Measuring Angle ϕ_3

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1. CP Violation and Unitarity of CKM Matrix
2. $B \to DK$
3. $B \to D^{(*)}\pi$
4. $B \to D^{*}\rho$
5. $B \to K\pi, \pi\pi$

General left-handed quark-W Interaction

$$L_{\text{int}}(t) = \int d^3x \big(\mathcal{L}_{qW}(x) + \mathcal{L}^{\dagger}_{qW}(x)\big)$$

$$\mathcal{L}_{qW}(x) = \frac{g}{\sqrt{8}} \sum_{i,j=1,3} V_{i\,j}\, \bar{U}_i\, \gamma_\mu (1-\gamma_5) D_j\, W^\mu$$

$$U_i(x) \equiv \begin{pmatrix} u(x) \\ c(x) \\ t(x) \end{pmatrix}, \quad D_j(x) \equiv \begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \text{(CKM matrix)}$$

Experimentally, V has a hierarchical structure. Approximately,

$$|V_{i\,j}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda \sim 0.22$$

Transformation of L_{int} under CP

$$CP: \quad \begin{array}{l} \text{exchanges particle } (n) \leftrightarrow \text{antiparticle } (\bar{n}) \\ \text{flips momentum sign } (\vec{p} \leftrightarrow -\vec{p}) \\ \text{keeps the spin } z\text{-component } (\sigma) \text{ the same} \end{array} \qquad \text{(a)}$$

Such CP operator in Hilbert space is not unique:

$$\mathcal{CP} a^{\dagger}_{n,\vec{p},\sigma} \mathcal{P}^{\dagger} \mathcal{C}^{\dagger} = \eta_n a^{\dagger}_{\bar{n},-\vec{p},\sigma}$$

η_n: 'CP phase': arbitrary, depends on n
(for antiparticle: $\eta_{\bar{n}} = (-)^{2J} \eta_n^*$)

The choice of η_n amounts to choosing a specific operator in Hilbert space among those satisfying (a).

Then, a pure algebra leads to

$$\mathcal{CP}\ \bar{u}(x)\gamma_\mu(1-\gamma_5)d(x)W^\mu(x)\ \mathcal{P}^\dagger\mathcal{C}^\dagger$$
$$= \eta_u \eta_d^* \eta_W^* \left(\bar{u}(x')\gamma^\mu(1-\gamma_5)d(x')W_\mu(x')\right)^\dagger$$

$$x' \equiv (t, -\vec{x})$$

$\mathcal{L}_{qW}$ transforms as (taking $\eta_W = 1$)

$$\mathcal{CP}\ \mathcal{L}_{qW}(x)\ \mathcal{P}^\dagger\mathcal{C}^\dagger = \frac{g}{\sqrt{8}} \sum_{i,\,j=1,3} \eta_{U_i}\eta^*_{D_j} V_{i\,j} \left(\bar{U}_i(x')\ \gamma^\mu(1-\gamma_5)D_j(x')W_\mu(x')\right)^\dagger$$

IF $\eta_{U_i}\eta^*_{D_j}$ can be chosen s.t.

$$\eta_{U_i}\eta^*_{D_j} V_{i\,j} = V^*_{i\,j} \quad (2)\,,$$

then, $L_{\text{int}}(t)$ becomes invariant under CP:

$$\mathcal{CP}\ \mathcal{L}_{qW}(x)\ \mathcal{P}^\dagger\mathcal{C}^\dagger = \mathcal{L}^\dagger_{qW}(x') \quad (x' = (t, -\vec{x}))$$

$$\begin{aligned} \to \mathcal{CP}\ L_{\text{int}}(t)\ \mathcal{P}^\dagger\mathcal{C}^\dagger \\ &= \int d^3x\ \mathcal{CP}\left[\mathcal{L}_{qW}(x) + \mathcal{L}^\dagger_{qW}(x)\right]\mathcal{P}^\dagger\mathcal{C}^\dagger \\ &= \int d^3x\ \left[\mathcal{L}^\dagger_{qW}(x') + \mathcal{L}_{qW}(x')\right] \\ &= L_{\text{int}}(t) \end{aligned}$$

$\to$ S operator is invariant under CP
(through Dyson series)

Condition for CP Invariance

Rewrite the condition (2):

$$\frac{\eta_{D_j}}{\eta_{U_i}} = 2 \arg V_{i,j}$$

Thus, for a given (arbitrary) matrix $V_{i,j}$, if the CP phases η's can be chosen so that the phase difference between η_{D_j} and η_{U_i} is twice the arbitrary phase of $V_{i,j}$, then the physics is invariant under CP.

This is equivalent to rotate the quark phases to make $V_{i,j}$ all real.

In general, there are 5 phase differences for 6 quarks $\rightarrow$ 5 elements of V can be set to real always.

For example.,

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$V_{i,j}$: real
$V_{i,j}$: complex

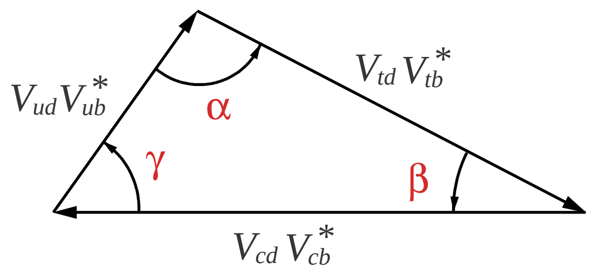
(No unitarity condition imposed)

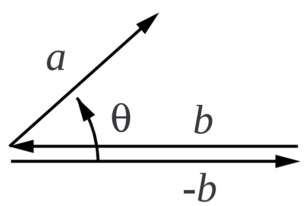
Any of the four red elements is not real $\rightarrow$ CP violation

A Main Question of the CPV Study in B: 'Is V unitary?'

e.g: orthogonality of d-column and b-column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$





$$\theta = \arg \frac{a}{-b}$$

$$\phi_2 \equiv \arg\left(\frac{V_{td}V_{tb}^*}{-V_{ud}V_{ub}^*}\right), \ \phi_1 \equiv \arg\left(\frac{V_{cd}V_{cb}^*}{-V_{td}V_{tb}^*}\right), \ \phi_3 \equiv \arg\left(\frac{V_{ud}V_{ub}^*}{-V_{cd}V_{cb}^*}\right) \tag{3}$$

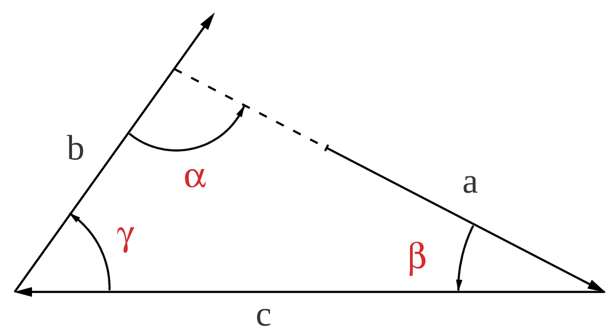
With our phase convention:

$$\phi_2 \equiv \arg\left(\frac{V_{td}V_{tb}^*}{-V_{ub}^*}\right), \ \phi_1 \equiv \arg\left(V_{td}^*V_{tb}\right), \ \phi_3 \equiv \arg\left(V_{ub}^*\right)$$

For **any** complex numbers a, b, c, trivially

$$\alpha + \beta + \gamma = \pi \quad (\text{mod } 2\pi)$$

$$\alpha \equiv \text{arg}\left(\frac{a}{-b}\right), \quad \beta \equiv \text{arg}\left(\frac{b}{-c}\right), \quad \gamma \equiv \text{arg}\left(\frac{c}{-a}\right).$$



$\rightarrow$ The condition $\alpha + \beta + \gamma = \pi$ (mod2π) holds even if the triangle does not close.
It does **not** test the unitarity of V_{CKM}.

It simply tests if the angles measured are as defined in (3) in terms of V_{CKM}.

$\rightarrow$ It is critical to measure the length of the sides.

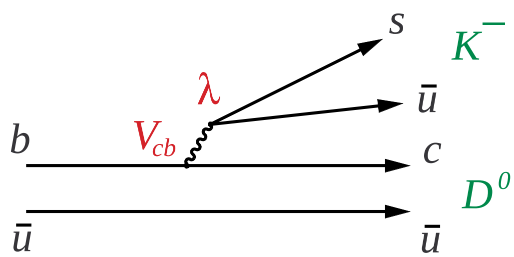
$B \to DK$

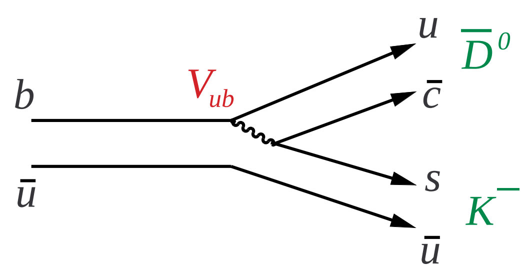
Gronau-London-Wyler (GLW) method for ϕ_3

$$B^- \to D^0_{CP} K^-$$

D^0_{CP} : CP eigenstate. e.g. $K_S\pi^0, K^+K^- \cdots$

Both D^0 and $\bar{D}^0$ decay to a CP eigenstate.
$\to$ 2 diagrams



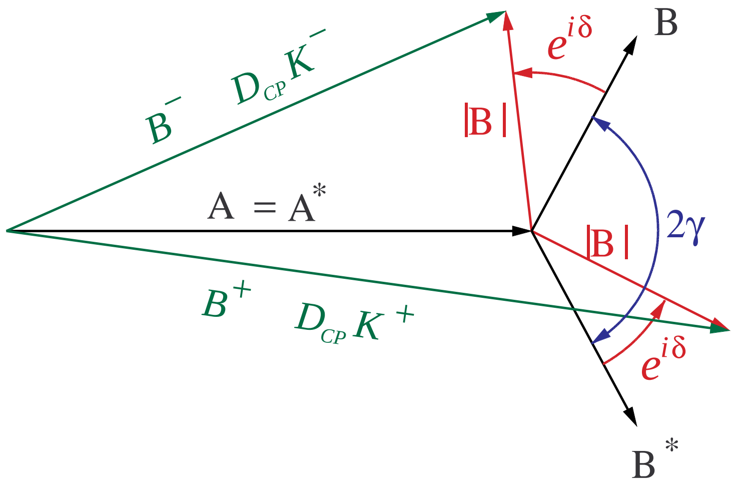


$A \equiv Amp(B^- \to D^0 K^-)$	$B \equiv Amp(B^- \to \bar{D}^0 K^-)$
λV_{cb}	$V_{ub} \sim 0.4 \lambda V_{cb}$
Color-favored	Color-suppressed
$(a_1 + a_2 \sim 1.24)$	$(a_2 \sim 0.24)$
$\bar{A} \equiv Amp(B^+ \to \bar{D}^0 K^+)$	$\bar{B} \equiv Amp(B^+ \to D^0 K^+)$
$\bar{A} = A^*$	$\bar{B} = B^*$

($\lambda \sim 0.22$: Cabbibo factor)

Strong final-state-interaction phase:
B relative to A : $e^{i\delta}$ (δ could be complex)

Phase convention: $A = A^*$



$\phi_3 \equiv \arg B^* = \arg V_{ub}^*$

Measure 4 lengths:

$Amp(B^- \to D^0_{CP} K^-)$

$Amp(B^+ \to D^0_{CP} K^+)$

$|A|$ by $B^- \to D^0 K^-,\ D^0 \to K^- \pi^+$

$|B|$ by $B^- \to \bar{D}^0 K^-,\ \bar{D}^0 \to K^+ \pi^-$

Reconstruct two triangles $\to \phi_3$

CP asymmetry expected:

$$a_{cp} \equiv \frac{\Gamma[B^- \to (K_S\pi^0)K^-] - \Gamma[B^+ \to (K_S\pi^0)K^+]}{\Gamma[B^- \to (K_S\pi^0)K^-] + \Gamma[B^+ \to (K_S\pi^0)K^+]}$$

$$\frac{|B|}{|A|} \sim \underbrace{(\text{color factor})}_{\frac{a_2}{a_1+a_2} \sim 0.2} \underbrace{(\text{CKM factor})}_{\frac{V_{ub}}{\lambda V_{cb}} \sim 0.4} \sim 0.08$$

$\rightarrow$ a_{cp} is of order 10%.

(γ can be measured even if $a_{cp} = 0$)

Relevant D^0 decay modes:

CP eigenstates	$K_S\,\pi^0$	$1.06 \pm 0.11\%$	$CP-$
	$K_S\,\rho^0$	$0.60 \pm 0.09\%$	$CP-$
	$K_S\,\phi$	$0.84 \pm 0.10\%$	$CP-$
	K^+K^-	$0.43 \pm 0.03\%$	$CP+$
	$\pi^+\pi^-$	$0.15 \pm 0.01\%$	$CP+$
calibration	$K^-\pi^+$	$3.83 \pm 0.12\%$	

D^0 decay FSI phase does not contribute.
$\rightarrow$ can be combined.

Problem with the GLW method and Solution

[Atwood, Dunietz, Soni (ADS)]

How to measure $B = Amp(B^- \to \bar{D}^0 K^-)$?

$$B^- \xrightarrow{B} \bar{D}^0\, K^- \quad \text{but also} \quad B^- \xrightarrow{A} D^0\, K^-$$
$$\hookrightarrow K^+\pi^- \qquad\qquad\qquad\qquad \hookrightarrow K^+\pi^- \; (DCSD)$$

The ratio of the two amplitudes (R_{DCSD}):

$$R_{DCSD} = \underbrace{\frac{A}{B}}_{\sim \frac{1}{0.08}} \underbrace{\frac{Amp(D^0 \to K^+\pi^-)}{Amp(D^0 \to K^-\pi^+)}}_{\substack{0.088 \pm 0.020 \\ \text{(CLEO 94)}}} \sim 1$$

Phase of R_{DCSD} not known $\to$ cannot measure $|B|$.

(Difficult to detect $D^0 \to X_s^- \ell^+ \bar{\nu}$)

The interference of DCSD and B-amplitude causes CP asymmetry of **order unity** in the wrong-sign $K\pi$ modes:

ADS method to extract ϕ_3

Measure $B^- \to DK^-$ in two decay modes of D: wrong-sign flavor-specific modes or CP eigenstates, say $K^+\pi^-$ and $K_S\,\pi^0$ (and their conjugate modes).

$$\Gamma[B^- \to (K^+\pi^-)K^-] \quad \Gamma[B^+ \to (K^-\pi^+)K^+]$$
$$\Gamma[B^- \to (K_S\,\pi^0)K^-] \quad \Gamma[B^+ \to (K_S\,\pi^0)K^+]$$

Assume we know $|A|$ and D branching fractions
$\to$ 4 unknowns:

$$\phi_3\,, \quad \delta_{K^-\pi^+}\,, \quad \delta_{K_S\pi^0}\,, \quad \frac{|B|}{|A|}$$

$\to$ can be solved.

Statistics: Possible at B-factories
(300 fb^{-1} needed for $\sigma_{\phi_3} \sim 0.3$ rad.)

Avoid using wrong-sign $B^+ \to D^0 K^+$

External input (experiment, theory):

$$r = \left|\frac{B}{A}\right| = \left|\frac{\bar{B}}{\bar{A}}\right| \sim 0.08$$

Measure

$$\begin{aligned}
\Gamma(B^- \to D_1 K^-) &= 1 + r^2 + 2r\cos(\phi_3 + \delta) \\
\Gamma(B^- \to D_2 K^-) &= 1 + r^2 - 2r\cos(\phi_3 + \delta) \\
\Gamma(B^+ \to D_1 K^+) &= 1 + r^2 + 2r\cos(\phi_3 - \delta) \\
\Gamma(B^+ \to D_2 K^+) &= 1 + r^2 - 2r\cos(\phi_3 - \delta)
\end{aligned}$$

in unit of $\Gamma(B^- \to D^0 K^-)$.

$\to$ fit for ϕ_3 and δ.

Ambiguity: the equations are symmetric under

$$\begin{cases} \phi_3 \to n\pi + \delta \\ \delta \to -n\pi + \gamma \end{cases} \text{or} \begin{cases} \phi_3 \to n\pi - \delta \\ \delta \to n\pi - \phi_3 \end{cases} \quad (n : \text{integer})$$

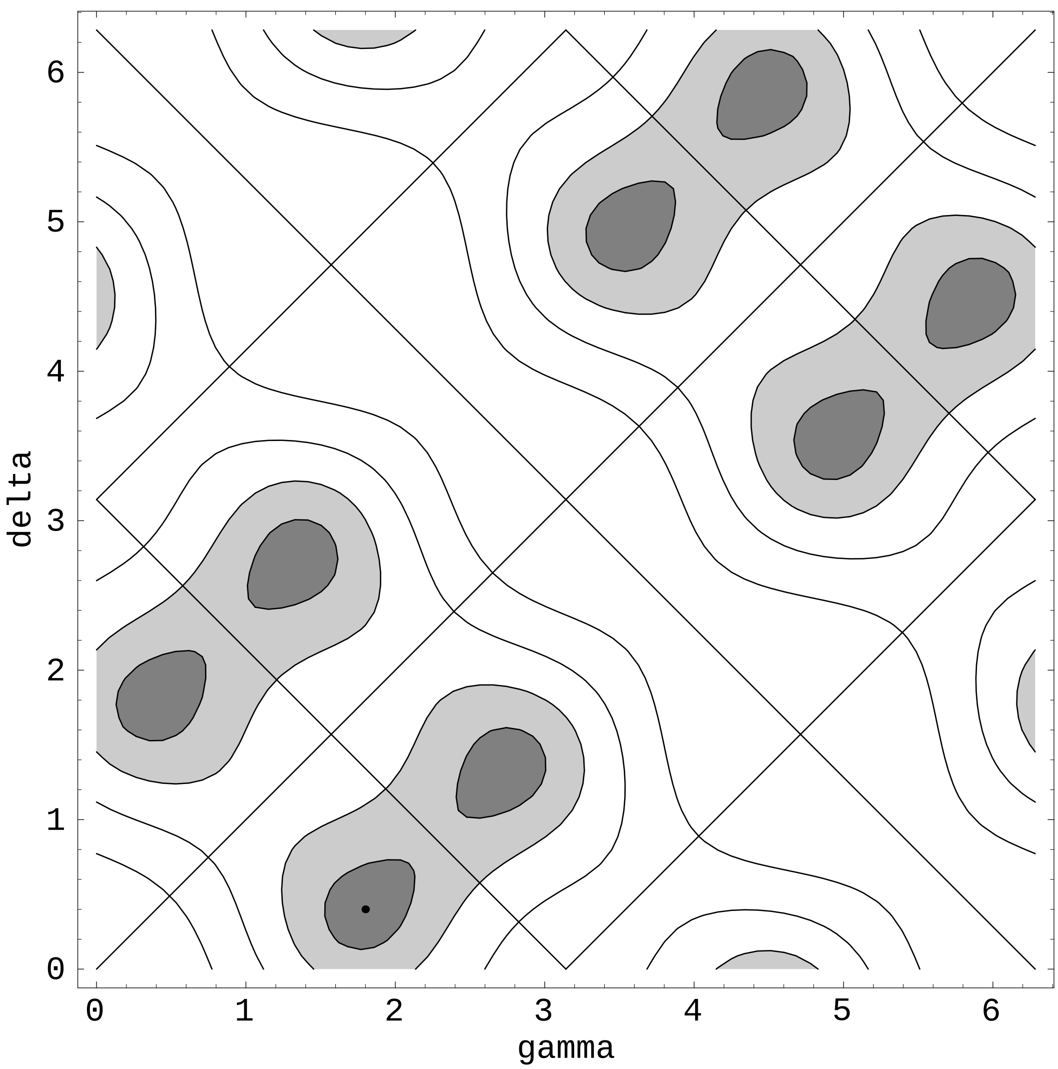
Fit result for ϕ_3 and δ

Input:

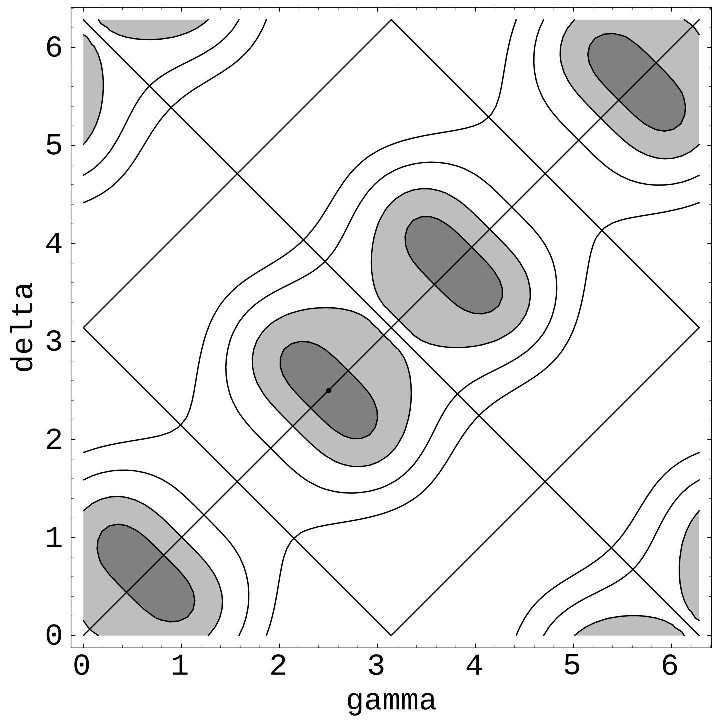
$\phi_3 = 1.8\,, \delta = 0.4$

$\sigma(\Gamma's) = 10\%$ (100 events each)

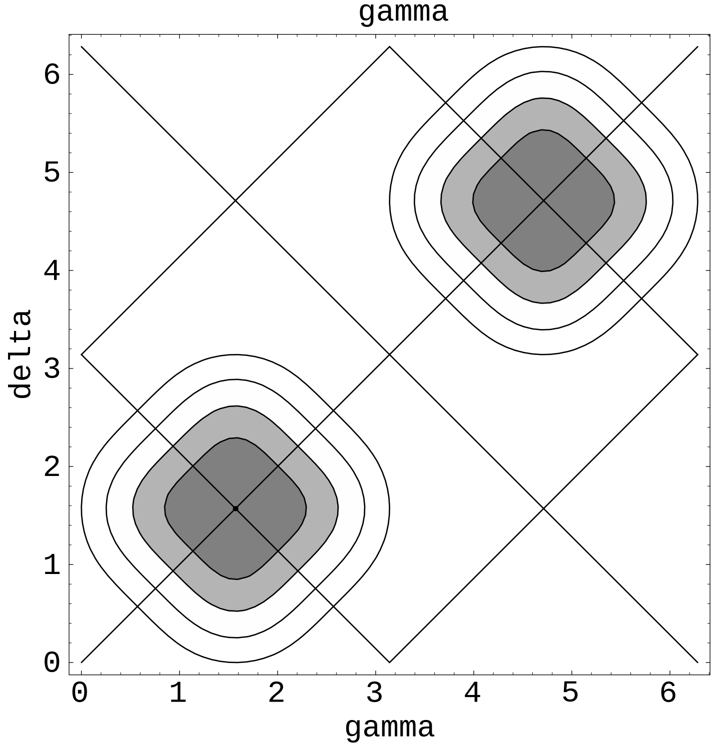
($300\mathrm{fb}^{-1}$)



Fit result for ϕ_3 and δ



$\phi_3 = 2.5$
$\delta = 2.5$



$\phi_3 = 1.57$
$\delta = 1.57$

Statistics Estimate

1. Relative yields (compare to $D^0 \to K^-\pi^+$)

   - $K^-\pi^+$(3.9%)
   - D_1: K^+K^-(0.43%) + $\pi^+\pi^-$(0.15%) = 0.58%.
   - D_2: $K_s\pi^0$(1.05%)×2/3(K_sBr) × 1/2(π^0) = 0.35%.

2. Yield of $B \to D^0K^-$, $D^0 \to K^-\pi^+$ at 3.1 fb^{-1}

   - CLEO: $N(D^0\pi^-) = 239$ at 3.1 fb^{-1}
   - Then, $N(D^0K^-) = 17.5$ at 3.1 fb^{-1}

3. Yields at 300 fb^{-1}

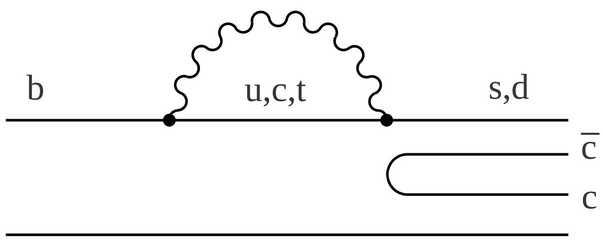
   - $N(D^0(K^-\pi^+)K^-) = 1694$
   - $N(D_1K^-) = 252$ (126 each for $B^\pm$)
   - $N(D_2K^-) = 152$ (76 each for $B^\pm$)

Background? Needs a good vertexing to reject continuum background.

$B \to DK$ **Modes**

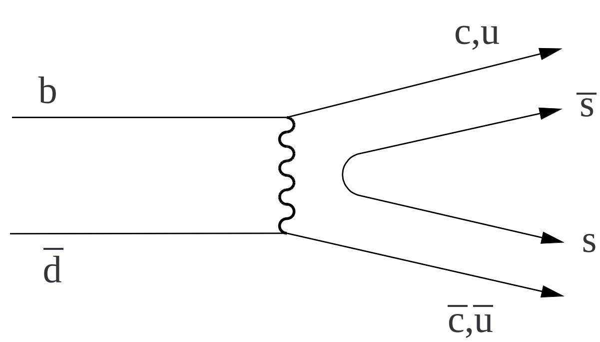
Final state: one charm, one strange.

- **No penguine contaminations**



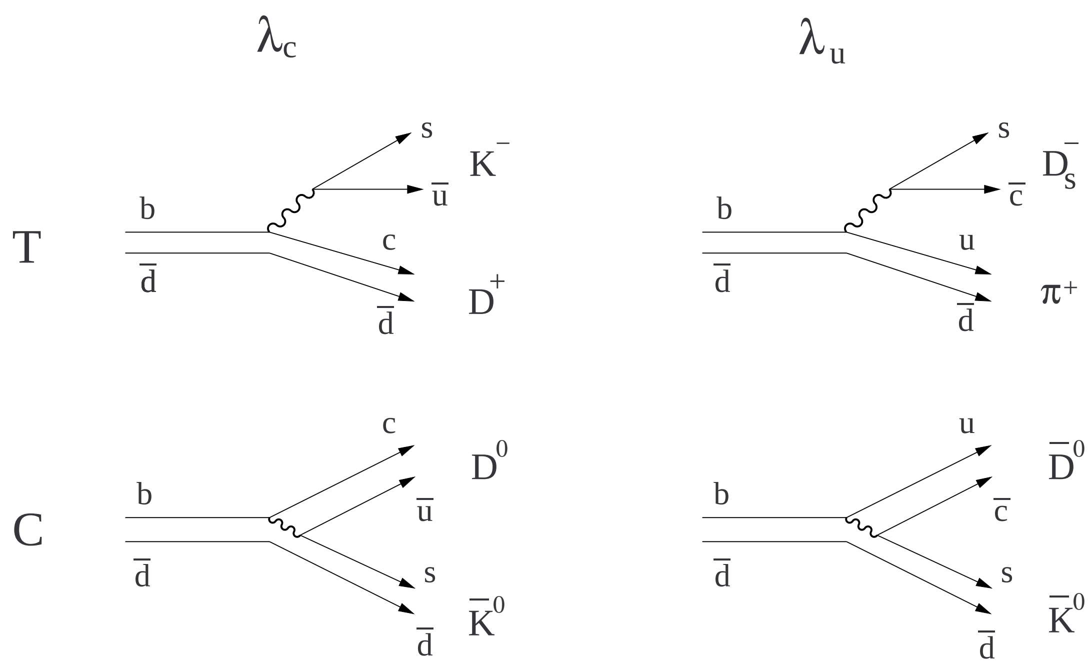
Penguine should have even number of charms.
(True for charged and neutral B)

- **Neutral B has no annihilations**



Annihilations should have even number of stranges.

Classification of $\bar{B}^0 \to DK$

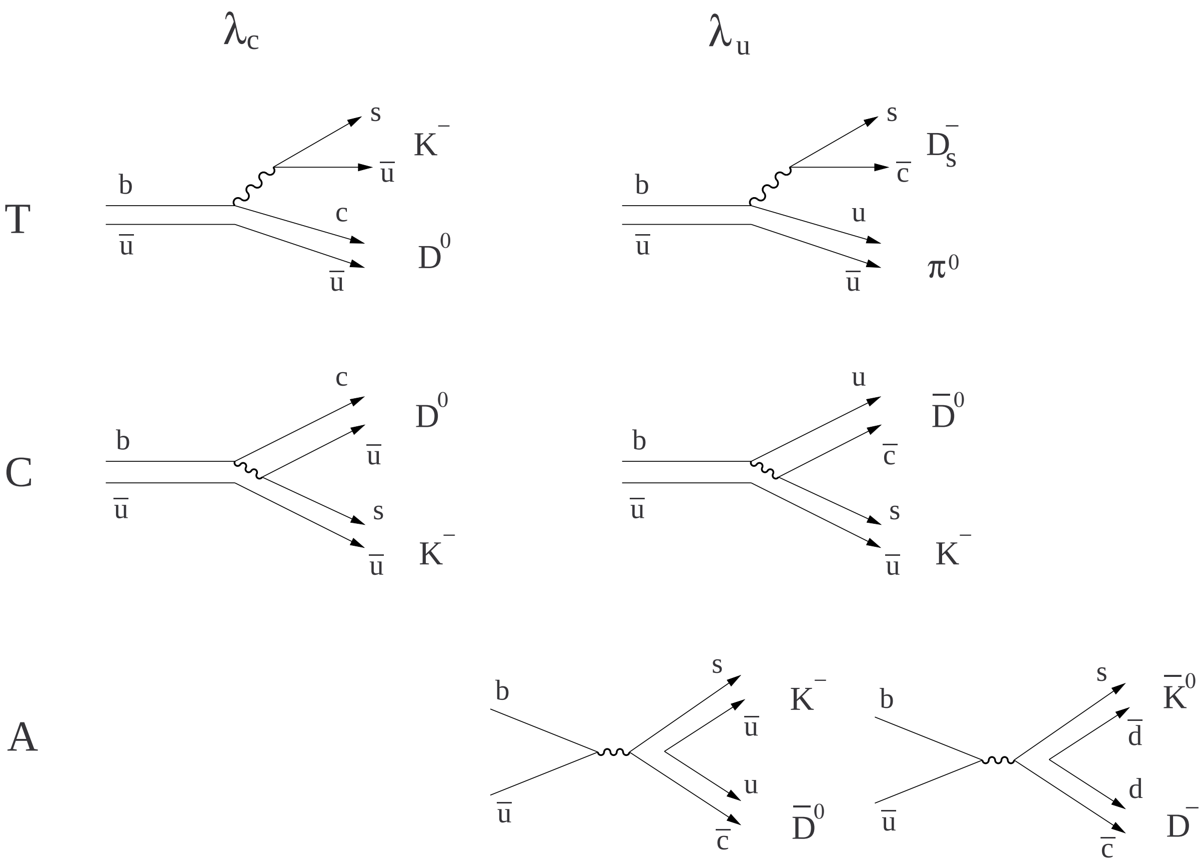


T: tree, C: color-suppressed, A: annihilation
(T, C: depends on $b \to c$ or $b \to u$)

$$\lambda_c = V_{cb} V_{cs}^*\,, \qquad \lambda_u = V_{ub} V_{us}^*\,.$$

$$\begin{aligned}
Amp(\bar{B}^0 \to D^+ K^-) &= \lambda_c T_c \\
Amp(\bar{B}^0 \to D^0 \bar{K}^0) &= \lambda_c C_c \\
Amp(\bar{B}^0 \to \bar{D}^0 \bar{K}^0) &= \lambda_u C_u \\
Amp(\bar{B}^0 \to D_s^- \pi^+) &= \lambda_u T_u
\end{aligned} \qquad (4)$$

Classification of $B^- \to DK$



$$Amp(B^- \to D^0 K^-) = \lambda_c T_c + \lambda_c C_c \qquad (5a)$$
$$Amp(B^- \to \bar{D}^0 K^-) = \lambda_u C_u + \lambda_u A \qquad (5b)$$
$$Amp(B^- \to D^- \bar{K}^0) = \lambda_u A \qquad (5c)$$
$$Amp(B^- \to D_s^- \pi^0) = \frac{1}{\sqrt{2}} \lambda_u T_u \qquad (5d)$$

Final-state Rescatterings

Final-state rescattering can occur:

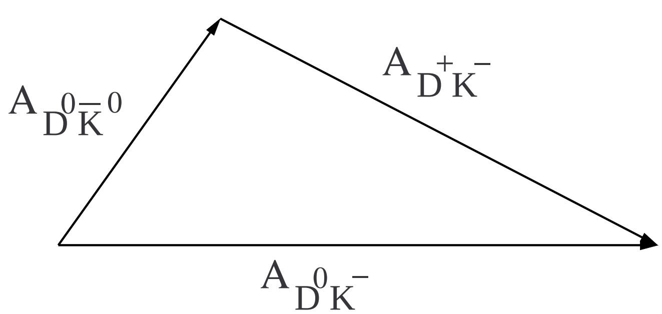
$$\bar{B}^0 \to D^+K^-(T_c) \to D^0\bar{K}^0(C_c)$$
$$\bar{B}^0 \to D_s^-\pi^+(T_u) \to \bar{D}^0\bar{K}^0(C_u)$$

We **define** T_c, C_c, T_u, C_u by (4) including rescattering effects.

Then, is (5a) still true?

$$\begin{aligned} Amp(B^- \to D^0K^-) &= \lambda_c T_c + \lambda_c C_c \\ &= Amp(\bar{B}^0 \to D^+K^-) + Amp(\bar{B}^0 \to D^0\bar{K}^0) \end{aligned}$$

which is nothing but the isospin relation for H_{eff} having $|1/2, -1/2\rangle$ structure: (good to all orders as long as $m_u = m_d$)

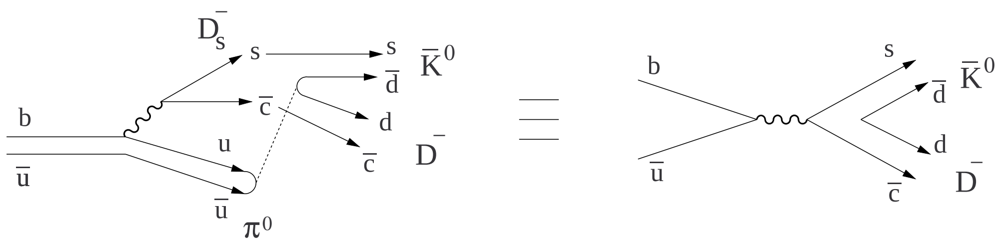


Final-state Rescatterings - annihilation

Final-state $D^-\bar{K}^0$ can be reached by

$$B^- \to D_s^-\pi^0 \to D^-\bar{K}^0$$

This is a ‘long-distance’ annihilation:

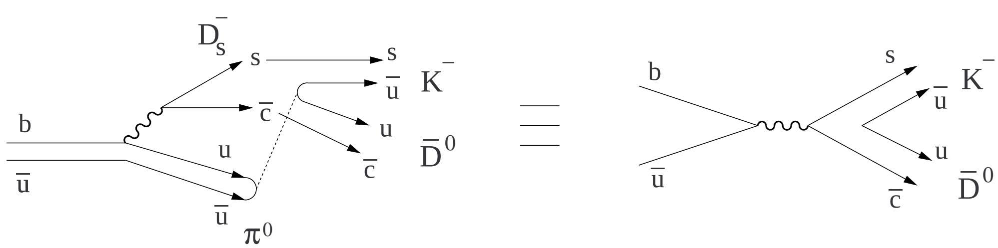


We thus **define** A by

$$Amp(B^- \to D^-\bar{K}^0) = \lambda_u A \quad (5c)$$

including the rescattering effect.

Then, the annihilation in $B^- \to \bar{D}^0K^-$ (5b) has exactly the same rescattering contribution:



GLW, its variant, ADS methods:

Still work after including rescattering
and annihilation effects:

$$A \equiv \lambda_c(T_c + C_c)$$
$$B \equiv \lambda_u(C_u + A)$$

where T_c, C_c, C_u, and A as redefined above.

Then, in particular,

$$r = \frac{\lambda_u}{\lambda_c}\frac{T_c + C_c}{C_u + A}\,.$$

A scenario:

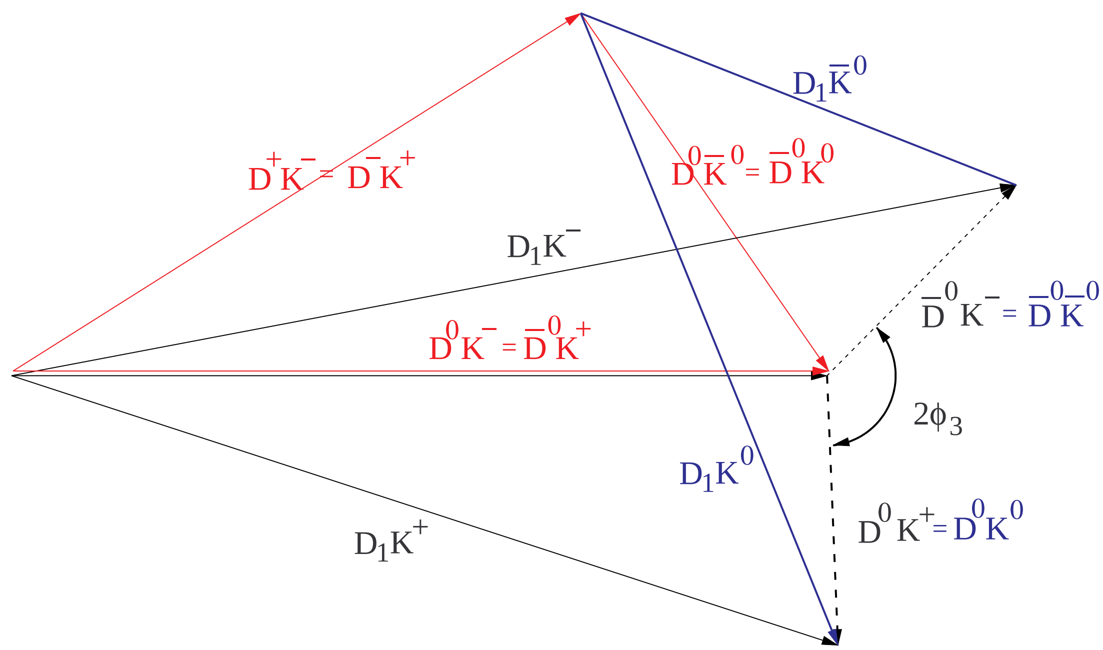
Non observation of $D^- K_s \rightarrow$ smallness of A

$D^0\pi^0$, ΨK^-, $D_s^-\pi^0 \rightarrow r$

If Annihilation is small

(Jang, Ko, 1998)

$$A = 0 \rightarrow \begin{cases} Amp(B^- \rightarrow \bar{D}^0 K^-) = Amp(\bar{B}^0 \rightarrow \bar{D}^0 \bar{K}^0) \\ Amp(B^+ \rightarrow D^0 K^+) = Amp(B^0 \rightarrow D^0 K^0) \end{cases}$$



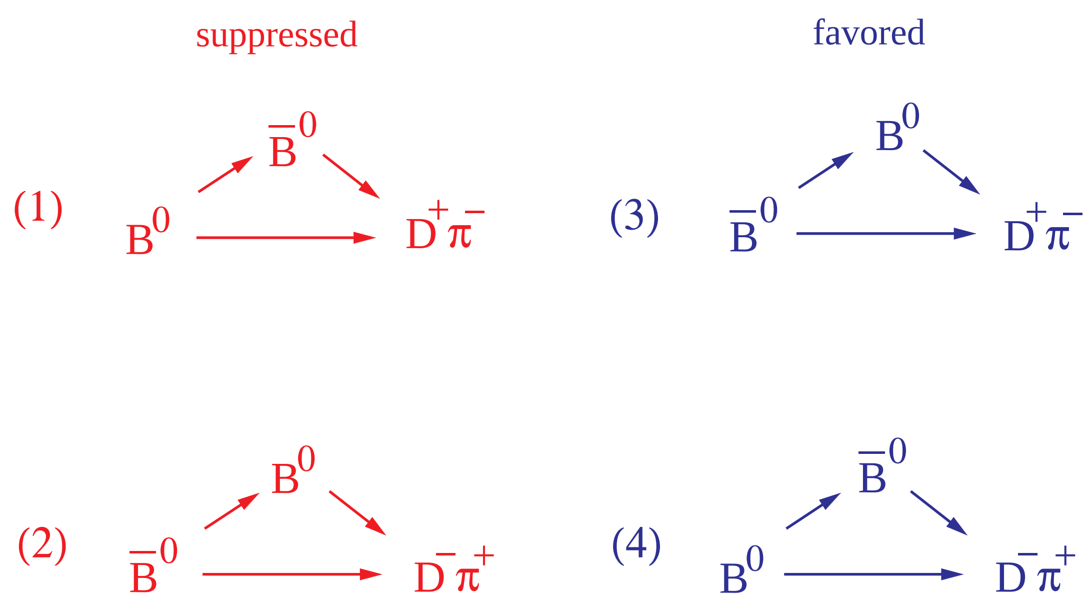
Black: The original double triangle of GLW.

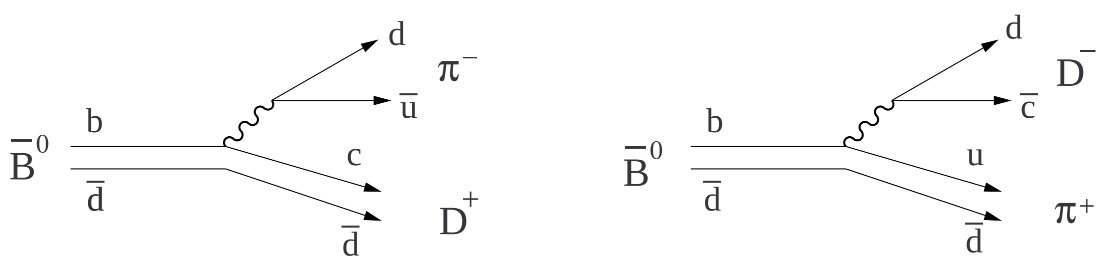
Red: The isospin triangle.

Blue: Measure $D_{1,2}K^{\pm}$ instead of the suppressed B amplitudes.

$B \to D^{(*)+}\pi^-$: Mixing $\to$ non-CP

Sachs (1985), Dunietz, Rosner PRD34 (1986) 1404.





$V_{cb}V_{ud}^*$ $\qquad$ $V_{ub}V_{cd}^*$

|Amplitude ratio| $r \sim \left|\frac{V_{ub}V_{cd}^*}{V_{cb}V_{ud}^*}\right| \sim 0.4\lambda^2 \sim 0.02$

In unit of $|A(B^0 \to D^-\pi^+)A(B^0 \to \ell^+)|^2$

$$(1)\ \Gamma(D^+\pi^-, \ell^-) = \frac{1}{8}\left|\frac{q}{p}\right| e^{-\gamma_+|t_-|}\Big[(1+|\rho'|^2)\mathrm{ch}_{\gamma_+ t_-} - 2\,\Re\rho'\,\mathrm{sh}_{\gamma_- t_-} -(1-|\rho'|^2)\mathrm{c}_{\delta m t_-} - 2\Im\rho'\mathrm{s}_{\delta m t_-}\Big]$$

$$(2)\ \Gamma(D^-\pi^+, \ell^+) = \frac{1}{8}\left|\frac{p}{q}\right| e^{-\gamma_+|t_-|}\Big[(1+|\rho|^2)\mathrm{ch}_{\gamma_+ t_-} - 2\,\Re\rho\,\mathrm{sh}_{\gamma_- t_-} -(1-|\rho|^2)\mathrm{c}_{\delta m t_-} - 2\Im\rho\mathrm{s}_{\delta m t_-}\Big]$$

$$(3)\ \Gamma(D^+\pi^-, \ell^+) = \frac{1}{8} e^{-\gamma_+|t_-|}\Big[(1+|\rho'|^2)\mathrm{ch}_{\gamma_+ t_-} - 2\,\Re\rho'\,\mathrm{sh}_{\gamma_- t_-} +(1-|\rho'|^2)\mathrm{c}_{\delta m t_-} + 2\Im\rho'\mathrm{s}_{\delta m t_-}\Big]$$

$$(4)\ \Gamma(D^-\pi^+, \ell^-) = \frac{1}{8} e^{-\gamma_+|t_-|}\Big[(1+|\rho|^2)\mathrm{ch}_{\gamma_+ t_-} - 2\,\Re\rho\,\mathrm{sh}_{\gamma_- t_-} +(1-|\rho|^2)\mathrm{c}_{\delta m t_-} + 2\Im\rho\mathrm{s}_{\delta m t_-}\Big]$$

$(\mathrm{sh}_x \equiv \sinh(x)\,, \quad \mathrm{s}_x \equiv \sin(x) \quad$ e.t.c.)

$$t_\pm \equiv t_{\mathrm{sig}} \pm t_{\mathrm{tag}}, \qquad \gamma_\pm \equiv \frac{\gamma_a \pm \gamma_b}{2}$$

$$\rho \equiv \frac{qA(\bar{B}^0 \to D^-\pi^+)}{pA(B^0 \to D^-\pi^+)} \sim re^{i(2\phi_1+\phi_3+\delta)}$$

$$\rho' \equiv \frac{pA(B^0 \to D^+\pi^-)}{qA(\bar{B}^0 \to D^+\pi^-)} \sim re^{i(-2\phi_1-\phi_3+\delta)}$$

(δ: strong phase difference: common to ρ&ρ')

$\rightarrow$ Measures $2\phi_1 + \phi_3$

Assume $\gamma_- = (\gamma_a - \gamma_b)/2 = 0$, $|p/q| = 1$,
and $\delta = 0$ for simplicity.

(In unit of $|A(B^0 \to D^- \pi^+) A(B^0 \to \ell^+)|^2$)

$$(1)\ \Gamma(D^+\pi^-, \ell^-) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) - (1-r^2)\mathrm{c}_{\delta m t_-} - 2r\,\xi\,\mathrm{s}_{\delta m t_-}\Big]$$

$$(2)\ \Gamma(D^-\pi^+, \ell^+) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) - (1-r^2)\mathrm{c}_{\delta m t_-} + 2r\,\xi\,\mathrm{s}_{\delta m t_-}\Big]$$

$$(3)\ \Gamma(D^+\pi^-, \ell^+) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) + (1-r^2)\mathrm{c}_{\delta m t_-} + 2r\,\xi\,\mathrm{s}_{\delta m t_-}\Big]$$

$$(4)\ \Gamma(D^-\pi^+, \ell^-) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+}\Big[(1+r^2) + (1-r^2)\mathrm{c}_{\delta m t_-} - 2r\,\xi\,\mathrm{s}_{\delta m t_-}\Big]$$

$$t_- \equiv t_{\mathrm{sig}} - t_{\mathrm{tag}}, \quad r \sim 0.02, \quad \xi \equiv \sin(2\phi_1 + \phi_3)$$

Asymmetry in the suppressed modes (1) $\leftrightarrow$ (2)

Smaller asymmetry in the favored modes (3) $\leftrightarrow$ (4)

Asymmetry is essentially rate asymmetries:
(1), (2) have similar shapes
(3), (4) have similar shapes

Gain in $\#\sigma$ by fitting t_-: ~ 1.5? (study)

Integrate for $t_- > 0$ to see the size of asymmetries.
(In unit of $|A(B^0 \to D^-\pi^+)A(B^0 \to \ell^+)|^2/(4\gamma_+^2)$)

$$(1)\ \Gamma(D^+\pi^-, \ell^-) = (1+r^2) - \frac{1-r^2}{1+x^2} - \frac{2\,\xi\,r\,x}{1+x^2}$$

$$(2)\ \Gamma(D^-\pi^+, \ell^+) = (1+r^2) - \frac{1-r^2}{1+x^2} + \frac{2\,\xi\,r\,x}{1+x^2}$$

$$(1)\ \Gamma(D^+\pi^-, \ell^+) = (1+r^2) + \frac{1-r^2}{1+x^2} + \frac{2\,\xi\,r\,x}{1+x^2}$$

$$(1)\ \Gamma(D^-\pi^+, \ell^-) = (1+r^2) + \frac{1-r^2}{1+x^2} - \frac{2\,\xi\,r\,x}{1+x^2}$$

$$x \equiv \frac{\delta m}{\gamma_+} \sim 0.7\,, \quad r \sim 0.02\,, \quad \xi = \sin(2\phi_1 + \phi_3)$$

Asymmetry in the suppressed ('mixed') modes:

$$A_s \equiv \frac{(1)-(2)}{(1)+(2)} \sim -\frac{2r}{x}\,\xi \sim -0.057\,\xi$$

Asymmetry in the favored ('unmixed') modes:

$$A_f \equiv \frac{(3)-(4)}{(3)+(4)} \sim \frac{2rx}{2+x^2}\,\xi \sim 0.011\,\xi$$

The favored modes has 5 times stat, but 5 times less asym. $\to \sqrt{5}$ times less in $\#\sigma$.

Most of the info is in the suppressed modes.

Statistics needed for $D^{(*)}\pi$

$\sigma_\xi = 0.1 \rightarrow \sigma_{A_s} = 0.0057 \rightarrow N_s = 30K$
(suppressed modes)

We need $6 \times 30K = 180K$ total tagged $D\pi$'s.

Yongheng: 3.7 fb^{-1} $\rightarrow$ 282 ± 25 lepton-tagged $D^*\pi$'s
(partial reconstruction)

No-bkg equivalent: $\left(\frac{282}{25}\right)^2 \sim 127$

300 fb^{-1} $\rightarrow 10K$ to be compared with 180K needed.

- Need to improve background.
- Need to improve tagging efficiency.
- Add various modes (exclusive and partial).
  (strong phases?)

Strong phases for $D^{(*)}\pi$

Recall the asymmetry term in the suppressed modes:
(1) $\cdots + \sin(2\phi_1 + \phi_3 + \delta)\cdots$
(2) $\cdots - \sin(2\phi_1 + \phi_3 - \delta)\cdots$.

In principle, real and imaginary part of $\rho = re^{i(2\phi_1+\phi_3+\delta)}$ and $\rho' = re^{-i(2\phi_1+\phi_3+\delta)}$ can be measured.

$$\rightarrow \frac{\rho}{\rho'} = e^{2i(2\phi_1+\phi_3)}$$

In practice the real parts are difficult to measure.
($\gamma_- \sim 0$ and $|\rho|^2 \sim 10^-4$)

But its is OK. Perform a fit for $2\phi_1 + \phi_3$ and δ on the time distributions for (1) and (2) or all four modes.

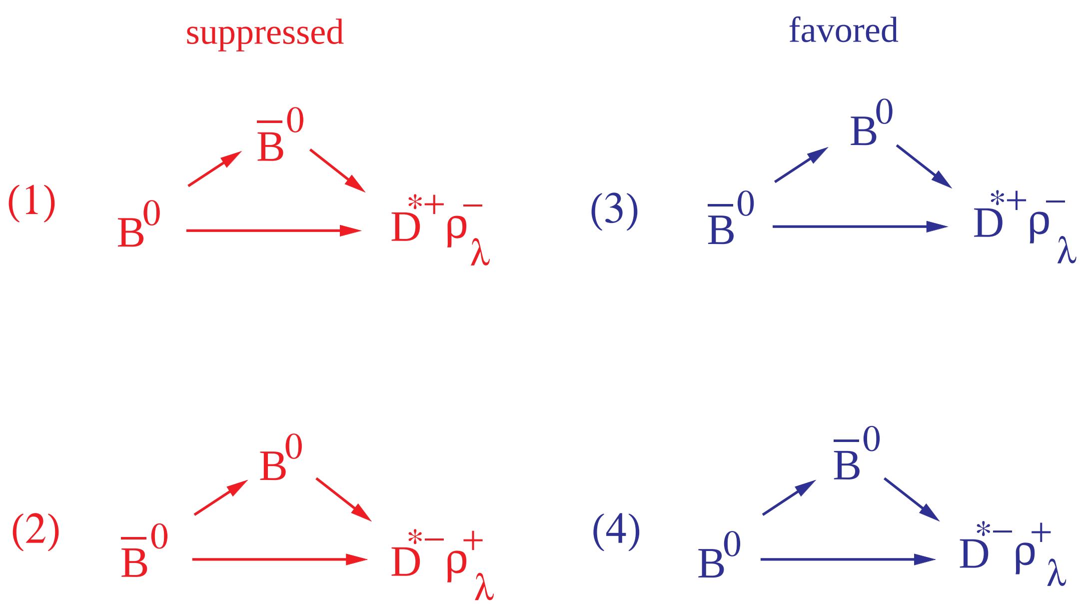
- Nonzero $\delta \rightarrow$ Asymmetry not prop. to $\sin(2\phi_1 + \phi_3)$
- δ is different for every final states
  (e.g. $D\pi$ different from $D^*\pi$ etc.) systematics?

$B \to D^{*+}\rho^-$

Mixing $\to$ non-CP eigenstate + angular correlation

London, Sinha, Sinha, hep-ph/0005248.

Similar to $B \to D\pi$ (needs to be flavor-tagged):
(Measures $2\phi_1 + \phi_3$)



Repeats for each helicity final state.

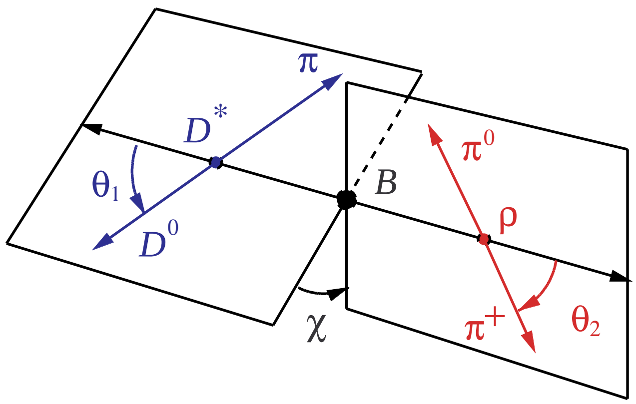
$\lambda = +, -, 0\,,$ or $||, \perp, 0$

|Amplitude ratio| $r \sim 0.02$

$\to$ asymmetry in each $\lambda \sim 0.02$

Angular correlation in $B \to D^*\rho$

(helicity basis)



$$\frac{1}{\Gamma}\frac{d^3\Gamma}{dc_{\theta_1}dc_{\theta_2}d\chi} =$$

$$\frac{9}{32\pi}\Big\{4|H_0|^2 c^2_{\theta_1} c^2_{\theta_2} + (|H_+|^2 + |H_-|^2) s^2_{\theta_1} s^2_{\theta_2}$$

$$+[\Re(H_- H_+^*) c_{2\chi} + \Im(H_- H_+^*) s_{2\chi}] 2 s^2_{\theta_1} s^2_{\theta_2}$$

$$+[\Re(H_- H_0^* - H_+ H_0^*) c_\chi + \Im(H_- H_0^* - H_+ H_0^*) c_\chi] s_{2\theta_1} s_{2\theta_2}\Big\}$$

New ingredients in $D^*\rho$:

Interference between different polarization states

$$\Gamma(B^0 \to D^{*+}\rho^-) =$$
$$e^{-\gamma_+ t} \sum_{\lambda<\lambda'} \Big[\Lambda_{\lambda\lambda'} + \Sigma_{\lambda\lambda'} \mathsf{c}_{\delta m t} - \rho_{\lambda\lambda'} \mathsf{s}_{\delta m t}\Big] g_\lambda g_{\lambda'}$$

(g_λ : function of angles)

The term with $\lambda = \lambda'$ corresponds to the CP vilating terms we have seen in $D\pi$:

$$\rho_{\lambda\lambda'} = \Im\left(\frac{q}{p}(A^*(B^0 \to D^{*+}\rho^-_\lambda)A(\bar{B}^0 \to D^{*+}\rho^-_\lambda)\right)$$

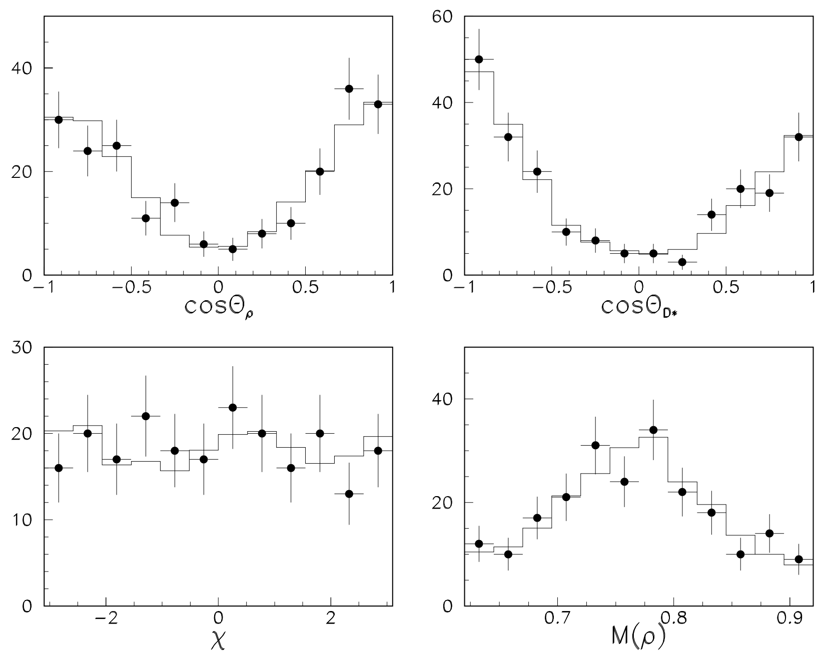
The interference term of ρ have similar size: ($\lambda \neq \lambda'$)

$$\rho_{\lambda\lambda} = \Im\Big(\frac{q}{p}(A^*(B^0 \to D^{*+}\rho^-_\lambda)A(\bar{B}^0 \to D^{*+}\rho^-_{\lambda'})$$
$$+A^*(B^0 \to D^{*+}\rho^-_{\lambda'})A(\bar{B}^0 \to D^{*+}\rho^-_\lambda))\Big)$$

$\rightarrow$ If similar stat as $D\pi$, simiar sensitivity to $2\phi_1 + \phi_1$.
But has more degrees of freedom to measure.
(more powerful resolving ambiguities.
but more sys. study needed)

$D^*\rho$: **CLEO Preliminary**

3.1 fb^{-1}. 197 ± 15 signal events.



$$|H_0|^2 + |H_+|^2 + |H_-|^2 = 1\,, \qquad H_0 = \text{real}$$

$\bar{B}^0 \to D^{*+}\rho^-$

	$\lvert H\rvert$	arg $H(rad)$
H_+	$0.153 \pm 0.052 \pm 0.013$	$1.36 \pm 0.36 \pm 0.32$
H_-	$0.311 \pm 0.048 \pm 0.036$	$0.19 \pm 0.23 \pm 0.13$

$B^- \to D^{*0}\rho^-$

	$\lvert H\rvert$	arg $H(rad)$
H_+	$0.221 \pm 0.064 \pm 0.035$	$0.98 \pm 0.30 \pm 0.08$
H_-	$0.290 \pm 0.066 \pm 0.038$	$1.12 \pm 0.26 \pm 0.09$

Statistics for $D^*\rho$

CLEO: 3.1 fb^{-1} $\rightarrow$ 197 ± 15 signal events.

300 fb^{-1} $\rightarrow$ 19K events. With the high-p_t lepton tag efficiency of 12%, we have 2.3K tagged $D^*\rho$.

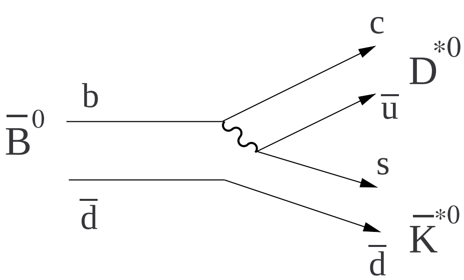
This is compared with 10K (bkg-free equivalent for 300 fb^{-1}) of $D^*\rho$ partial reconstruction analysis. Or compared with 180K needed for $\sigma_\xi = 0.1$.

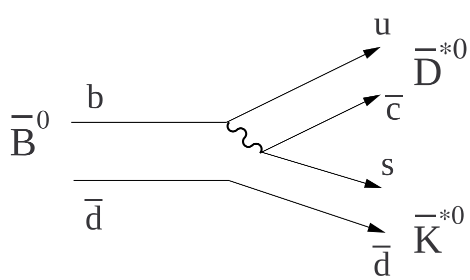
Comments:

- Partical reconstruction cannot be used. This may not be too big a problem since partial reconstruction efficiency is not that good.
- Need to tackle with the systematics of non-resonant component of ρ.
- Also check the sys. of ρ mass dependence of amplitudes.

How about $D^* K^*$?

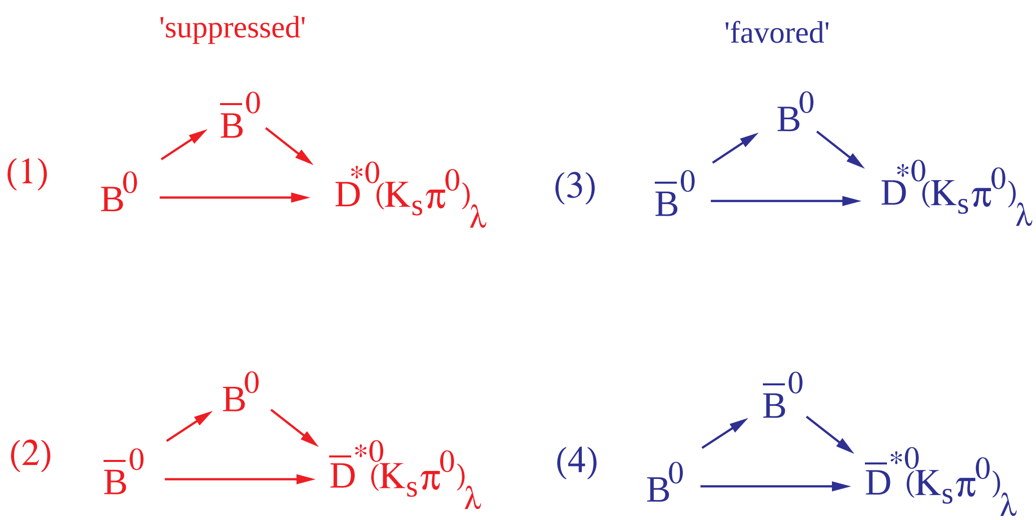
Less stat.than $D^*\rho$, but asym$\sim$1.





$V_{cb}V_{us}^*$ $\qquad$ $V_{ub}V_{cs}^*$

Amplitude ratio ~ 0.4 (compare to $0.4\lambda^2$ for $D^*\rho$)
(note: color-suppressed)



K^{*0} and $\bar{K}^{*0}$ should be detected as $K_s\pi^0$ for the interference to occur.

Sensitivity of D^*K^* wrt $D^*\rho$

Compared to $D^*\rho$, the D^*K^* mode has

1. $\times 1/\lambda^2$ larger asymmetry.

2. $(r_{\text{col}}\,\lambda)^2$ less statistics
   r_{col}: color suppression factor ~ 0.2
   (no Br's, nor det. eff. included)

Thus, $\#\sigma$ is $\dfrac{r_{\text{col}}\,\lambda}{\lambda^2} \sim 1$ times that of $D^*\rho$.

Then, we need to include the BR's and det.eff.
the difference is

$$\frac{eff(K^{*0} \to K_s\pi^0, K_s \to \pi^+\pi^-)}{eff(\rho^+ \to \pi^+\pi^0)}$$

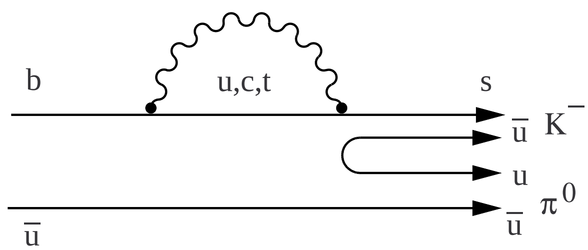
$$\sim \frac{1}{6}(K^{*0} \to K_s\pi^0) \times \frac{2}{3}(K_s \to \pi^+\pi^-) \times \frac{2}{3}(1\,\text{more}\,\pi) \sim 0.08$$

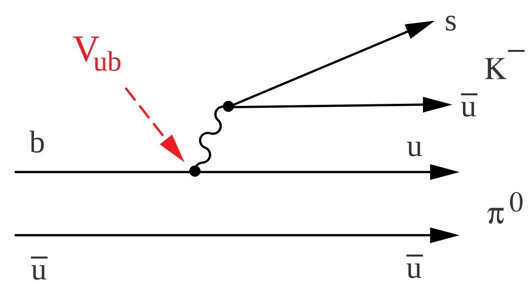
The equivalent stat of $B \to D^*K^*$ angular/time analysis
is $\sim 1/10$ of that of $B \to D^*\rho$.

Using $B \to K\pi, \pi\pi$ for ϕ_3

Tree-penguin interference
$\to$ large direct CP asymmetries expected.

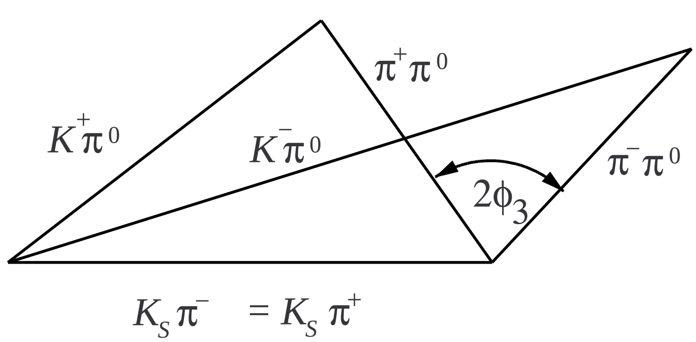
For example: $B^- \to K^-\pi^0$





Interference $\to$ asymmetry $B^- \to K^-\pi^0$ vs $B^+ \to K^+\pi^0$
(infromation on arg$V_{ub} = -\phi_3$.)

Need to remove unknown strong FSI phase.
One historical method:



Note:

- Charged B modes $\rightarrow$ self-tagging.
- SU(3) breaking effect are reasonably under control. Complication by EW penguins which breaks the isospin.
- Requires substantial development in theory.
  $\rightarrow$ QCD factorization formalism:
  Benecke, Buchalla, Neubert, Sachrajda hep-ph/0006124.

Probably the way to approach is to take theorist's predictions of branching ratios (ratios of branching ratios) for various modes and perform a global fit.

Summary

- $B \to K\pi, \pi\pi$ modes are statistically most powerful, but mired by theoretical uncertainties.

- $B \to DK$, $B \to D^{*+}\pi$, $B \to D^{*+}\rho^-$(angular) are all in the same ball park statistically, all worth pursuing.

- $B \to D^{*0}K^{*0}$ angular analysis is not very promising wrt. $B \to D^*\rho$.