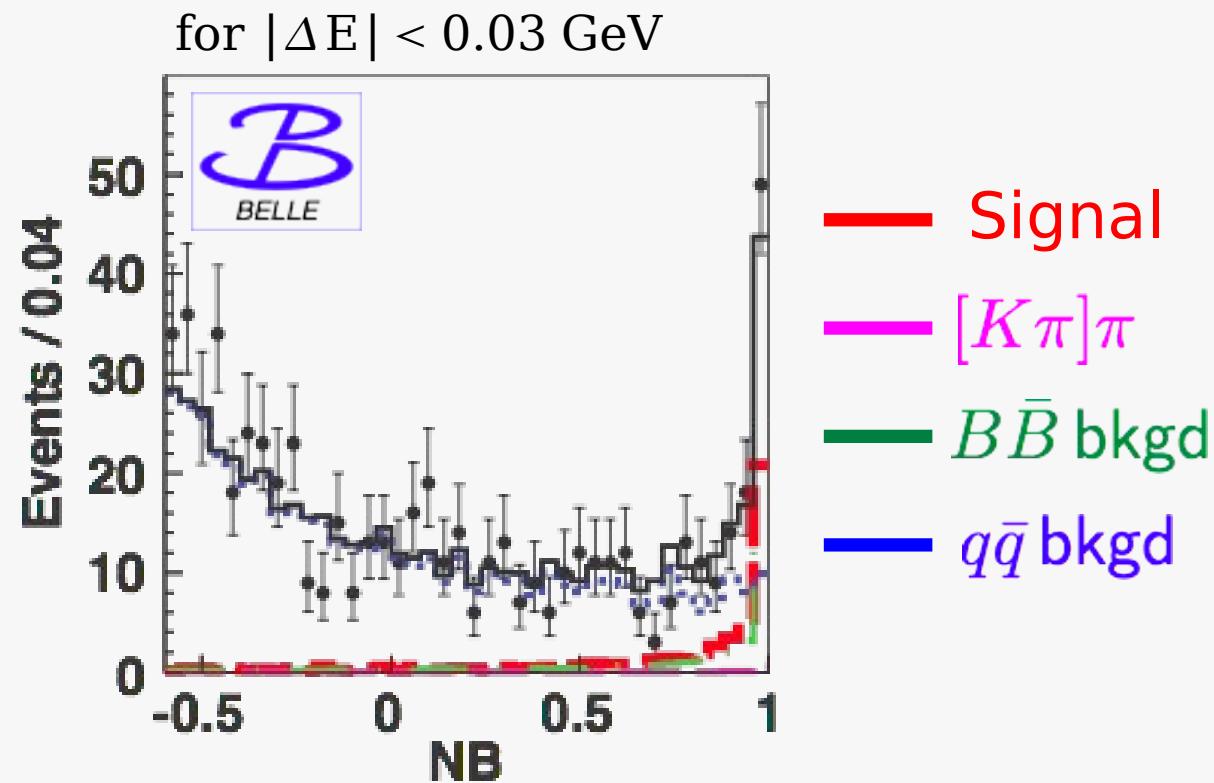
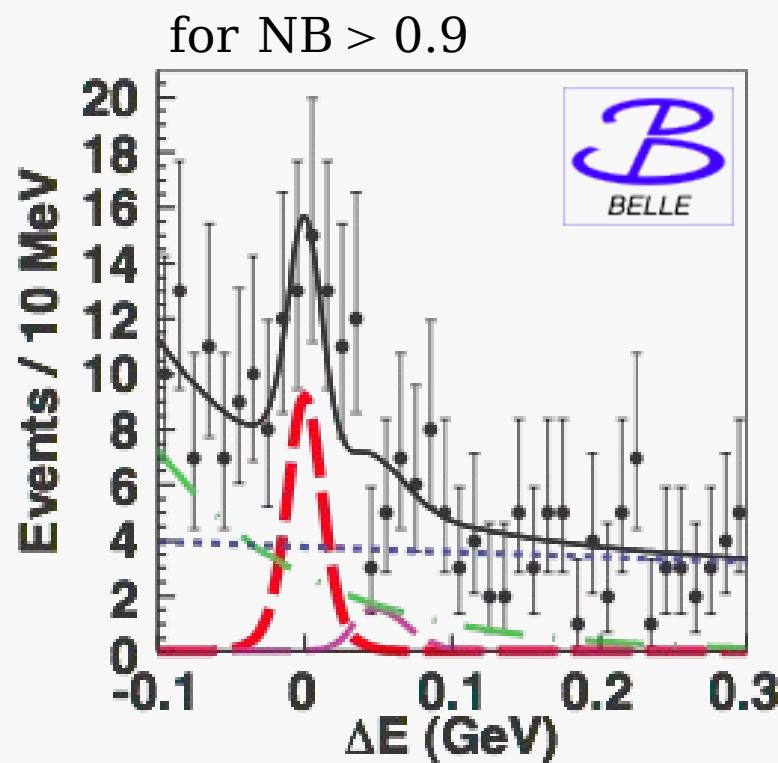
Decay angle of $B \rightarrow DK$.

10 variables combined
to obtain a single
NN output (NB)

**for example,
at 99 % bckg rej.
signal eff. = 42 %
now becomes 60 %**

Yields for the ADS mode $B^- \rightarrow [K^+ \pi^-]_D K^-$ from 772 million $B\bar{B}$ events
PRL 106, 231803 (2011)

Fit ΔE and NB distributions together to extract signal



56.0^{+15.1}_{-14.2} events

$$R_{DK} = (1.63^{+0.44}_{-0.41}{}^{+0.07}_{-0.13}) \times 10^{-2}$$

$$A_{DK} = -0.39^{+0.26}_{-0.28}{}^{+0.04}_{-0.03}$$

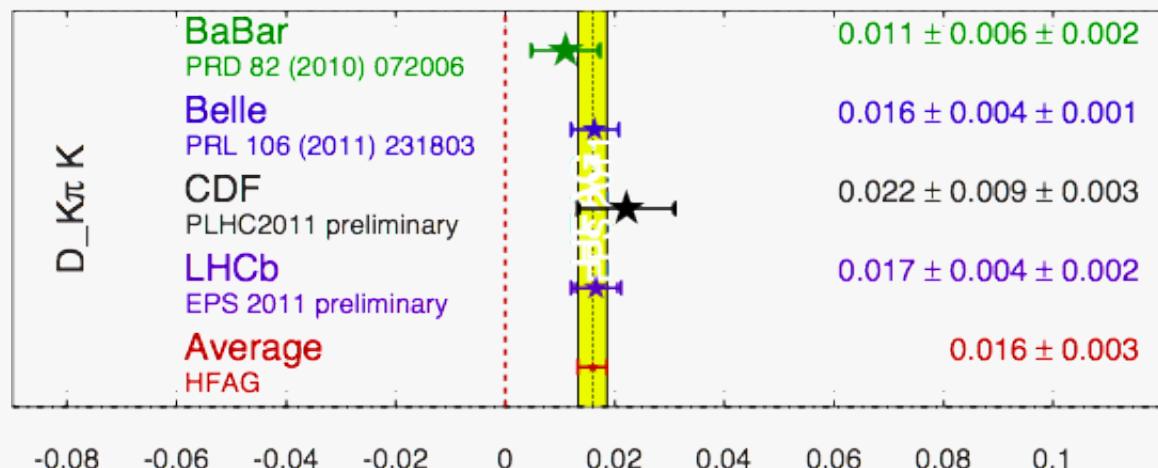
**First evidence obtained
with a significance of 3.8σ
(including syst.)**

Results for the ADS mode $B^- \rightarrow [K^+ \pi^-]_D K^-$ from 772 million $B\bar{B}$ events

PRL 106, 231803 (2011)

R_{ADS} Averages

HFAG
EPS 2011
PRELIMINARY



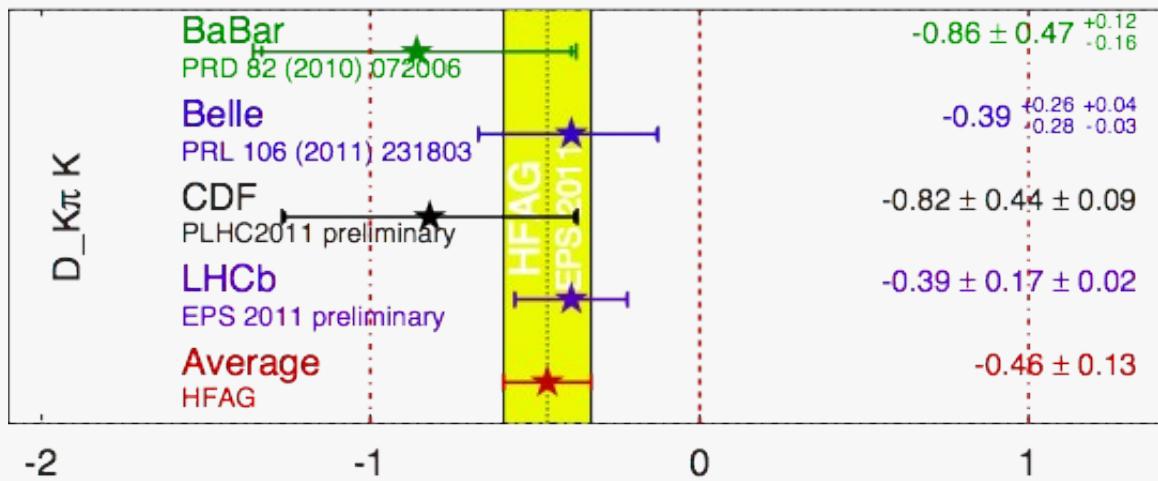
$$(1.63^{+0.44}_{-0.41}{}^{+0.07}_{-0.13}) \times 10^{-2}$$



$$\Rightarrow r_B \neq 0$$

A_{ADS} Averages

HFAG
EPS 2011
PRELIMINARY



$$-0.39^{+0.26}_{-0.28} {}^{+0.04}_{-0.03}$$



First evidence for the ADS mode $B^- \rightarrow [K^\pm \pi^\mp]_{D^*} K^\mp$ from Belle 772 million $B\bar{B}$ events

Preliminary
LP 2011

study both modes: $D^* \rightarrow D\pi^0$, $D\gamma$:

**Signal seen
with a significance of 3.5σ
for $D^* \rightarrow D\gamma$ mode**

Ratio to favored mode:

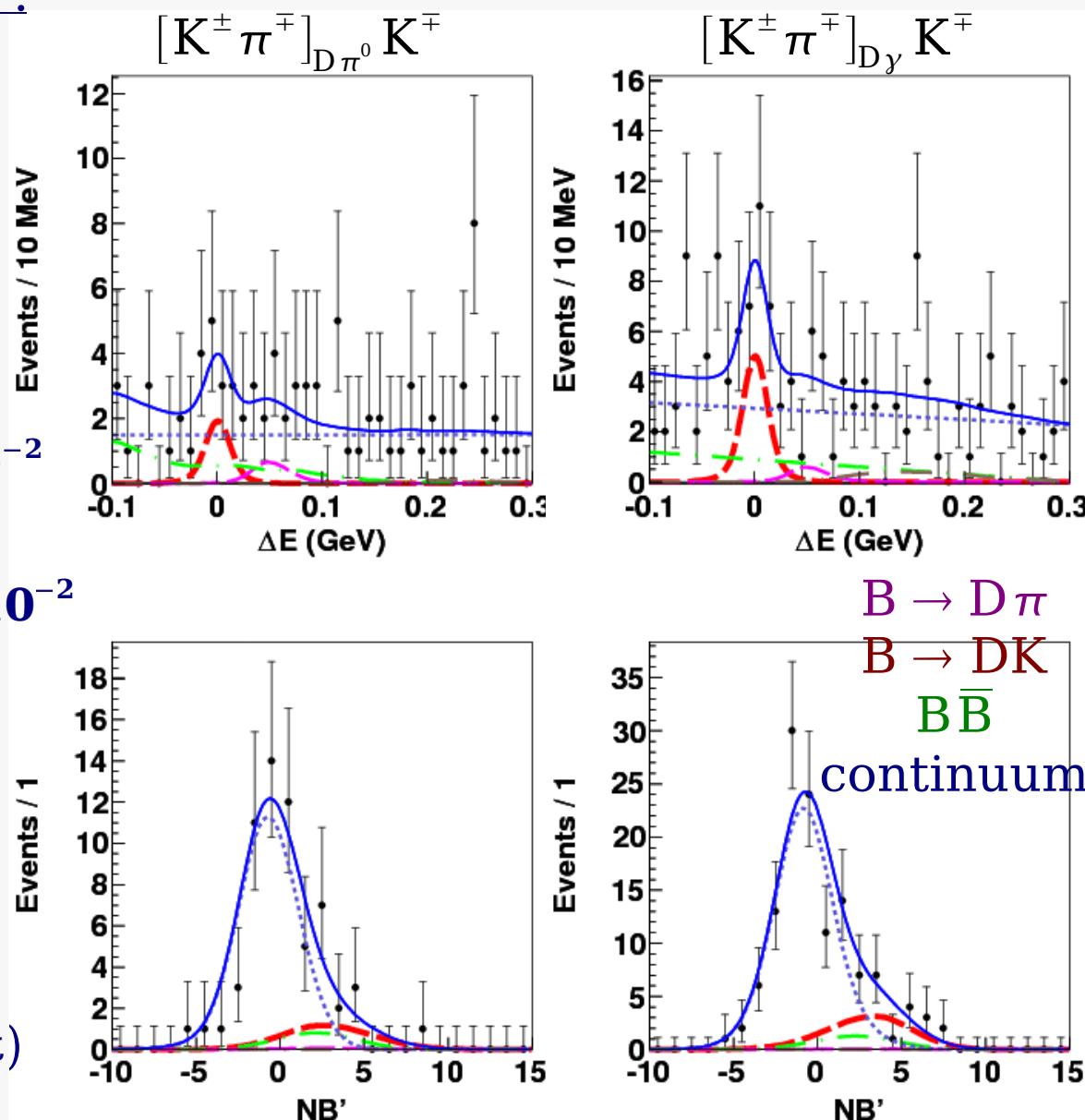
$$R_{D\pi^0} = (1.0^{+0.8}_{-0.7}(\text{stat})^{+0.1}_{-0.2}(\text{syst})) \times 10^{-2}$$

$$R_{D\gamma} = (3.6^{+1.4}_{-1.2}(\text{stat}) \pm 0.2(\text{syst})) \times 10^{-2}$$

asymmetry:

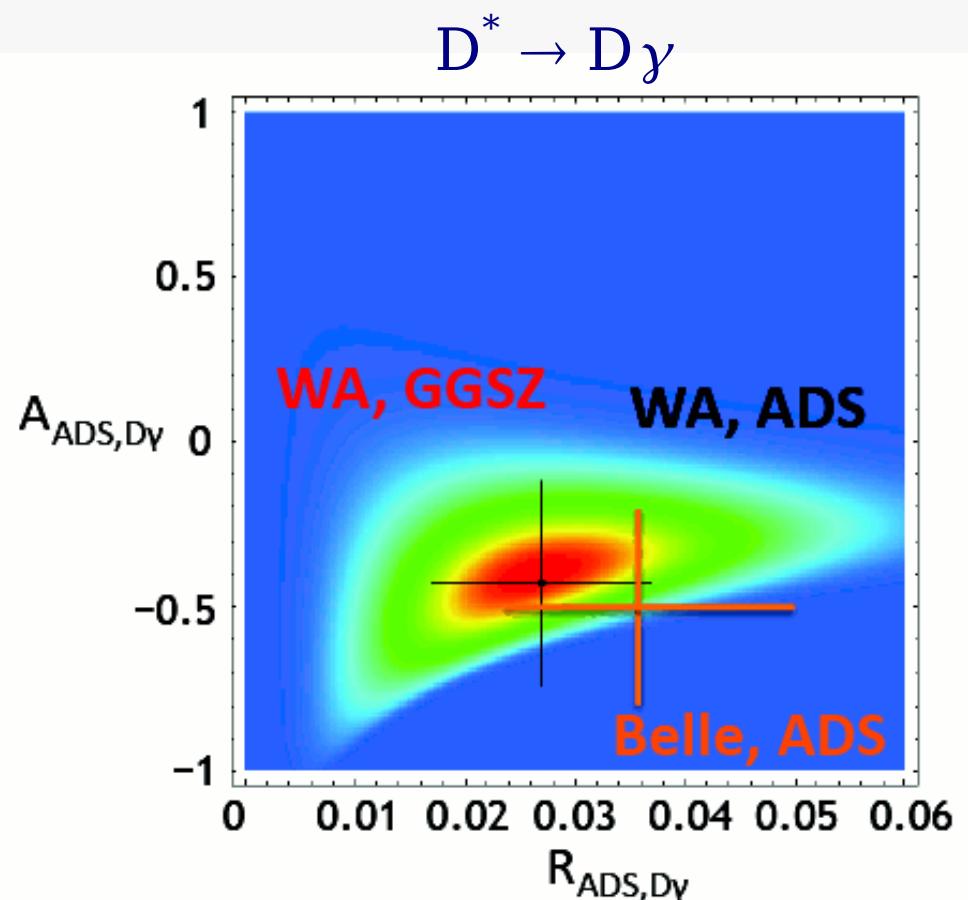
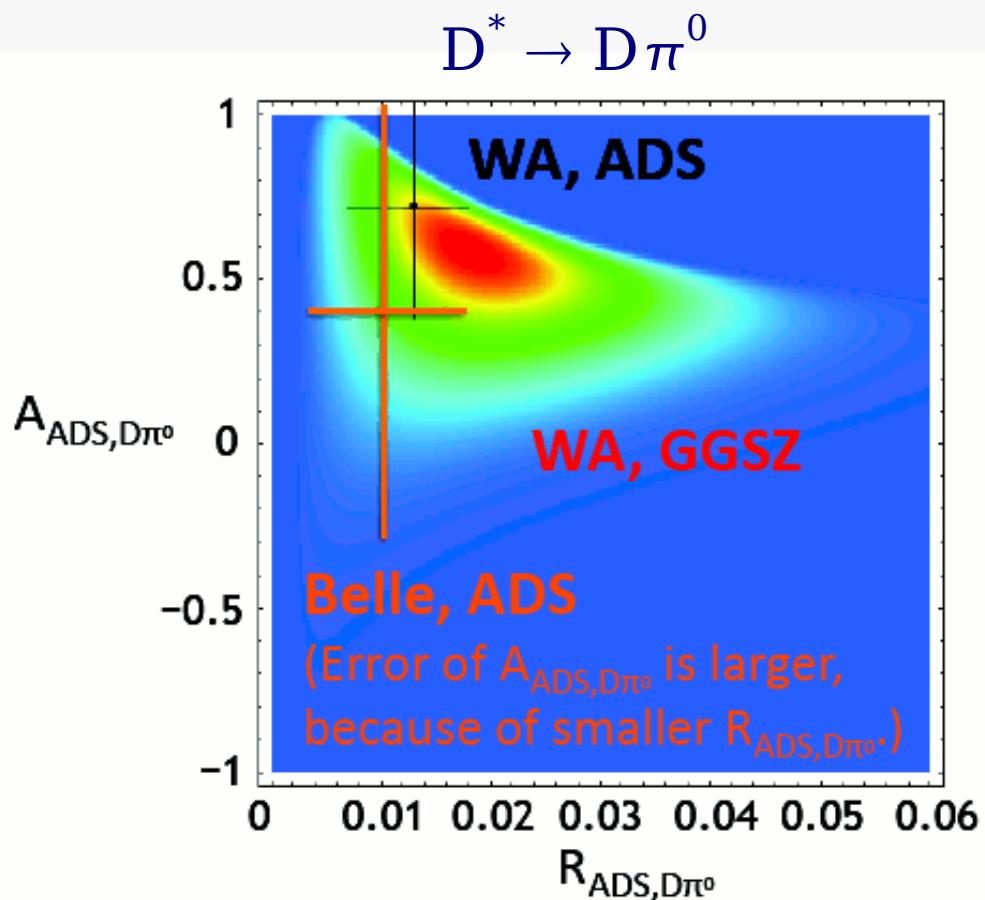
$$A_{D\pi^0} = 0.4^{+1.1}_{-0.7}(\text{stat})^{+0.2}_{-0.1}(\text{syst})$$

$$A_{D\gamma} = -0.51^{+0.33}_{-0.29}(\text{stat}) \pm 0.08(\text{syst})$$



Comparison of the results obtained for D^*K with expectations

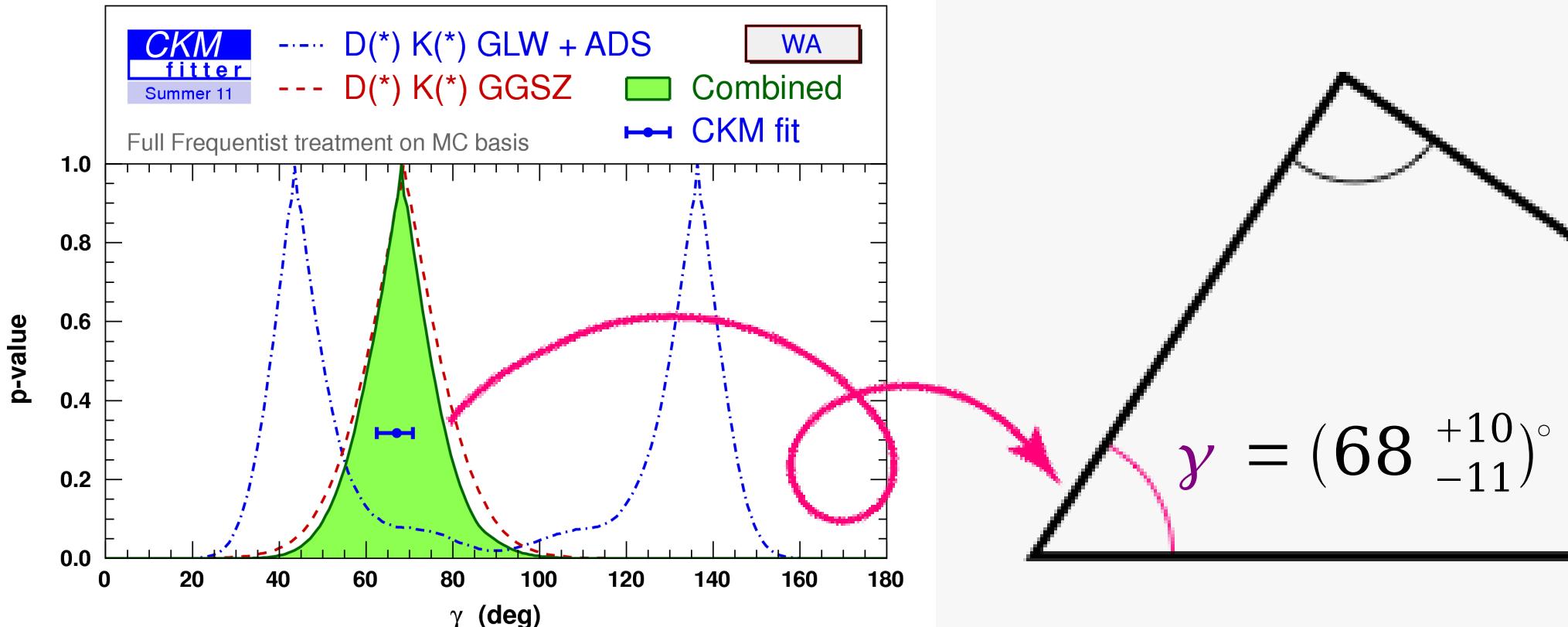
(where "expectations" are derived from the GGSZ observables)



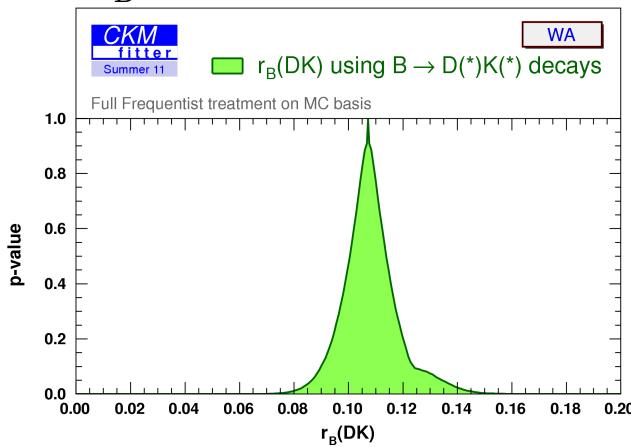
WA taken from HFAG 2011 summer.

Combined measurements for γ from all methods

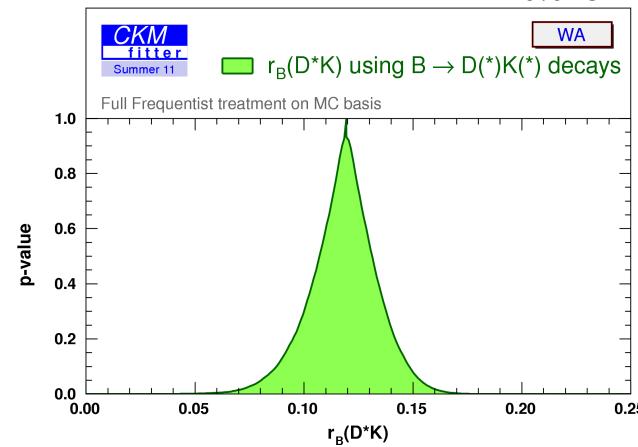
<http://ckmfitter.in2p3.fr/>



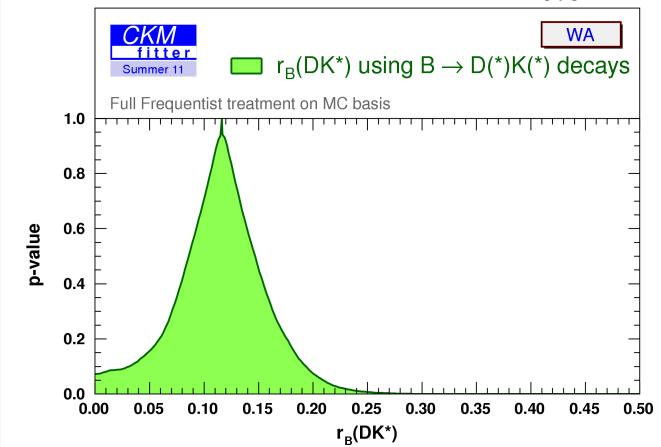
$$r_B(DK) = 0.107 \pm 0.010$$



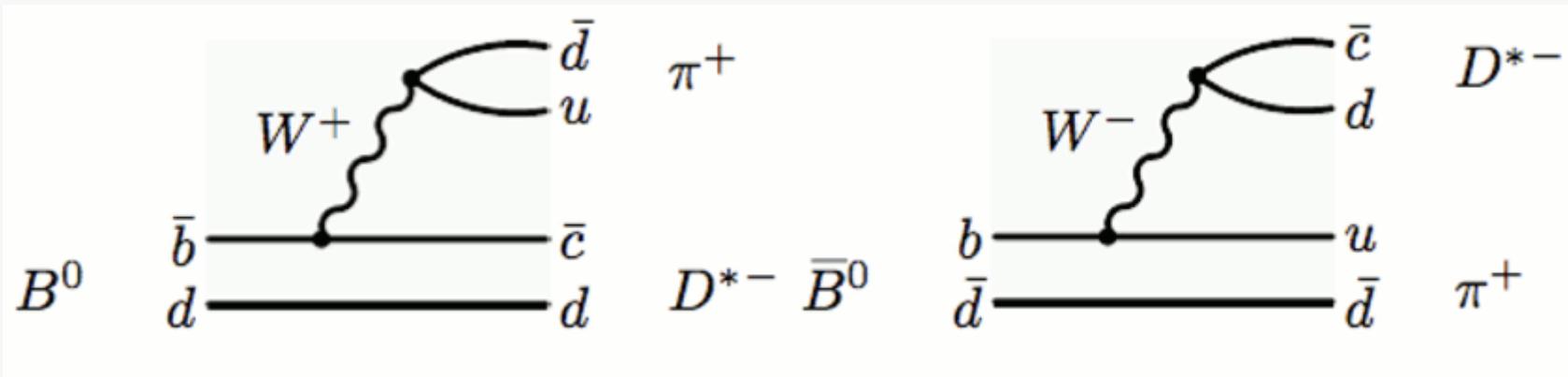
$$r_B(D^*K) = 0.119^{+0.018}_{-0.019}$$



$$r_B(D^*K) = 0.116^{+0.045}_{-0.044}$$



$r \sin(2\phi_1 + \phi_3)$ from $B^0 \rightarrow D^{(*)} \pi$ decay



Use B flavor tag, measure time-dependent decay rates

$$P(B^0 \rightarrow D^{(*)\pm} \pi^\mp) = \frac{1}{8\tau_B} e^{-|\Delta t|/\tau_B} [1 \mp \mathbf{C} \cos(\Delta m \Delta t) - \mathbf{S}^\pm \sin(\Delta m \Delta t)]$$

$$P(\bar{B}^0 \rightarrow D^{(*)\pm} \pi^\mp) = \frac{1}{8\tau_B} e^{-|\Delta t|/\tau_B} [1 \pm \mathbf{C} \cos(\Delta m \Delta t) + \mathbf{S}^\pm \sin(\Delta m \Delta t)]$$

$$\mathbf{S}^\pm = -2r \sin(2\phi_1 + \phi_3 \pm \delta_{D^{(*)}\pi})$$

$$\mathbf{C} = \frac{1-r^2}{1+r^2} \approx 1 \quad r \approx 0.02$$

⇒ large stat available, small CP violation effect

- partial reconstruction helps increase statistics **$657 \times 10^6 B\bar{B}$ pairs**
[ArXiv:0809.3203]
- lepton tag (50196 ± 286 signal evts)

$r \sin(2\phi_1 + \phi_3)$ from $B^0 \rightarrow D^{(*)}\pi$ decay $657 \times 10^6 B\bar{B}$ pairs [PRD 84 (2011) 021101]

HFAG notation

$$a = -(S^+ + S^-)/2$$

$$c = -(S^+ - S^-)/2$$

$$a = -0.046 \pm 0.013 \pm 0.015$$

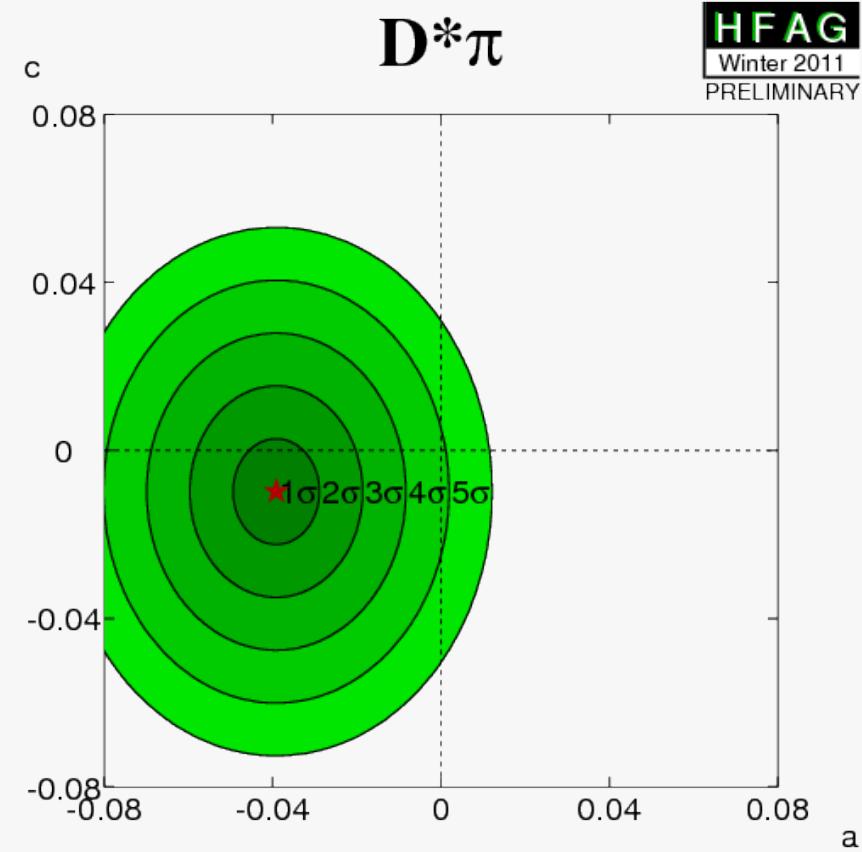
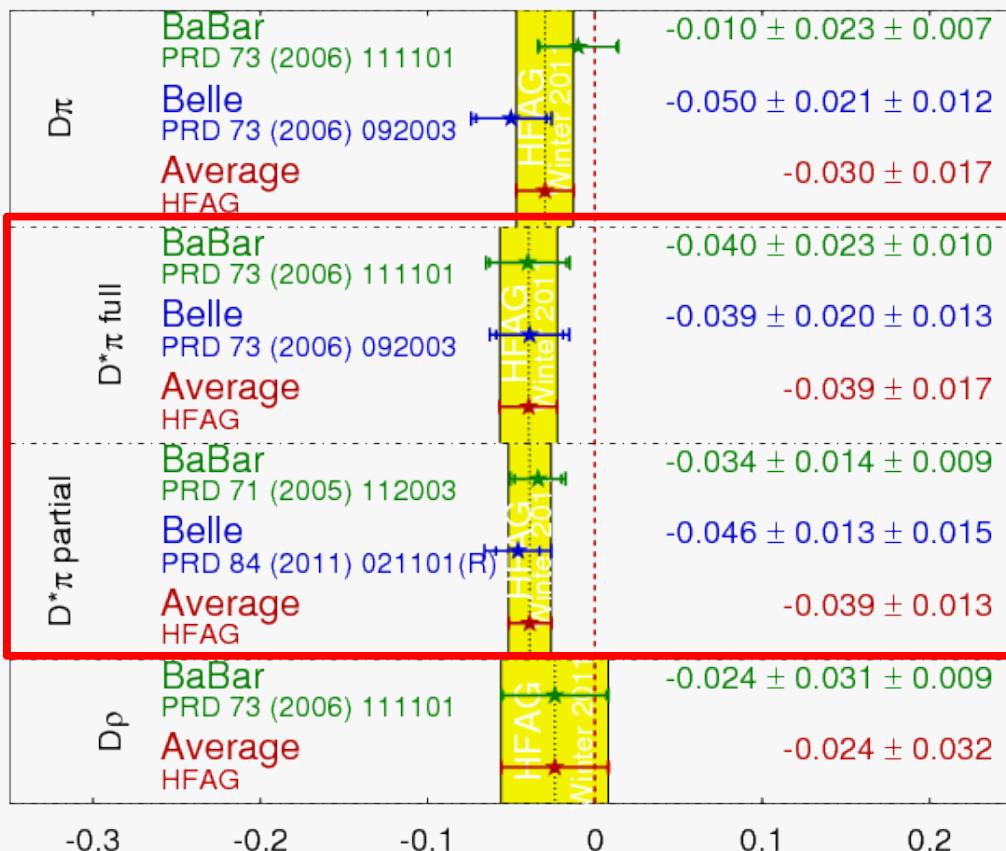
$$c = -0.015 \pm 0.013 \pm 0.015$$

significance of CPV is 2.6σ

summary for all measurements (partial and full)

a parameters

HFAG
Winter 2011
PRELIMINARY

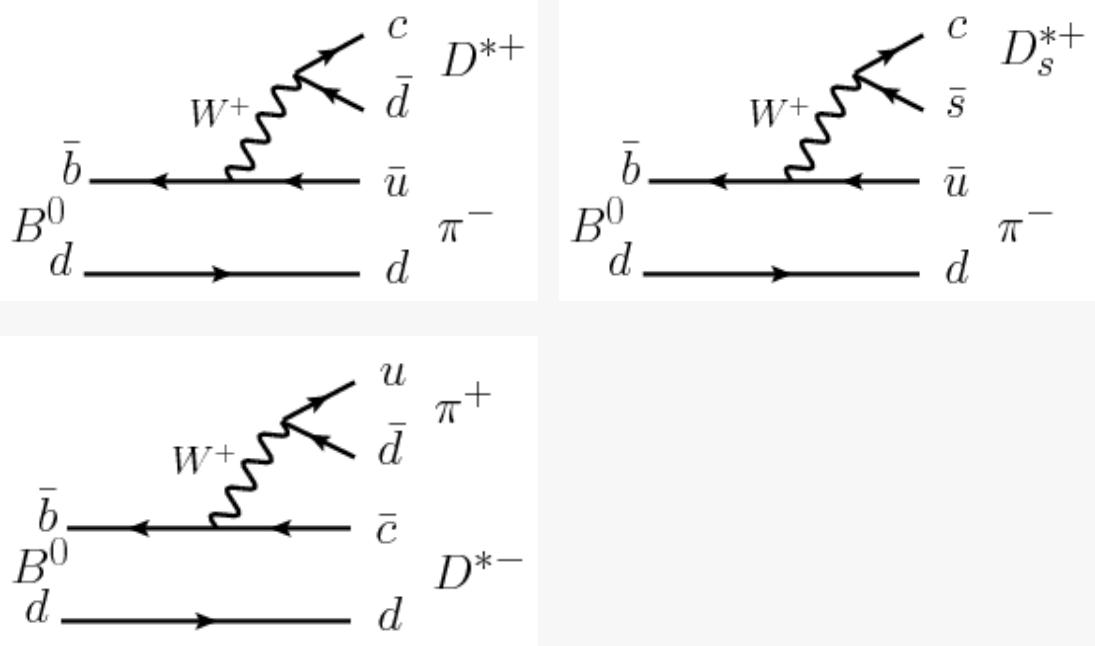


significance of CPV is $\sim 4\sigma$

r using $B^0 \rightarrow D_s^{*+} \pi^-$ decay

657×10⁶ B̄B pairs
[PRD 81, 031101 (2010)]

1st measurement in Belle

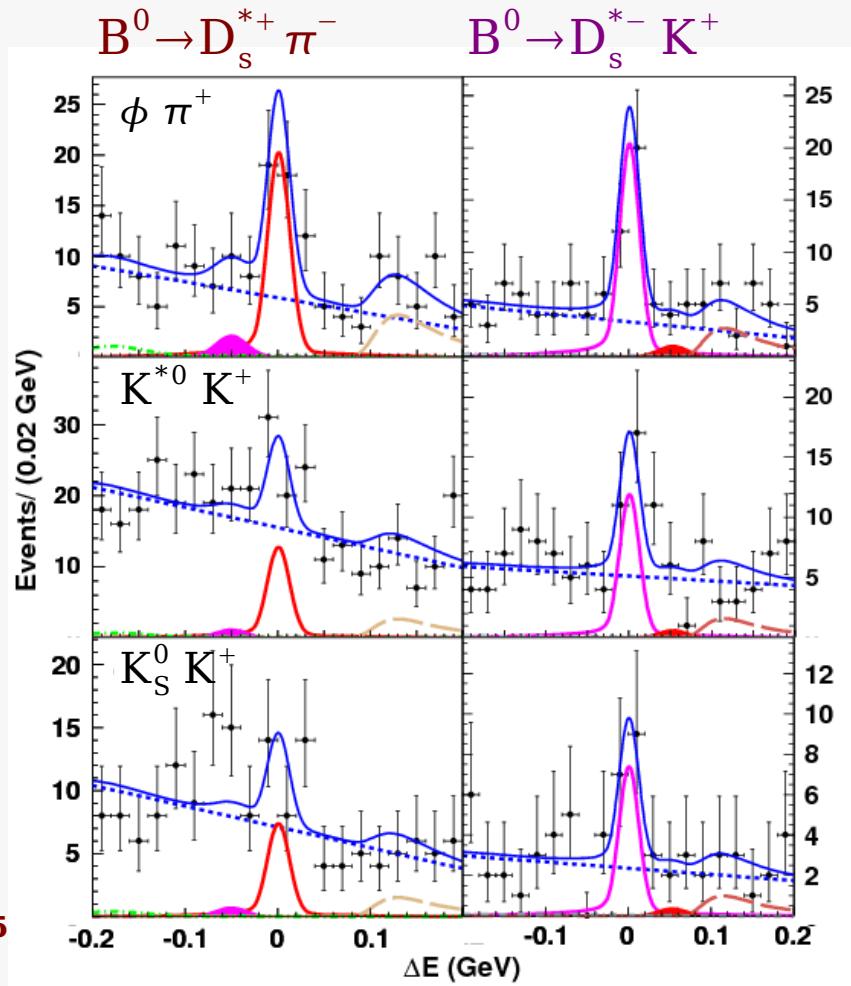


$$r = \tan \theta_C \left(\frac{f_{D_s^*}}{f_{D_s}} \right) \sqrt{\frac{\text{Br}(B^0 \rightarrow D_s^{*+} \pi^-)}{\text{Br}(B^0 \rightarrow D_s^{*-} \pi^+)}}$$

$$\Sigma = 6.1 \sigma$$

$$\text{Br}(D_s^{*+} \pi^-) = (1.75 \pm 0.34 \pm 0.17 \pm 0.10) \times 10^{-5}$$

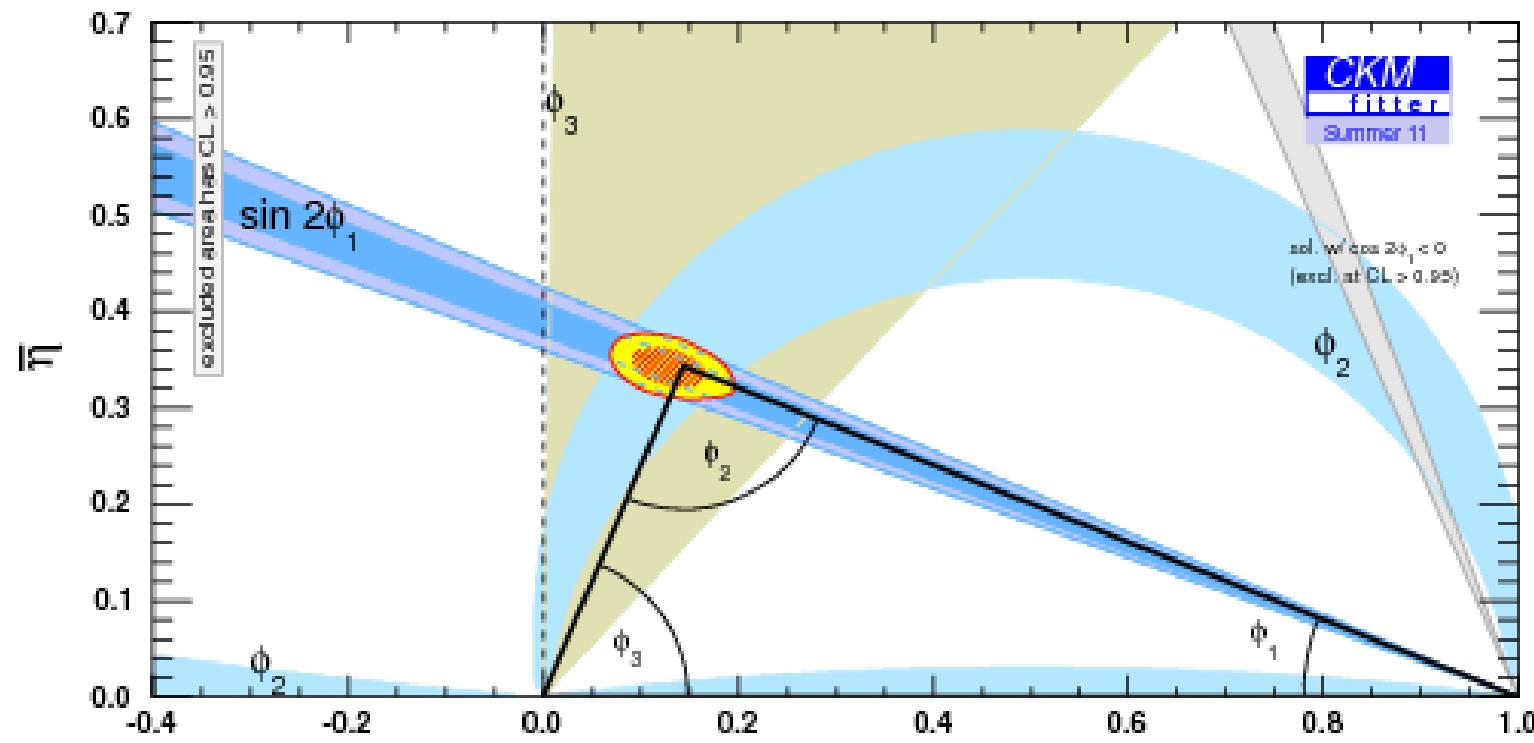
$$r = (1.58 \pm 0.15 \pm 0.10 \pm 0.03)\%$$



Summary

$$\alpha = (89.0^{+4.4}_{-4.2})^\circ$$

(WA, CKMfitter, Winter09)



$$\gamma = (68^{+10}_{-11})^\circ$$

(WA, CKMfitter)

$$\beta = (21.4 \pm 0.8)^\circ$$

(WA, HFAG, Winter11)