

B Factories

- search for the origin of CP violation -

Hitoshi Yamamoto

Univeristy of Hawaii

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B-factory:

e^+e^- collider running on $\Upsilon 4S$

$\Upsilon 4S$ (made of $b\bar{b}$ quark pairs) $\rightarrow B^0\bar{B}^0, B^+B^-$

Main goal:

Study of CP violation.

$\langle e^+e^- \text{ B-factory accelerators} \rangle$

$$e^+e^- \rightarrow \Upsilon_{4S}(10.58\text{GeV}) \rightarrow B^0\bar{B}^0 \text{ or } B^+B^-$$

B pairs nearly at rest in Υ_{4S}

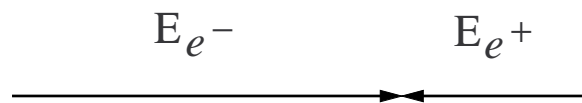
Symmetric energies

(CESR at Cornell)

$$E_{e^-} = E_{e^+} = \frac{M_{\Upsilon_{4S}}}{2} = 5.29\text{GeV}$$

Asymmetric energies

(PEP-II at Stanford, KEK-B at Tsukuba)



Υ_{4S} (and B 's) is moving in the lab frame.
→ B decay time measurements (Why? Later.)

$$E_{\text{CM}} = 2\sqrt{E_{e^+}E_{e^-}} = M_{\Upsilon_{4S}}$$

$$\begin{cases} E_{\Upsilon_{4S}} = E_{e^-} + E_{e^+} \\ P_{\Upsilon_{4S}} = E_{e^-} - E_{e^+} \end{cases}$$

$$\rightarrow \beta_{\Upsilon_{4S}} = \frac{P_{\Upsilon_{4S}}}{E_{\Upsilon_{4S}}} = \frac{E_{e^-} - E_{e^+}}{E_{e^-} + E_{e^+}}$$

Beam separation

Charge in each beam bunch cannot be too large
→ **many bunches**

Want collision to occur only at one location
→ **beam separation**
(avoid parasitic crossings)

CESR: Pretzel orbit

Interweaving e^+e^- orbits within a single ring

Crossing angle = ± 2.3 mrad

PEP-II: Separation by bending magnet

$$E_{e^+} \neq E_{e^-}$$

→ e^+, e^- beams bend differently

Head-on collision

KEK-B: Finite-angle crossing

Crossing angle = ± 11 mrad

Large crossing angle

→ Beam instability
→ Luminosity reduction (geometrical)

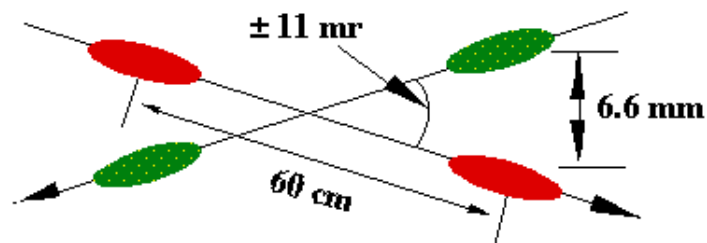
Looks OK for now.

Crab crossing

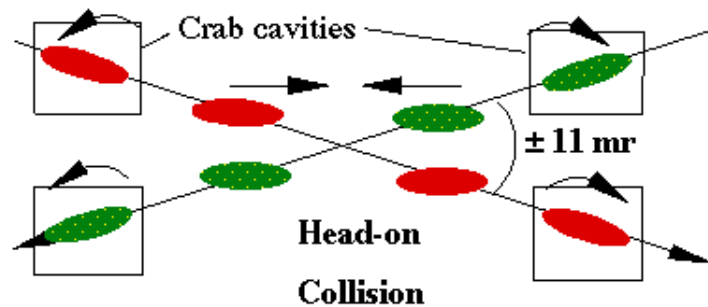
(KEK-B: installation in a few years)

In case finite-angle crossing causes problems

□ Without crab cavities

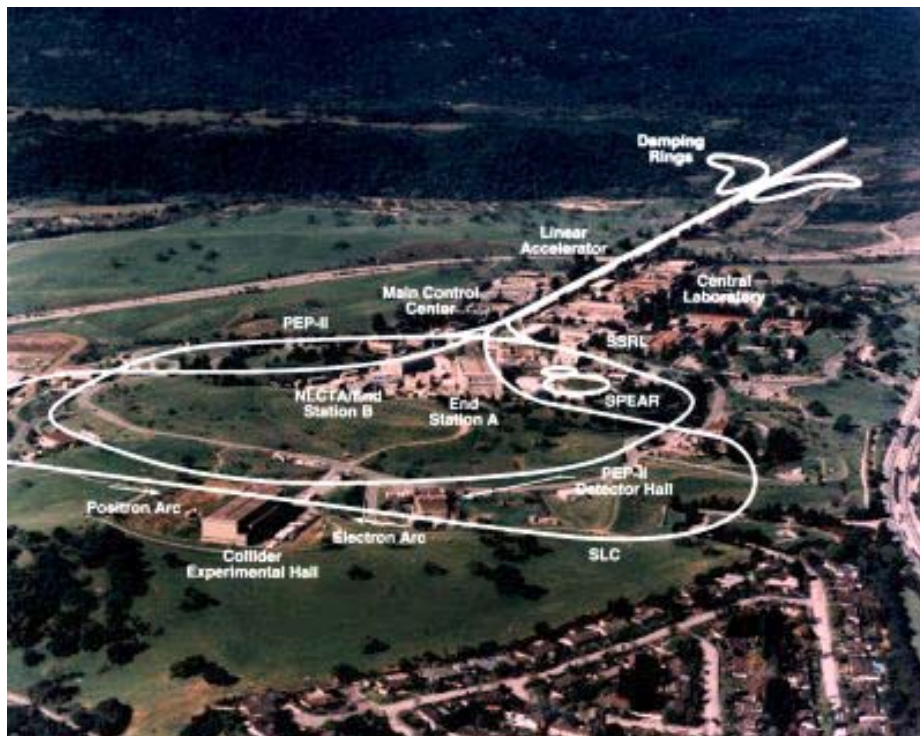
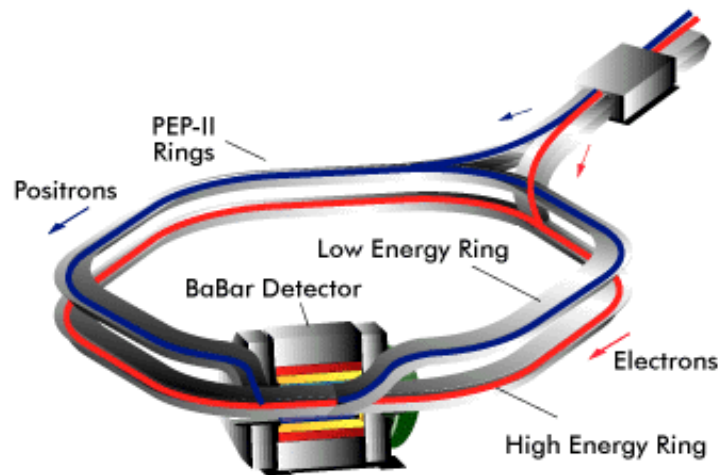


□ With crab cavities

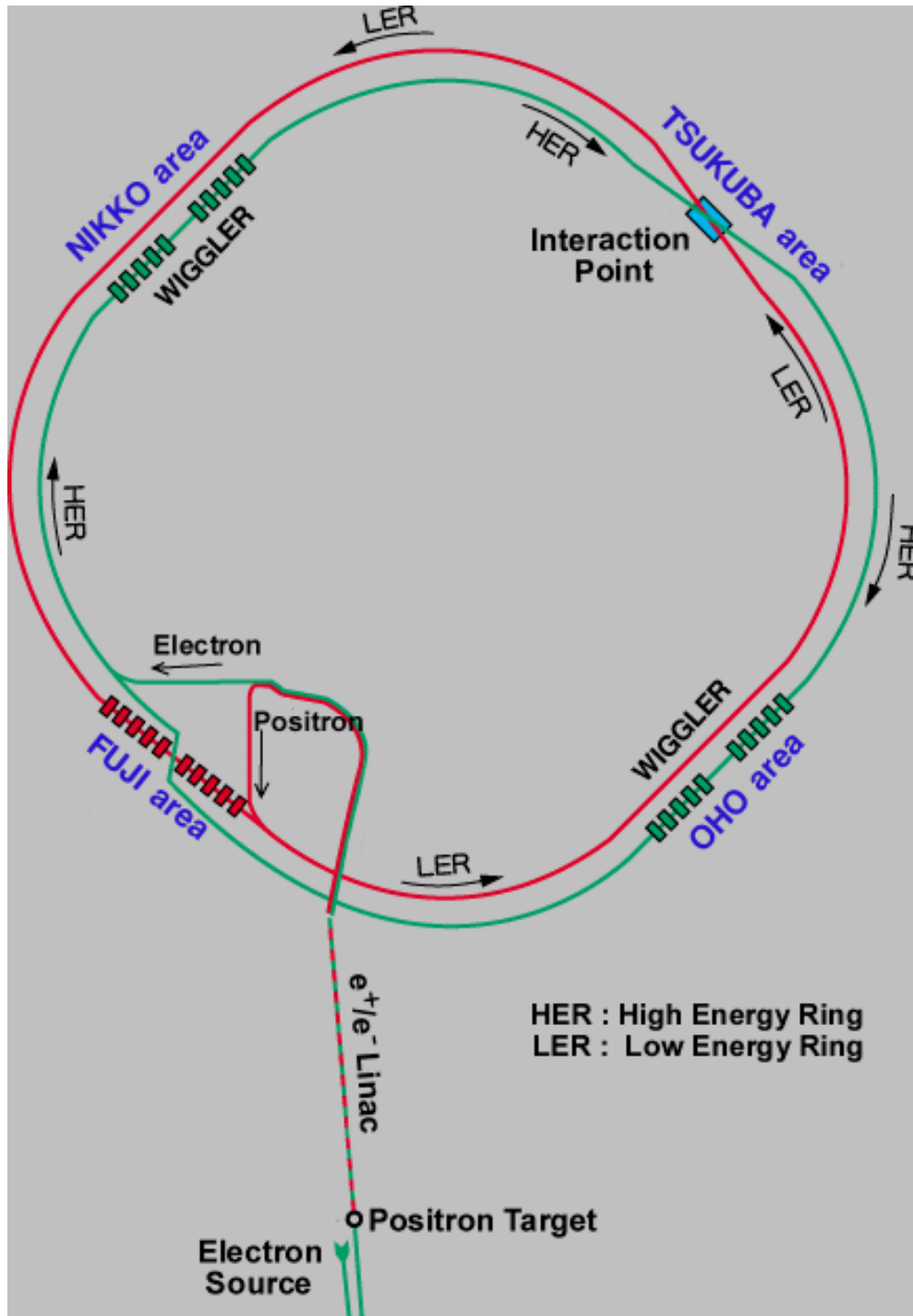


→ complete overlap of beams
(No geometrical luminosity loss.
Suppresses beam-beam instability)

PEP-II (SLAC)



KEK-B (KEK, Japan)



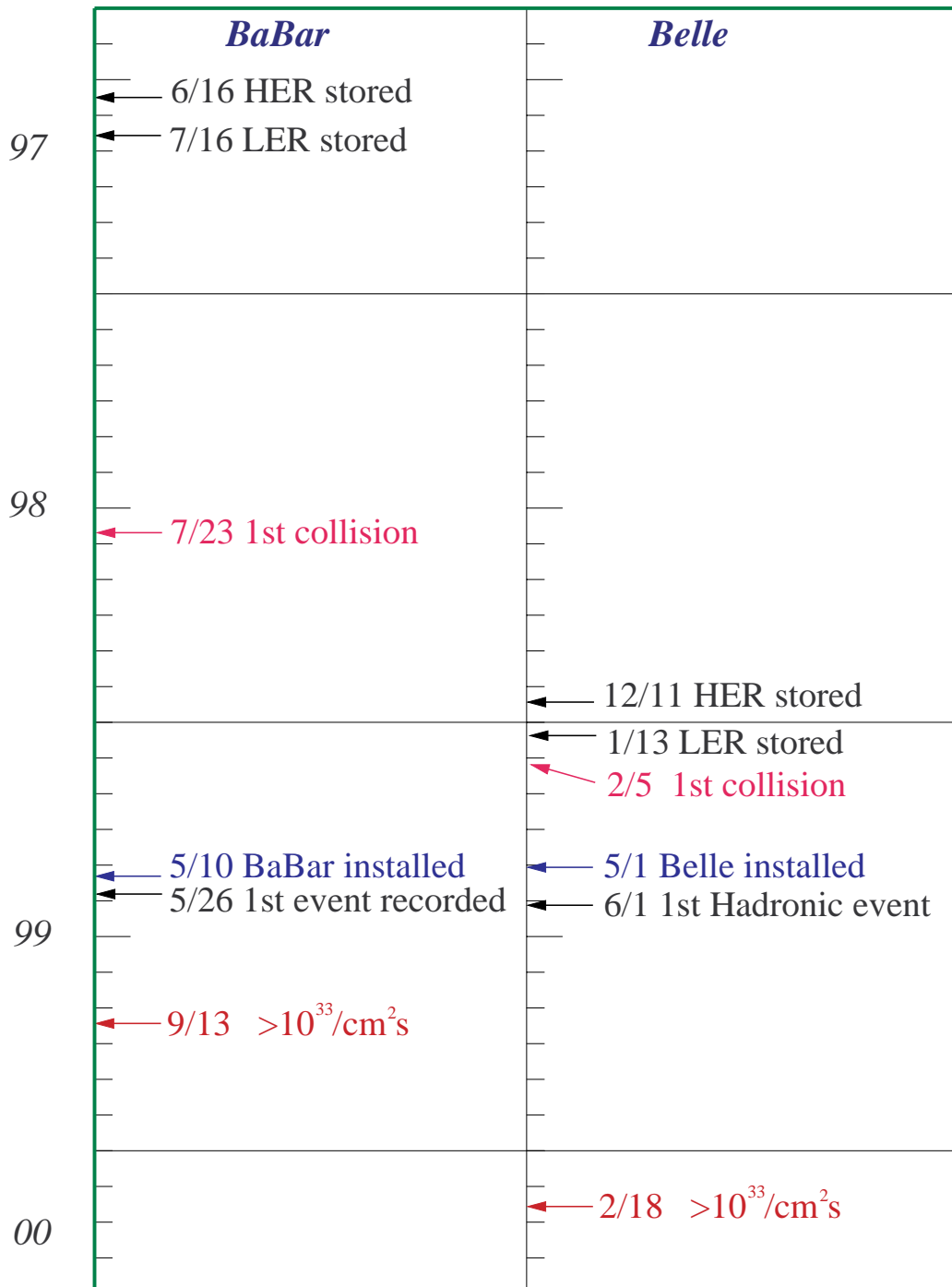
machine	<i>CESR</i>	<i>PEP-II</i>	<i>KEK-B</i>
detector	<i>CLEO</i>	<i>BaBar</i>	<i>Belle</i>
circumference (km)	0.768	2.199	3.016
# of rings	1	2	2
E_{e^+} (GeV)	5.3	3.1	3.5
E_{e^-} (GeV)	5.3	9.0	8.0
$\beta\gamma_{4S}$	~ 0	0.49	0.39
$\delta E/E$	6×10^{-4}	7×10^{-4}	7×10^{-4}
Δt_{bunch}	14ns	4.2ns	2ns
bunch size(w)	500 μ	181 μ	77 μ
" (h)	10 μ	5.4 μ	1.9 μ
" (l)	1.8cm	1.0cm	0.4cm
crossing angle(mrad)	± 2.3	0	± 11
Luminosity($cm^{-2}s^{-1}$)	1.5×10^{33}	3×10^{33}	10×10^{33}
$\#B\bar{B}/s$	1.5	3	10

achievements so far

Lum(peak)	12×10^{32}	31×10^{32}	38×10^{32}
$\int Ldt$ (fb $^{-1}$)	<i>xxx</i>	31.0	24.0

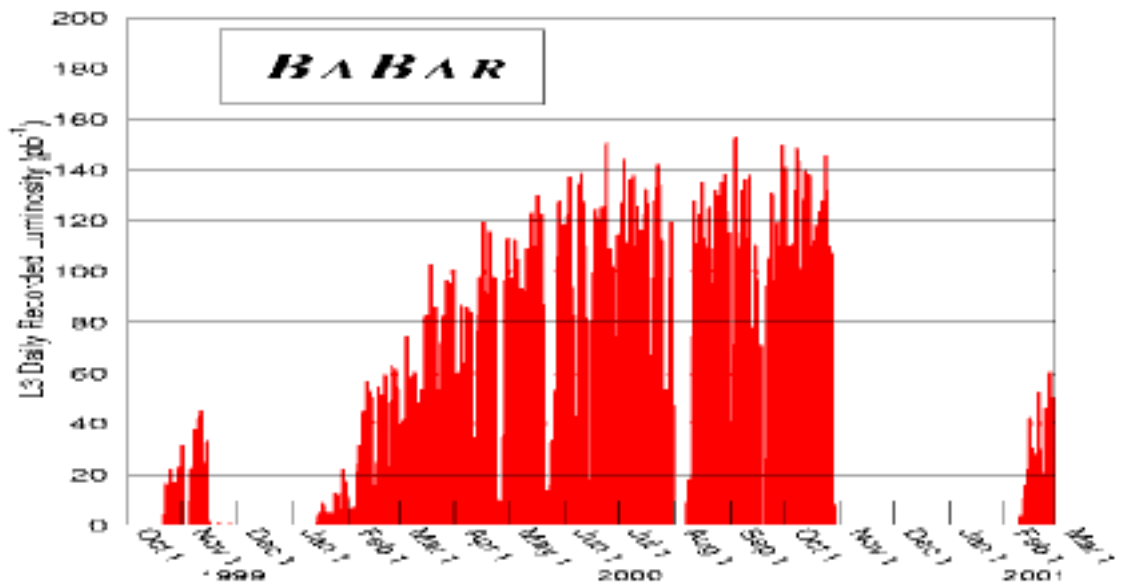
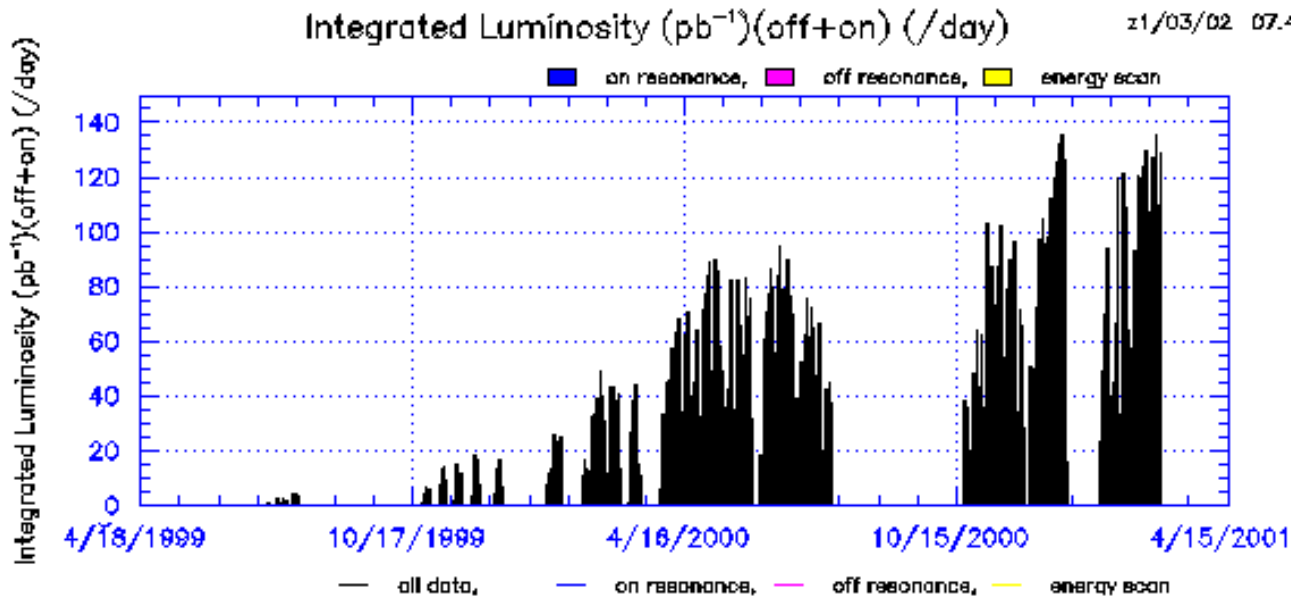
$1 \text{ fb}^{-1} \sim 10^6 B\bar{B}$ pairs

Benchmarks



Luminosity

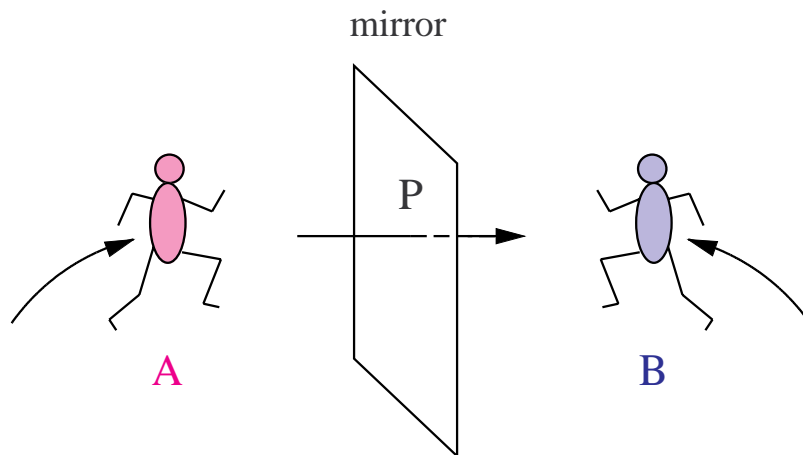
Belle



Introduction to CP violation

Symmetry in physical laws

- Parity (mirror inversion) as an example -



- Suppose motion A satisfies a law of physics.
- Reflect A in the mirror, and think that the motion in the mirror (B) is actually happening.
- **Does B satisfy the same law of physics?**

If YES, and so for all motions that satisfy the law, then the law of physics is **symmetric** under parity, or it **conserves** parity.

Symmetry \leftrightarrow **Conservation**
(Noether's theorem)

Parity symmetry and conservation of parity (Quantum mechanics)

Transition amplitude $S_{f,i}$ from state i to state f :

$$S_{f,i} = \langle f|S|i\rangle$$

Parity inverted states:

$$|Pi\rangle \equiv P|i\rangle, \quad |Pf\rangle \equiv P|f\rangle.$$

If S operator is invariant under (commute with) P

$$\boxed{PSP^\dagger = S}$$

$$\rightarrow S_{Pf, Pi} = \langle f|P^\dagger \underbrace{S}_{PSP^\dagger} P|i\rangle = \langle f|S|i\rangle = S_{f,i}$$

Namely, the parity-inverted process occurs at the same rate, and the physics is thus **symmetric under parity**.

If $PSP^\dagger = S$ and i and f are eigenstates of P :

$$P|i\rangle = \eta_i|i\rangle, \quad P|f\rangle = \eta_f|f\rangle, \quad (\eta_{i,f} = \pm 1)$$

$$S_{f,i} = \langle f| \underbrace{S}_{PSP^\dagger} |i\rangle = \eta_f\eta_i\langle f|S|i\rangle = \eta_f\eta_i S_{f,i}$$

Namely, $S_{f,i} = 0$ unless $\eta_i = \eta_f$:

\rightarrow **parity quantum number is conserved.**

$$\boxed{P \text{ conservation } \overset{PSP^\dagger}{\longleftrightarrow} S \text{ Symmetry under } P}$$

CP symmetry: similarly formulated.

$K^0-\bar{K}^0$ system: the only place CPV (CP Violation) have been seen (before B^0 meson).

1964, K_L (as well as K_S) $\rightarrow \pi^+\pi^-$ (CP+)

This is CPV because:

$Br(K_S \rightarrow \pi\pi) \sim 1 \Rightarrow$ natural to identify

$$K_S = K_1(\text{CP+}), \quad K_L = K_2(\text{CP-})$$

- If $K_L = K_2$ really,

$$K_2(\text{CP-}) \rightarrow \pi^+\pi^-(\text{CP+})$$

(CPV in decay- or - direct CPV)

- If $K_L \neq K_2$, \rightarrow CP+ component in K_L

$$K_2(\text{CP-}) = K_L - \epsilon_0 K_S \\ \xrightarrow{t} c_1 K_L + c_2 K_S \text{ (CP mixture)}$$

(CPV in mixing- or - indirect CPV)

Either case, both $K_L \& K_S \rightarrow \pi^+\pi^-$ is CPV

(A neutron is a physical state and not a CP eigenstate. But it does not mix (evolve) as far as we know.
 \Rightarrow not CPV)

All CPV effects so far are consistent with the hypothesis that

CPV in the $K^0-\bar{K}^0$ system is purely indirect (mixing) with

$$\epsilon_0 = (2.26 \times 10^{-3}) e^{i44^\circ}$$

Direct CPV in K found by:

$KTeV(Fermilab)$ $NA48(CERN)$

Hypothetical interaction that causes indirect CPV :
'Superweak' (Wolfenstein, 1964)

We now have a 'real' theoretical model of CPV :
the Standard Model

Standard-Model quark-W Interaction

$$L_{\text{int}}(t) = \int d^3x (\mathcal{L}_{qW}(x) + \mathcal{L}_{qW}^\dagger(x))$$

$$\mathcal{L}_{qW}(x) = \frac{g}{\sqrt{8}} \sum_{i,j=1,3} V_{ij} \bar{U}_i \gamma_\mu (1 - \gamma_5) D_j W^\mu$$

$$U_i(x) \equiv \begin{pmatrix} u(x) \\ c(x) \\ t(x) \end{pmatrix}, \quad D_j(x) \equiv \begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo-Kobayashi-Masukawa (CKM) matrix
(Unitary)

Experimentally, V has a hierarchical structure.
Approximately,

$$|V_{ij}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$\lambda \sim 0.22$$

Transformation of L_{int} under CP

exchanges particle (n) \leftrightarrow antiparticle (\bar{n})
 CP : flips momentum sign ($\vec{p} \leftrightarrow -\vec{p}$) (a)
 keeps the spin z -component (σ) the same

Such CP operator in Hilbert space is not unique:

$$CP a_{n,\vec{p},\sigma}^\dagger \mathcal{P}^\dagger \mathcal{C}^\dagger = \eta_n a_{\bar{n},-\vec{p},\sigma}^\dagger$$

η_n : 'CP phase': arbitrary, depends on n
 (for antiparticle: $\eta_{\bar{n}} = (-)^{2J} \eta_n^*$, $J = \text{spin}$)

The choice of η_n amounts to choosing a specific operator in Hilbert space among those satisfying (a).

Then, a pure algebra leads to

$$\begin{aligned} CP \bar{u}(x) \gamma_\mu (1 - \gamma_5) d(x) W^\mu(x) \mathcal{P}^\dagger \mathcal{C}^\dagger \\ = \eta_u \eta_d^* \eta_W^* \left(\bar{u}(x') \gamma^\mu (1 - \gamma_5) d(x') W_\mu(x') \right)^\dagger \\ x' \equiv (t, -\vec{x}) \end{aligned}$$

\mathcal{L}_{qW} transforms as (taking $\eta_W = 1$)

$$\begin{aligned} & \mathcal{CP} \mathcal{L}_{qW}(x) \mathcal{P}^\dagger \mathcal{C}^\dagger \\ &= \frac{g}{\sqrt{8}} \sum_{i,j=1,3} \eta_{U_i} \eta_{D_j}^* V_{ij} \left(\bar{U}_i(x') \gamma^\mu (1 - \gamma_5) D_j(x') W_\mu(x') \right)^\dagger \end{aligned}$$

IF $\eta_{U_i} \eta_{D_j}^*$ can be chosen such that

$$\boxed{\eta_{U_i} \eta_{D_j}^* V_{ij} = V_{ij}^*},$$

then, $\mathcal{CP} \mathcal{L}_{qW}(x) \mathcal{P}^\dagger \mathcal{C}^\dagger = \mathcal{L}_{qW}^\dagger(x')$

Namely,

$$\mathcal{L}_{qW} \xleftrightarrow{CP} \mathcal{L}_{qW}^\dagger$$

→ $L_{\text{int}}(t)$ becomes invariant under CP:

$$\begin{aligned} & \mathcal{CP} L_{\text{int}}(t) \mathcal{P}^\dagger \mathcal{C}^\dagger \\ &= \int d^3x \mathcal{CP} [\mathcal{L}_{qW}(x) + \mathcal{L}_{qW}^\dagger(x)] \mathcal{P}^\dagger \mathcal{C}^\dagger \\ &= \int d^3x [\mathcal{L}_{qW}^\dagger(x') + \mathcal{L}_{qW}(x')] \\ &= L_{\text{int}}(t) \end{aligned}$$

→ S matrix is invariant under CP

Condition for CP Invariance

The CP invariance condition (2) is equivalent to

rotate the quark phases to make $V_{i,j}$ all real.

Can the CKM matrix be made real
by rotating quark phases?

Count the degrees of freedom

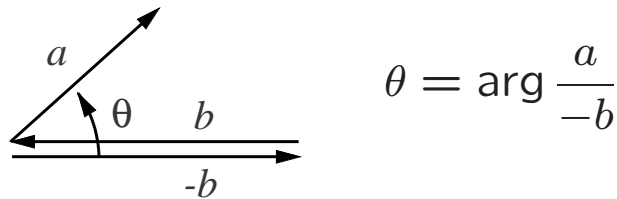
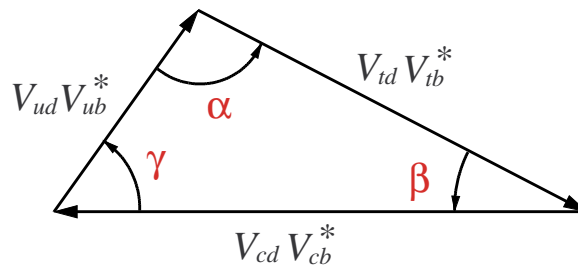
3×3 complex matrix	18
Unitarity condition $V^\dagger V = I$	-9
Quark relative phases	-5
Eular angles (real)	-3
Complex phase	1

One irreducible complex phase \rightarrow CP violation
(Kobayashi, Masukawa)

One complex phase \rightarrow Unitarity Triangle

e.g: orthogonality of d -column and b -column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



$$\theta = \arg \frac{a}{b}$$

$$\alpha \equiv \arg \left(\frac{V_{td}V_{tb}^*}{-V_{ud}V_{ub}^*} \right), \quad \beta \equiv \arg \left(\frac{V_{cd}V_{cb}^*}{-V_{td}V_{tb}^*} \right), \quad \gamma \equiv \arg \left(\frac{V_{ud}V_{ub}^*}{-V_{cd}V_{cb}^*} \right)$$

(Another notation: $\alpha \equiv \phi_2, \beta \equiv \phi_1, \gamma \equiv \phi_3$)

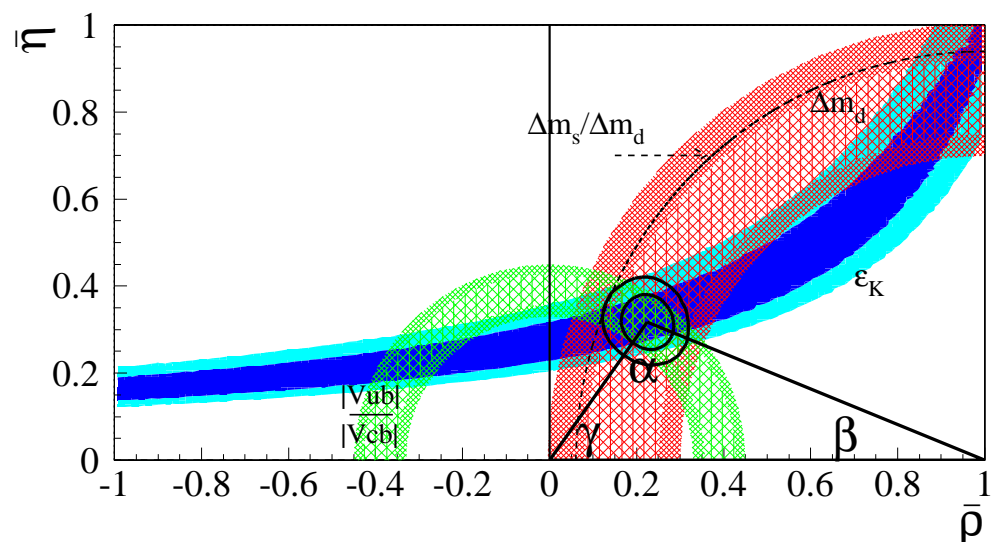
If the CKM matrix is real, the triangle is a line.

How does the CKM unitarity triangle look?

Experimental inputs:

1. $|V_{ub}/V_{cb}|$ (by $b \rightarrow ue\nu$)
2. $B^0-\bar{B}^0$ mixing $\rightarrow |V_{td}|$
3. ϵ_K (from Kaon system)

Many people have performed a fit.
One recent example: Ciuchini et.al.:



Normalized to the bottom length of the triangle.
(two bands for each are 68% and 95% c.l.)

Three bands cross at one point
 \rightarrow already a triumph of the standard model.

**If the K-M model of CP violation is correct,
 → CP Violation (CPV) in B meson decays**

1. **CPV in mixing. (neutral B)**

Particle-antiparticle imbalance in physical neutral B states ($B_{a,b}$): $|\langle B^0 | B_{a,b} \rangle|^2 \neq |\langle \bar{B}^0 | B_{a,b} \rangle|^2$

Expected to be small.

2. **CPV in decay. (neutral and charged B)**

Partial decay rate asymmetries.

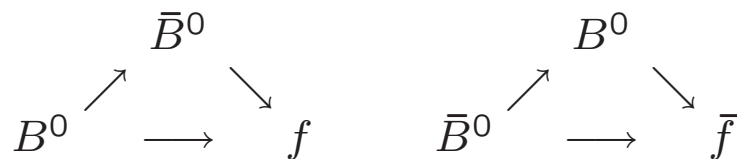
$$|Amp(B \rightarrow f)| \neq |Amp(\bar{B} \rightarrow \bar{f})|$$

($Amp(B^0 \rightarrow f)$: instantaneous decay amplitude.)

Difficult to analyse → 2nd-round measurements.

3. **CPV by mixing-decay interference. (neutral B)**

When both B^0 & \bar{B}^0 can decay to the same final state f :



the interference results in

$$\Gamma_{B^0 \rightarrow f}(t) \neq \Gamma_{\bar{B}^0 \rightarrow \bar{f}}(t).$$

Most promising.

Mixing-decay interference

Eigenstates of mass & decay rate (assume CPT):

$$(*) \quad \begin{cases} B_a = pB^0 + q\bar{B}^0 \\ B_b = pB^0 - q\bar{B}^0 \end{cases}$$

B_a (mass: m_a , decay rate: γ_a)

B_b (mass: m_b , decay rate: γ_b)

In good approximation, $\gamma_a = \gamma_b (\equiv \gamma)$

→ decay rate decouple from the arguments below.

(Also, p, q becomes pure phases)

Time evolution:

$$B_a \rightarrow B_a e^{-im_a t}, \quad B_b \rightarrow B_b e^{-im_b t}.$$

Solving (*) for B^0 and \bar{B}^0 ,

the time evolutions of pure B^0 or \bar{B}^0 at $t = 0$ are

$$\begin{cases} B^0 \rightarrow B^0 \cos \frac{\delta m}{2} t - \frac{q}{p} \bar{B}^0 i \sin \frac{\delta m}{2} t \\ \bar{B}^0 \rightarrow \bar{B}^0 \cos \frac{\delta m}{2} t - \frac{p}{q} B^0 i \sin \frac{\delta m}{2} t \end{cases}$$

$$(\delta m \equiv m_a - m_b)$$

Amplitude for pure B^0 or \bar{B}^0 at $t = 0$
to decay to f at t :

$$\begin{cases} A_{B^0 \rightarrow f}(t) = A \cos \frac{\delta m}{2} t - \frac{q}{p} \bar{A} i \sin \frac{\delta m}{2} t \\ A_{\bar{B}^0 \rightarrow f}(t) = \bar{A} \cos \frac{\delta m}{2} t - \frac{p}{q} A i \sin \frac{\delta m}{2} t \end{cases},$$

$$A \equiv \text{Amp}(B^0 \rightarrow f), \quad \bar{A} \equiv \text{Amp}(\bar{B}^0 \rightarrow f).$$

The probability that a pure $B^0(\bar{B}^0)$ at $t = 0$ decays to
a final state f at t : (for f : CP eigenstate):

$$\Gamma_{B^0(\bar{B}^0) \rightarrow f}(t) = e^{-\gamma t} |pA|^2 \left[1 \pm \Im \left(\frac{q\bar{A}}{pA} \right) \sin \delta m t \right]$$

→ CPV by def.

The Time-dependent asymmetry is

$$A_{CP}(t) = \Im \left(\frac{q\bar{A}}{pA} \right) \sin \delta m t$$

How to prepare ('tag') pure B^0/\bar{B}^0 on $\Upsilon 4S$?

Since \bar{B}^0 decays to $e^- + X$ but not $e^+ + X$
(X : something), look at the other side:

$$\Upsilon 4S \rightarrow \begin{cases} B^0 \rightarrow f \\ \bar{B}^0 \rightarrow e^- X \end{cases}$$

If the other side decays to e^- , then f came from B^0 - ?

In reality, $B^0\bar{B}^0$ pair is created in a coherent $L = 1$ state which is asymmetric. The time evolution in the $\Upsilon 4S$ system is (with a simple algebra)

$$\begin{aligned} \Upsilon 4S &\rightarrow (B^0\bar{B}^0 - \bar{B}^0B^0) \\ &\rightarrow e^{-\gamma t}(B^0\bar{B}^0 - \bar{B}^0B^0) \end{aligned}$$

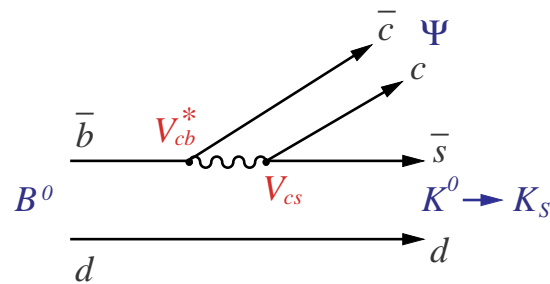
If one finds one side to be \bar{B}^0 at t , then the other side is pure B^0 **at the same time** t , then it will evolve as before.

$\rightarrow \Gamma_{B^0(\bar{B}^0) \rightarrow f}(t)$ applies to $\Upsilon 4S$ with

$$t \rightarrow \Delta t \equiv t(f) - t(\text{tag}),$$

$$(\text{and } e^{-\gamma t} \rightarrow e^{-\gamma|\Delta t|})$$

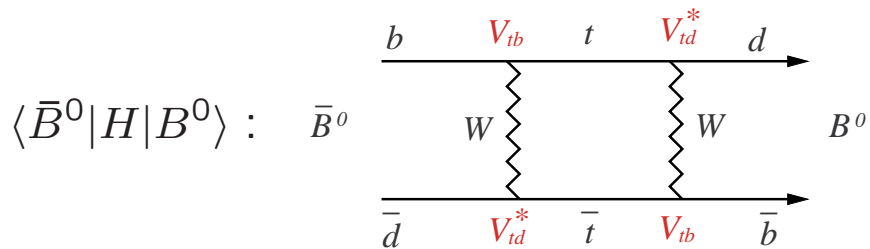
The gold-plated mode $f = \Psi K_S$



What is $\Im \frac{q\bar{A}}{pA}$ for this mode?

q/p is obtained by diagonalizing

$$H_{\text{eff}} = \begin{pmatrix} \langle B^0 | H | B^0 \rangle & \langle B^0 | H | \bar{B}^0 \rangle \\ \langle \bar{B}^0 | H | B^0 \rangle & \langle \bar{B}^0 | H | \bar{B}^0 \rangle \end{pmatrix}$$



$$\rightarrow \frac{q}{p} = \frac{\langle \bar{B}^0 | H | B^0 \rangle}{|\langle \bar{B}^0 | H | B^0 \rangle|} = -\frac{V_{td} V_{tb}^*}{V_{td}^* V_{tb}}$$

$$f = \Psi K_S$$

$$\frac{\bar{A}}{A} = \frac{\langle K_S | \bar{K}^0 \rangle \langle \Psi \bar{K}^0 | H | \bar{B}^0 \rangle}{\langle K_S | K^0 \rangle \langle \Psi K^0 | H | B^0 \rangle} = \frac{-q_K^* V_{cb} V_{cs}^*}{p_K^* V_{cb}^* V_{cs}}$$

With a similar procedure to the B case,

$$\frac{q_K}{p_K} = -\frac{V_{cd} V_{cs}^*}{V_{cd}^* V_{cs}}$$

Combining all,

$$\frac{q\bar{A}}{pA} = \left(\frac{V_{cd} V_{cb}^*}{-V_{td} V_{tb}^*} \right)^* / \left(\frac{V_{cd} V_{cb}^*}{-V_{td} V_{tb}^*} \right)$$

With the definition of the angle β :

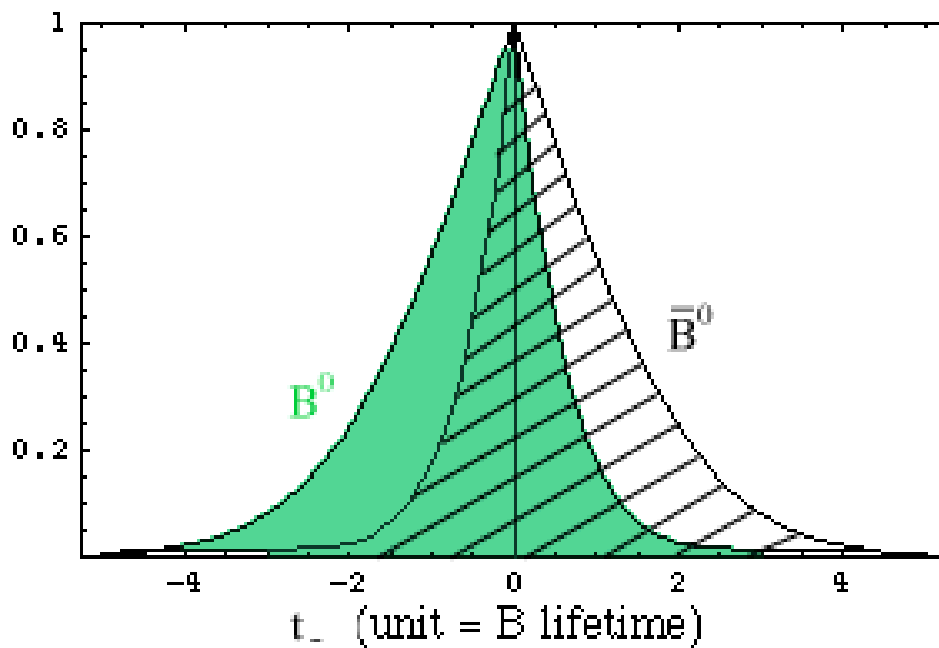
$$\beta = \arg \left(\frac{V_{cd} V_{cb}^*}{-V_{td} V_{tb}^*} \right)$$

$$\rightarrow \Im \frac{q\bar{A}}{pA} = -\sin 2\beta \quad (\Psi K_S)$$

$$\Upsilon 4S \rightarrow \begin{cases} B_1 \rightarrow \Psi K_S (t_1) \\ B_2 \rightarrow e^\mp X (t_2) \end{cases}$$

$$\Gamma_{e^\mp, \Psi K_S}(\Delta t) = N(1 \mp \sin 2\beta \sin \delta m \Delta t)$$

$$(\Delta t \equiv t_1 - t_2)$$



$$B^0 \equiv e^- \text{ tag}, \quad \bar{B}^0 \equiv e^+ \text{ tag},$$

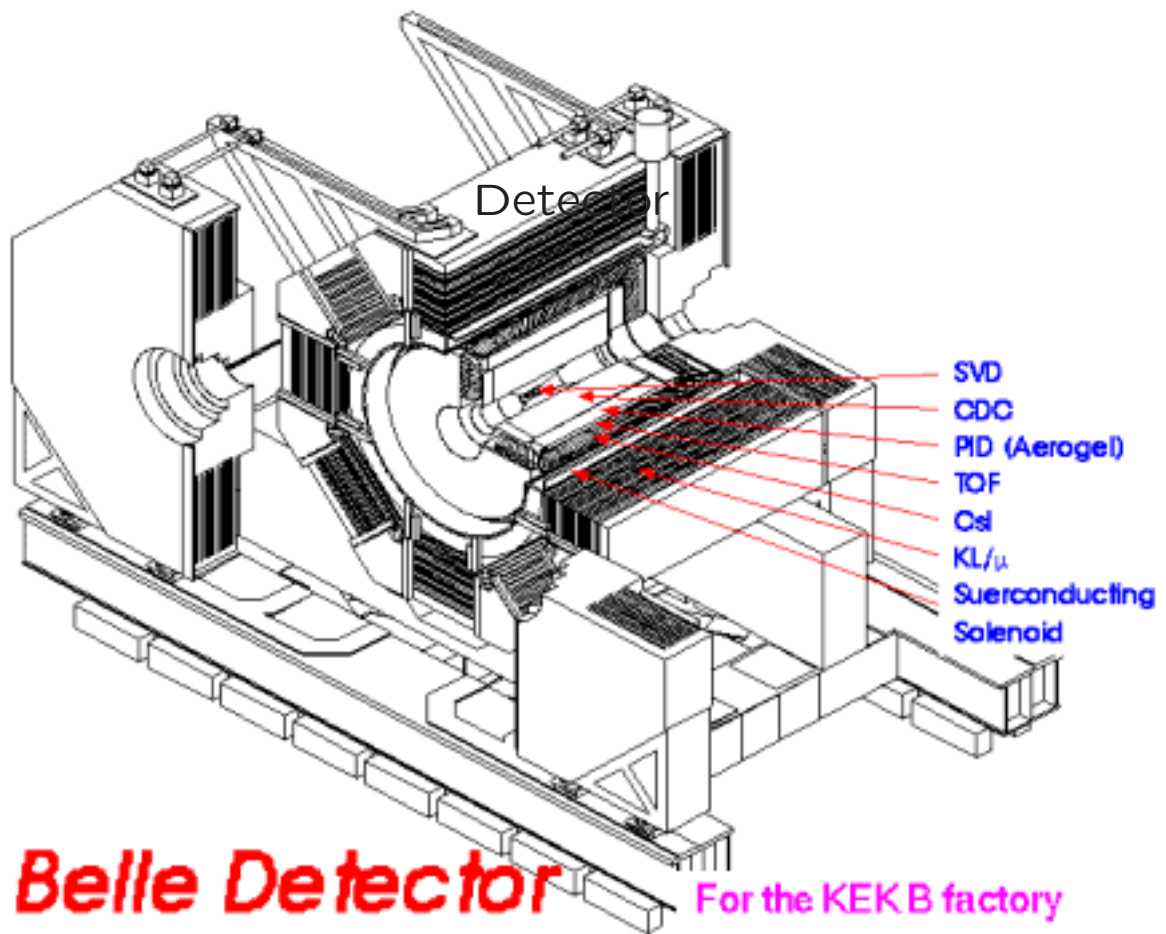
Total area is the same for B^0/\bar{B}^0

→ need to measure Δt

⇒ **Asymmetric B-factory**

(moving B 's → decay vertexes → Δt)

[At CLEO, $B^0\bar{B}^0$ are nearly at rest]



- SVD: Silicon vertex detector.
resolution $\sim 100 \mu\text{m}$ (B lifetime = $200 \mu\text{m}$)
- CDC: charged particle tracking
- PID: π/K separation
- TOF: π/K separation
- Cal: electron/photon detection
- KL/μ : K_L /muon detection

<Full reconstruction on $\Upsilon 4S$ >

$$B \rightarrow f_1 \cdots f_n$$

Energy and absolute momentum of B in the $\Upsilon 4S$ frame are known:

$$E_B = E_{\text{beam}} = 5.290 \text{ GeV}$$
$$|\vec{P}_B| = \sqrt{E_{\text{beam}}^2 - M_B^2} = 0.34 \text{ GeV}/c$$

→ Move to the $\Upsilon 4S$ rest frame and require that candidates satisfy

$$E_{\text{tot}} = E_{\text{beam}}, \quad |\vec{P}_{\text{tot}}| = |\vec{P}_B|$$

where

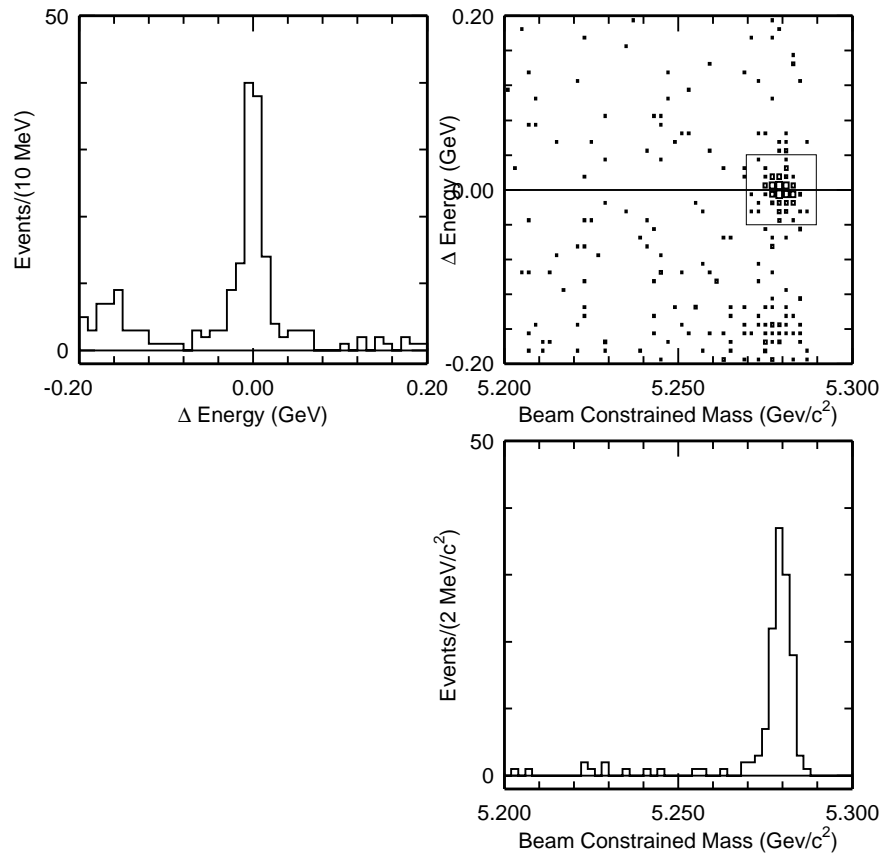
$$E_{\text{tot}} \equiv \sum_{i=1}^n E_i, \quad \vec{P}_{\text{tot}} \equiv \sum_{i=1}^n \vec{P}_i$$

Instead of E_{tot} and $|\vec{P}_{\text{tot}}|$, we often use

$$\Delta E \equiv E_{\text{tot}} - E_B \quad (\text{energy difference})$$

$$M_{\text{bc}} \equiv \sqrt{E_{\text{beam}}^2 - \vec{P}_{\text{tot}}^2} \quad (\text{beam-constrained mass})$$

$$\Psi \rightarrow e^+e^-, \mu^+\mu^-, \quad K_S \rightarrow \pi^+\pi^-$$

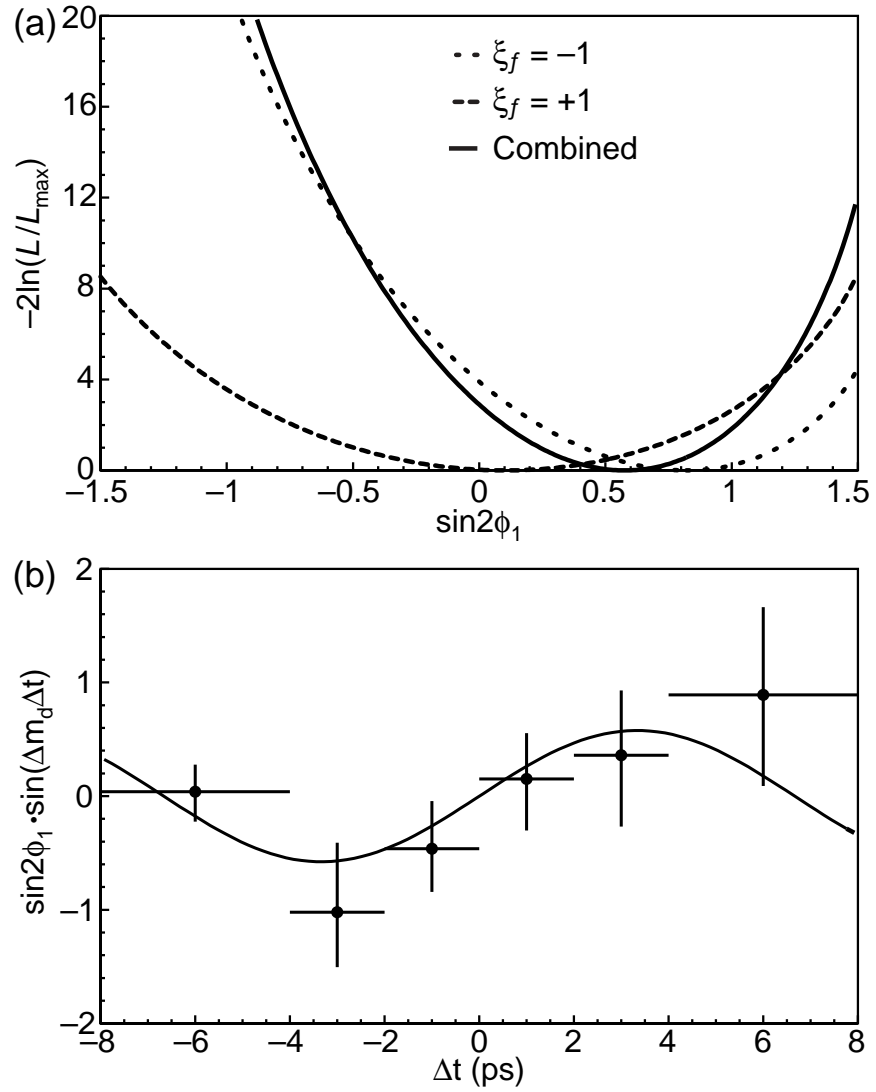


Also use

- $\Psi'K_S, \chi_{c1}K_S, \eta_cK_S$ ($\xi_f = -1$)
- $\Psi\pi^0, \Psi K_L$ ($\xi_f = +1$)
- Tagging by μ^\pm, K^\pm, π^\pm

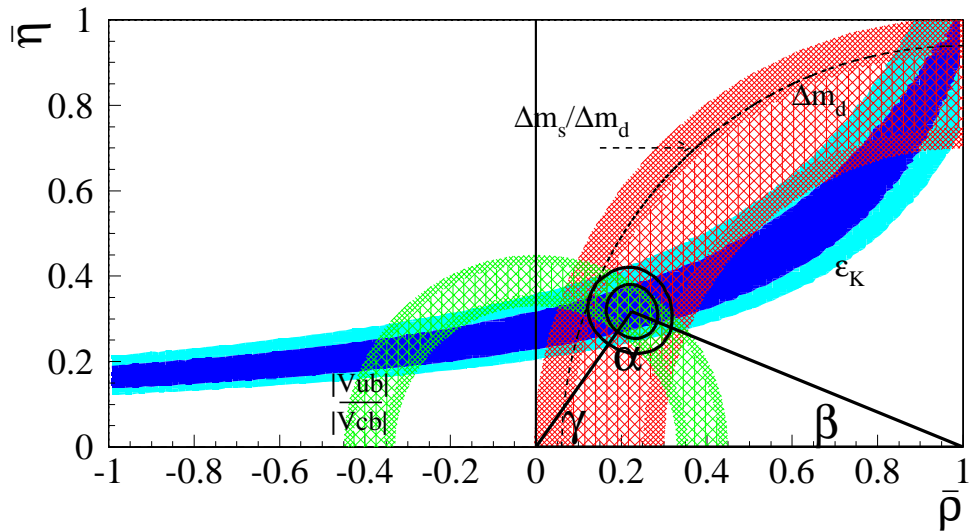
ξ_f : CP of the final state (sign in front of $\sin 2\beta$)

Result on $\sin 2\beta$ ($\equiv \sin 2\phi_1$)



$$\sin 2\beta = 0.58^{+0.32}_{-0.34}(\text{stat})^{+0.09}_{-0.10}(\text{sys})$$

In terms of the unitarity triangle)



The Belle result is consistent with the standard model.

Other experiments:

Experiment	year	$\sin 2\beta$
BaBar	2001	$0.34 \pm 0.22 \pm 0.05$
CDF ($p\bar{p}$)	2000	$0.79^{+0.41}_{-0.44}$
Aleph (Z^0)	2000	$0.93^{+0.64+0.36}_{-0.88-0.24}$

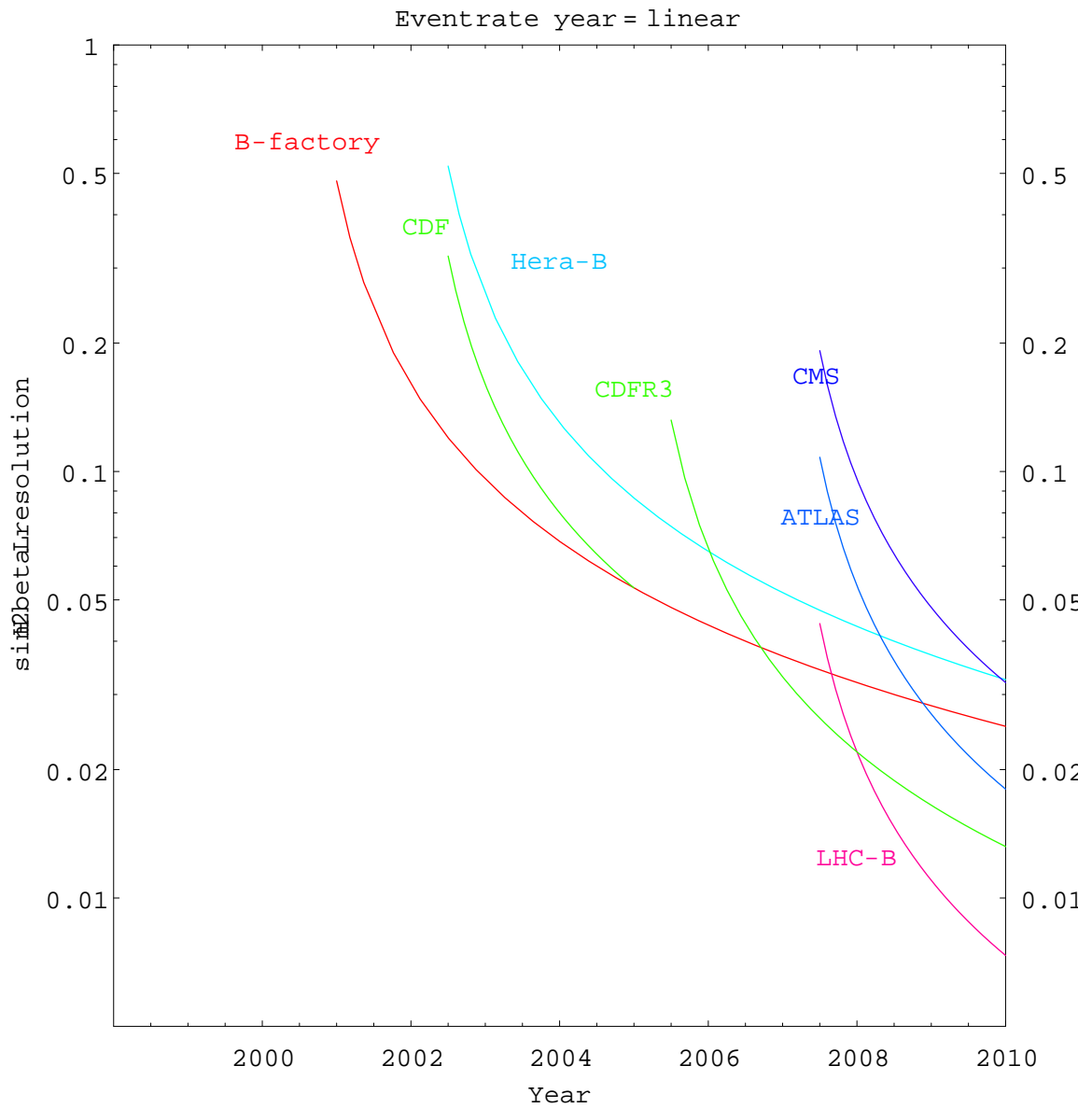
Prospects for Belle

- Reduce the beampipe size \rightarrow $\times 2$ improvement of vertex resolution (2002).
- Collect $\sim 30 \text{ fb}^{-1}$ per year for a few years.
- Installation of crab cavities, ante-chambers (2004?)
- Measure angles α/ϕ_2 and γ/ϕ_3 .
($\pi^+\pi^-$, DK , $D^*\pi$ modes etc.)
- Measure the sizes of the unitarity triangle better.

General prospects in B-physics

- CDF/D0 (Fermilab $p\bar{p}$) will join the game in 2002.
- HERA-B may join the game soon.
- LHC detectors (LHC-b and ATLAS, CMS) will start in 2006.
- BTeV (Fermilab) will start in 2008

EXtrapolation of $\sin 2\beta$ sensitivities (rough, irresponsible guesses)



Assumed linear increase of luminosity with time.