CP Violation in B Decays

Hitoshi Yamamoto
University of Hawaii
Kyoto, February 2001

1. CP Violation and CKM Matrix
2. CP Violation in Mixing
3. CP Violation by Mixing-Decay Interference
4. CP Violation in decay
**CPV and CKM Matrix**

General left-handed quark-W Interaction

\[ L_{\text{int}}(t) = \int d^3x \left( L_{qW}(x) + L_{qW}^\dagger(x) \right) \]

\[ L_{qW}(x) = \frac{g}{\sqrt{8}} \sum_{i,j=1,3} V_{ij} \bar{U}_i \gamma_\mu (1 - \gamma_5) D_j W^\mu \]

\[ U_i(x) \equiv \begin{pmatrix} u(x) \\ c(x) \\ t(x) \end{pmatrix}, \quad D_j(x) \equiv \begin{pmatrix} d(x) \\ s(x) \\ b(x) \end{pmatrix} \]

\[ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad \text{(CKM matrix)} \]

Experimentally, \( V \) has a hierarchical structure.

Approximately,

\[ |V_{ij}| \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \]

\[ \lambda \sim 0.22 \]
Transformation of $L_{\text{int}}$ under CP

exchanges particle ($n$) $\leftrightarrow$ antiparticle ($\bar{n}$)

$CP$: flips momentum sign ($\vec{p} \leftrightarrow -\vec{p}$)  \hspace{1cm} (a)

keeps the spin $z$-component ($\sigma$) the same

Such $CP$ operator in Hilbert space is not unique:

$$CP \alpha_{n,\vec{p},\sigma}^\dagger \mathcal{P}^\dagger \mathcal{C}^\dagger = \eta_n \alpha_{\bar{n},-\vec{p},\sigma}^\dagger$$

$\eta_n$: ‘CP phase’: arbitrary, depends on $n$
(for antiparticle: $\eta_{\bar{n}} = (-)^{2J} \eta_n^*$, $J =$ spin)

The choice of $\eta_n$ amounts to choosing a specific operator in Hilbert space among those satisfying (a).

Then, a pure algebra leads to

$$CP \bar{u}(x) \gamma_\mu (1 - \gamma_5) d(x) W^\mu(x) \mathcal{P}^\dagger \mathcal{C}^\dagger$$

$$= \eta_u \eta_d^* \eta_W^* \left( \bar{u}(x') \gamma_\mu (1 - \gamma_5) d(x') W_\mu(x') \right)^\dagger$$

$x' \equiv (t, -\vec{x})$
$\mathcal{L}_{qW}$ transforms as (taking $\eta_W = 1$)

$$
CP \mathcal{L}_{qW}(x) \mathcal{P}^\dagger \mathcal{C}^\dagger = \frac{g}{\sqrt{8}} \sum_{i,j=1,3} \eta_U \eta^*_D V_{ij} \left( \bar{U}_i(x') \gamma^\mu (1 - \gamma_5) D_j(x') W_\mu(x') \right)^\dagger
$$

IF $\eta_U \eta^*_D$ can be chosen s.t.

$$
\eta_U \eta^*_D V_{ij} = V_{ij}^* \quad (2),
$$

then, $L_{\text{int}}(t)$ becomes invariant under $CP$:

$$
CP \mathcal{L}_{qW}(x) \mathcal{P}^\dagger \mathcal{C}^\dagger = \mathcal{L}^\dagger_{qW}(x') \quad (x' = (t, -\bar{x}))
$$

$$
\rightarrow CP \mathcal{L}_{\text{int}}(t) \mathcal{P}^\dagger \mathcal{C}^\dagger
$$

$$
= \int d^3 x \ CP \left[ \mathcal{L}_{qW}(x) + \mathcal{L}^\dagger_{qW}(x) \right] \mathcal{P}^\dagger \mathcal{C}^\dagger
$$

$$
= \int d^3 x \left[ \mathcal{L}^\dagger_{qW}(x') + \mathcal{L}_{qW}(x') \right]
$$

$$
= L_{\text{int}}(t)
$$

$\rightarrow S$ operator is invariant under CP
(through Dyson series)
Condition for CP Invariance

Rewrite the condition (2):

\[
\frac{\eta_{Dj}}{\eta_{Ui}} = 2\arg V_{i,j}
\]

Thus, for a given matrix \( V_{i,j} \), if the CP phases \( \eta \)'s can be chosen so that the phase difference between \( \eta_{Dj} \) and \( \eta_{Ui} \) is twice the arbitrary phase of \( V_{i,j} \), then the physics is invariant under CP.

This is equivalent to rotate the quark phases to make \( V_{i,j} \) all real.

In general, there are 5 phase differences for 6 quarks \( \rightarrow 5 \) elements of \( V \) can be set to real always.

For example,

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}, \quad V_{i,j} : \text{real}
\]

\[
V_{i,j} : \text{complex}
\]

(No unitarity condition imposed)

Any of the four red elements is not real \( \rightarrow \) CP violation
A Main Question of the CPV Study in B: ‘Is $V$ unitary?’

e.g: orthogonality of $d$-column and $b$-column:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$\alpha \equiv \arg \left( \frac{V_{td}V_{tb}^*}{-V_{ud}V_{ub}^*} \right), \quad \beta \equiv \arg \left( \frac{V_{cd}V_{cb}^*}{-V_{td}V_{tb}^*} \right), \quad \gamma \equiv \arg \left( \frac{V_{ud}V_{ub}^*}{-V_{cd}V_{cb}^*} \right)$$

(Another notation: $\alpha \equiv \phi_2$, $\beta \equiv \phi_1$, $\gamma \equiv \phi_3$ )
Fit of the CKM unitarity triangle

Experimental inputs:
1. $|V_{ub}/V_{cb}|$
2. $B_d$ mixing $(\delta m_d) \rightarrow |V_{td}|$
3. $\epsilon_K$
4. $B_s$ mixing $\rightarrow \delta m_s/\delta m_s \rightarrow |V_{ts}/V_{td}|$, $|V_{ts}|$ known from unitarity of CKM $\rightarrow |V_{td}|$

Many people have performed a fit. One recent example: Ciuchini et.al.:

Normalized to the bottom length of the triangle. (two bands for each are 68% and 95% c.l.)

Three bands cross at one point $\rightarrow$ already a triumph of the standard model.
For any complex numbers $a, b, c$, trivially
\[ \alpha + \beta + \gamma = \pi \pmod{2\pi} \]
\[ \alpha \equiv \arg \left( \frac{a}{-b} \right), \quad \beta \equiv \arg \left( \frac{b}{-c} \right), \quad \gamma \equiv \arg \left( \frac{c}{-a} \right). \]

→ The condition $\alpha + \beta + \gamma = \pi \pmod{2\pi}$ holds even if the triangle does not close. It does not test the unitarity of $V_{\text{CKM}}$.

It simply tests if the angles measured are as defined in (3) in terms of $V_{\text{CKM}}$.

→ It is critical to measure the length of the sides.
3 types of CPV in $B$ decays

1. **CPV in mixing.** (neutral $B$)
   Particle-antiparticle imbalance in physical neutral $B$ states ($B_{a,b}$):
   \[
   |\langle B^0 | B_{a,b} \rangle|^2 \neq |\langle \bar{B}^0 | B_{a,b} \rangle|^2
   \]

2. **CPV by mixing-decay interference.** (neutral $B$)
   When both $B^0$ & $\bar{B}^0$ can decay to the same final state $f$:
   \[
   \bar{B}^0 \rightarrow f \\
   B^0 \rightarrow \bar{f}
   \]
   the interference results in
   \[
   \Gamma_{B^0 \rightarrow f}(t) \neq \Gamma_{\bar{B}^0 \rightarrow \bar{f}}(t).
   \]
   ($\Gamma_{B^0 \rightarrow f}(t)$: pure $B^0$ at $t = 0$, decaying to $f$ at $t$.)

3. **CPV in decay.** (neutral and charged $B$)
   Partial decay rate asymmetries.
   \[
   |Amp(B \rightarrow f)| \neq |Amp(\bar{B} \rightarrow \bar{f})|
   \]
   ($Amp(B^0 \rightarrow f)$: instantaneous decay amplitude.)
**CPV in mixing**

Eigenstates of mass & decay rate (assume CPT):

\[
\begin{align*}
B_a &= pB^0 + q\overline{B}^0 \\
B_b &= pB^0 - q\overline{B}^0,
\end{align*}
\]

- $B_a$ (mass: $m_a$, decay rate: $\gamma_a$)
- $B_b$ (mass: $m_b$, decay rate: $\gamma_b$)

→ Particle-antiparticle asymmetry in $B_{a,b}$:

\[
\delta \equiv \frac{|\langle B^0|B_{a,b}\rangle|^2 - |\langle \overline{B}^0|B_{a,b}\rangle|^2}{|\langle B^0|B_{a,b}\rangle|^2 + |\langle \overline{B}^0|B_{a,b}\rangle|^2} = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2}
\]

*CPT* → $B_a$ and $B_b$ have the same $\delta$ (incl. sign)
Use $B^0 \to \ell^+$, $\bar{B}^0 \to \ell^−$ to distinguish $B^0$ and $\bar{B}^0$.

For the neutral $K$ system

$$
\delta_K \equiv \frac{Br(K_L \to \pi^−\ell^+\nu) - Br(K_L \to \pi^+\ell^−\nu)}{Br(K_L \to \pi^−\ell^+\nu) + Br(K_L \to \pi^+\ell^−\nu)} = (3.27 \pm 0.12) \times 10^{-3}
$$

$\gamma_a \sim \gamma_b \to B_a$ and $B_b$ cannot be separated easily. Measure same-sign di-lepton asymmetry in 
$\Upsilon 4S \to B^0 \bar{B}^0$ (Okun,Zakharov,Pontecorvo,1975):

$$
A_{\ell\ell} \equiv \frac{N(\ell^+\ell^+) - N(\ell^−\ell^−)}{N(\ell^+\ell^+) + N(\ell^−\ell^−)} = 2\delta
$$

CLEO 1993 (by $A_{\ell\ell}$ on $\Upsilon 4S$)

$$
\delta = 0.015 \pm 0.048 \pm 0.016
$$

OPAL 1997 (by fitting the time dependence of tagged semileptonic decays of $B$'s on $Z^0$)

$$
\delta = -0.004 \pm 0.014 \pm 0.006
$$
Standard Model prediction for $\delta(= A_{\ell\ell}/2)$

The dominant diagram for mixing:

\[
\begin{array}{cccccccc}
& b & V_{tb} & t & V_{td}^* & d \\
\bar{B}^0 & W & V_{ud}^* & \bar{t} & V_{tb} & \bar{b} \\
\bar{d} & W & V_{td}^* & t & V_{tb} & \bar{b} \\
\end{array}
\]

\[
\rightarrow \begin{cases} 
    p = \frac{1}{\sqrt{2}} e^{i\phi} \\
    q = \frac{1}{\sqrt{2}} e^{-i\phi} 
\end{cases} \quad \phi = \arg(V_{tb}V_{td}^*)
\]

This does not result in $|p| \neq |q|$ (or $A_{\ell\ell} \neq 0$).

The interference of the above diagram with the same one with $t$ replaced by $c$ gives

\[
A_{\ell\ell} \sim -4\pi \frac{m_c^2}{m_t^2} \Im \left( \frac{V_{cb}V_{cd}^*}{V_{tb}V_{td}^*} \right) \sim 10^{-3}
\]
Long-distance effects may dominate (hadronic intermediate states) (Altomari, Wolfenstein, Bjorken, 1988):

\[ B^0 \leftrightarrow \begin{pmatrix} D^0 \bar{D}^0 \\ D^+ D^- \end{pmatrix} \leftrightarrow \bar{B}^0 \]

\[ |A_{\ell\ell}| = 10^{-3} \sim 10^{-2}. \]

Large theoretical uncertainty.

\[ \rightarrow \text{Cannot determine CKM phases from } A_{\ell\ell}. \]

\[ \delta (\equiv A_{\ell\ell}/2) \text{ of } 10^{-2} \text{ or larger signals } \textbf{new physics}. \]

(Also, \( \delta = 0 \) assumed in most calculations. \( \rightarrow \text{engineering value.} \))
Progress expected in the near future

There is also \( CP \) asymmetry in single lepton yield, (assuming leptons from \( B^\pm \) cannot be separated)

\[
A_\ell \equiv \frac{N_{\tau(4S)\rightarrow\ell^+} - N_{\tau(4S)\rightarrow\ell^-}}{N_{\tau(4S)\rightarrow\ell^+} + N_{\tau(4S)\rightarrow\ell^-}} = \chi \delta
\]

\[
\chi \equiv \text{Br}(B^0 \text{ decays as } \bar{B}^0) \sim 0.17
\]

Time measurement increases sensitivity.

\( B \)-factories: \( N(B^0, \bar{B}^0) \sim 4 \times 10^7 \) already

\[
\sigma_\delta(\ell + \ell\ell) \sim 0.1\% \text{ (B-factories now)}
\]

Quite possible that leptonic \( CP \) asymmetry will be observed in near future.
**CPV by Mixing-Decay Interference**

$\Gamma_{B(\bar{B}) \rightarrow f}(t)$: the probability that a pure $B^0(\bar{B}^0)$ at $t = 0$ decays to a final state $f$ at $t$ is

$$
\Gamma_{B(\bar{B}) \rightarrow f}(t) = |pA|^2 e^{-\gamma t} \left[ 1 \pm \Im\left( \frac{qA}{pA} \right) \sin \delta m t \right]
$$

(for $|qA/pA| = 1$, or $f$: CP eigenstate):

$$
\begin{align*}
B_a &= pB^0 + q\bar{B}^0, \\
B_b &= pB^0 - q\bar{B}^0,
\end{align*}
$$

$$
\begin{align*}
A &\equiv Amp(B^0 \rightarrow f), \\
\bar{A} &\equiv Amp(\bar{B}^0 \rightarrow f),
\end{align*}
$$

$$
\begin{align*}
\gamma_a &= \gamma_b \equiv \gamma, \\
\delta m &\equiv m_a - m_b
\end{align*}
$$

Time-integrated asymmetry:

$$
A_f \equiv \frac{\Gamma_{B \rightarrow f} - \Gamma_{\bar{B} \rightarrow f}}{\Gamma_{B \rightarrow f} + \Gamma_{\bar{B} \rightarrow f}} = \frac{x}{1 + x^2} \Im\left( \frac{qA}{pA} \right)
$$

$$
x \equiv \frac{\delta m}{\gamma} \sim 0.71 \pm 0.06 \quad \rightarrow \quad \frac{x}{1 + x^2} \sim \frac{1}{2}
$$
On $\Upsilon 4S \to B^0 \bar{B}^0$

Tag 'the other side' by a lepton:

$$\ell^\pm X(t_{\text{tag}}) \leftarrow (B^0 \bar{B}^0) \to f(t_{\text{sig}})$$

$B^0 \bar{B}^0$ created in a coherent $L = 1$ state.
Quantum correlation:

$\ell^+$ tag at $t$ $\rightarrow$ Signal side is $\bar{B}^0$ at $t$

$\ell^-$ tag at $t$ $\rightarrow$ Signal side is $B^0$ at $t$

The decay time distribution is nearly identical to the single $B$ case with

$$t \rightarrow t_- \equiv t_{\text{sig}} - t_{\text{tag}}$$

(in fact, exactly identical for $t_- > 0$)

$$\Gamma_{\Upsilon 4S \to \ell^+ f}(t_-) \propto e^{-\gamma |t_-|} \left[ 1 \pm \Im \left( \frac{qA}{pA} \right) \sin \delta m t_- \right]$$

($f$: CP eigenstate):
Gold-plated mode $B \to \psi K_S$

What phases of $V_{CKM}$ do we measure?

Recall \[ \begin{align*}
  p & = \frac{1}{\sqrt{2}} e^{i\phi} \\
  q & = \frac{1}{\sqrt{2}} e^{-i\phi}, \quad \phi = \arg(V_{tb}V_{td}^*)
\end{align*} \]

Actually, we need to include the CP phase of $B^0$:

\[ \frac{q}{p} = -\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \eta_B, \quad (CP|B\rangle = \eta_B|\bar{B}\rangle) , \]

\[ \frac{\bar{A}}{A} = \frac{\langle Ks|\bar{K}\rangle \langle \psi \bar{K}|H|\bar{B}\rangle}{\langle Ks|K\rangle \langle \psi K|H|B\rangle} \]

\[ = \left[ V_{cd}^*V_{cs} \eta_K^* \right] \left[ (-)^{L_{\psi K}} \eta_\psi \eta_K \frac{V_{cb}V_{cs}^*}{V_{cd}V_{cs}^*} \eta_B^* \right] \]

\[ (CP|K\rangle = \eta_K|\bar{K}\rangle, \quad CP|\psi\rangle = \eta_\psi|\psi\rangle) , \]

\[ \eta_\psi = +1, L_{\psi K} = 1 \to \frac{qA}{pA} = \left( \frac{V_{cd}V_{cb}^*}{-V_{td}V_{tb}^*} \right)^* / \left( \frac{V_{cd}V_{cb}^*}{-V_{td}V_{tb}^*} \right) \]

\[ \Rightarrow \Im \left( \frac{qA}{pA} \right) = -\sin 2\beta \quad (\psi K_S) \]
\[ \Gamma_{4S \rightarrow \ell^+ f(t_-)} \quad f = \Psi K_S \]

\[ B^0 \equiv \ell^- \text{ tag}, \quad \bar{B}^0 \equiv \ell^+ \text{ tag}, \]

Total rate asymmetry = 0
\[ \rightarrow \text{need to measure } t_- \]
\[ (\Rightarrow \text{Asymmetric } B\text{-factory}) \]

[At CLEO, \( B^0\bar{B}^0 \) are nearly at rest]
Measurements of $\sin 2\phi_1 / \sin 2\beta$ at B-factories

ICHEP2000 (Osaka)
Summer, 2000

• Belle (KEK) 6.2 fb$^{-1}$
  $\sin 2\phi_1 = 0.45^{+0.44}_{-0.45} \text{(stat+sys)}$

• BaBar (SLAC) 9.0 fb$^{-1}$
  $\sin 2\beta = 0.12 \pm 0.37 \text{(stat)} \pm 0.09 \text{(sys)}$

BCP4 (Ise, Japan)
End of Feb, 2001

• Belle (KEK) 10.4 fb$^{-1}$
  $\sigma \sin 2\phi_1 \sim 0.34$

• BaBar (SLAC) 26 fb$^{-1}$
  $\sin 2\beta \sim 0.22$
$B \to \pi^+\pi^-$: measurement of $\alpha$

\[
\frac{q\bar{A}}{pA} = \left( -\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*} \frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}} \eta_B \right) \eta_B^*
\]

\[
= -\left( \frac{V_{tb}^*V_{td}}{-V_{ub}V_{ud}^*} \right) / \left( \frac{V_{tb}^*V_{td}}{-V_{ub}V_{ud}^*} \right)^*
\]

\[
= -e^{2i\alpha}
\]

Penguin contamination:

No penguin contribution to $I=2$.
Extract $I=2$ contribution by isospin analysis.
Requires $B \to \pi^+\pi^-, \pi^+\pi^0, \pi^0\pi^0$. 
$B \rightarrow D^{(*)+}\pi^{-}$: Mixing $\rightarrow$ non-$CP$


|Amplitude ratio| $r \sim \left| \frac{V_{ub}V_{cd}^{*}}{V_{cb}V_{ud}^{*}} \right| \sim 0.4\lambda^{2} \sim 0.02$

Strong phase difference $= \delta$
Assume $\gamma_a = \gamma_b$, $|p/q| = 1$,
(In unit of $|A(B^0 \to D^- \pi^+)A(B^0 \to \ell^+)|^2$)

(1) $\Gamma(D^+ \pi^-, \ell^-) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+} \left[ (1 + r^2) - (1 - r^2)c_{\delta m \ell -} - 2r \xi s_{\delta m \ell -} \right]$ 

(2) $\Gamma(D^- \pi^+, \ell^+) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+} \left[ (1 + r^2) - (1 - r^2)c_{\delta m \ell -} + 2r \xi' s_{\delta m \ell -} \right]$ 

(3) $\Gamma(D^+ \pi^-, \ell^+) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+} \left[ (1 + r^2) + (1 - r^2)c_{\delta m \ell -} + 2r \xi s_{\delta m \ell -} \right]$ 

(4) $\Gamma(D^- \pi^+, \ell^-) = \frac{e^{-\gamma_+|t_-|}}{4\gamma_+} \left[ (1 + r^2) + (1 - r^2)c_{\delta m \ell -} - 2r \xi' s_{\delta m \ell -} \right]$ 

$t_- \equiv t_{\text{sig}} - t_{\text{tag}}, \quad r \sim 0.02$

$\xi \equiv \sin(2\beta + \gamma + \delta), \quad \xi' \equiv \sin(2\beta + \gamma - \delta)$

Asymmetry in the suppressed modes (1) $\leftrightarrow$ (2)

Smaller asymmetry in the favored modes (3) $\leftrightarrow$ (4)

Asymmetry is essentially rate asymmetries:
(1), (2) have similar shapes
(3), (4) have similar shapes

Some gain in $\#\sigma$ by fitting $t_-$. 
Asymmetry in the suppressed ('mixed') modes: 
\( r = 0.02, \ x = \delta m/\gamma = 0.71 \)

\[ A_s \equiv \frac{(1) - (2)}{(1) + (2)} \sim -\frac{2r}{x} \xi \sim -0.057 \xi \]

Asymmetry in the favored ('unmixed') modes:

\[ A_f \equiv \frac{(3) - (4)}{(3) + (4)} \sim \frac{2rx}{2 + x^2} \xi \sim 0.011 \xi \]

The favored modes has 5 times stat, but 5 times less asym. \( \rightarrow \sqrt{5} \) times less in \( \#\sigma \).

Most of the info is in the suppressed modes.
Statistics needed for $D^{(*)}_\pi$

$\sigma_\xi = 0.1 \rightarrow \sigma_A = 0.0057 \rightarrow N_s = 30K$
(suppressed modes)

We need $6 \times 30K = 180K$ total tagged $D\pi$'s.

Belle preliminary:
$3.7 \text{ fb}^{-1} \rightarrow 282 \pm 25$ lepton-tagged $D^\ast\pi$'s
(partial reconstruction)

No-bkg equivalent: $\left( \frac{282}{25} \right)^2 \sim 127$

$300 \text{ fb}^{-1} \rightarrow 10K$ to be compared with $180K$ needed.

- Need to improve background.
- Need to improve tagging efficiency.
- Add various modes (exclusive and partial).
  (strong phases?)

$\sigma_{\sin(2\beta+\gamma)} \sim (4 \text{ to } 5) \times \sigma_{\sin 2\beta}$
\[ B \rightarrow D^{*+} \rho^- \]

Mixing → non-CP eigenstate + angular correlation


Similar to \( B \rightarrow D \pi \) (needs to be flavor-tagged):

(Measures \( 2\beta + \gamma \))

\begin{align*}
\text{(1)} & \quad B^0 \rightarrow \bar{B}^0 \rightarrow D^{*+} \rho_-^\lambda \\
\text{(2)} & \quad \bar{B}^0 \rightarrow B^0 \rightarrow D^{*-} \rho_+^\lambda \\
\text{(3)} & \quad \bar{B}^0 \rightarrow B^0 \rightarrow D^{*+} \rho_-^\lambda \\
\text{(4)} & \quad B^0 \rightarrow \bar{B}^0 \rightarrow D^{*-} \rho_+^\lambda
\end{align*}

Repeats for each helicity final state.

\[ \lambda = \{ +, -, 0 \} \quad \text{(helicity basis), or} \quad \{ ||, \perp, 0 \} \quad \text{(tranversity basis)} \]

\[ |\text{Amplitude ratio}| \quad r \sim 0.02 \]

→ asymmetry in each \( \lambda \sim 0.02 \)
Angular correlation in $B \rightarrow D^*\rho$

(helicity basis)

\[
\frac{1}{\Gamma} \frac{d^3\Gamma}{dc_{\theta_1}dc_{\theta_2}d\chi} =
\frac{9}{32\pi} \left\{ 4|H_0|^2 c_{\theta_1}^2 c_{\theta_2}^2 + (|H_+|^2 + |H_-|^2) s_{\theta_1}^2 s_{\theta_2}^2 
+ [\Re(H_+^* H_-) c_{2\chi} + \Im(H_+^* H_-) s_{2\chi}] 2 s_{\theta_1}^2 s_{\theta_2}^2
+ [\Re(H_+^* H_0 + H^* H_0) c_{\chi} + \Im(H_+^* H_0 - H^* H_0) s_{\chi}] s_{2\theta_1} s_{2\theta_2} \right\}
\]

($c_x \equiv \cos x$, $s_x \equiv \sin x$)
New ingredients in $D^*\rho$:

Interference between different polarization states 
$(\lambda = ||, 0, \perp)$

$$\Gamma(B^0 \rightarrow D^{*+} \rho^-) = e^{-\gamma t} \sum_{\lambda \leq \lambda'} \mathcal{A}_{\lambda\lambda'} + \mathcal{A}_{\lambda\lambda'} \mathcal{C}_{\delta mt} - \mathcal{A}_{\lambda\lambda'} \mathcal{S}_{\delta mt} \right) g_\lambda g_{\lambda'}$$

($g_\lambda$: real functions of angles)

The term with $\lambda = \lambda'$ corresponds to the CP violating terms we have seen in $D\pi$:

$$\rho_{\lambda\lambda} = \mathcal{I}_\mathcal{A} \left( \frac{q}{p} (A^*(B^0 \rightarrow D^{*+} \rho^-) A(B^0 \rightarrow D^{*+} \rho^-) \right)$$

The interference term of $\rho$ have similar size: $(\lambda \neq \lambda')$

$$\rho_{\lambda\lambda'} = \mathcal{I}_\mathcal{A} \left( \frac{q}{p} (A^*(B^0 \rightarrow D^{*+} \rho_{\lambda}^-) A(\bar{B}^0 \rightarrow D^{*+} \rho_{\lambda'}^-) + A^*(B^0 \rightarrow D^{*+} \rho_{\lambda'}^-) A(\bar{B}^0 \rightarrow D^{*+} \rho_{\lambda}^-) \right)$$

→ If similar stat as $D\pi$, similar sensitivity to $2\phi_1 + \phi_1$.
   But has more degrees of freedom to measure.
   (more powerful resolving ambiguities,
   but more sys. study needed)
Statistics for $D^*\rho$

CLEO: $3.1 \text{ fb}^{-1} \rightarrow 197 \pm 15$ signal events.

$300 \text{ fb}^{-1} \rightarrow 19K$ events. With the high-$p_t$ lepton tag efficiency of 12%, we have 2.3K tagged $D^*\rho$.

This is compared with 10K (bkg-free equivalent for $300 \text{ fb}^{-1}$) of $D^*\pi$ partial reconstruction analysis. Or compared with 180K needed for $\sigma\xi = 0.1$.

$\rightarrow$ Number of events is $\sim \frac{1}{4}$ of $D^*\pi$, but more parameters to measure.

Comments:

- Partial reconstruction cannot be used. This may not be too big a problem since partial reconstruction efficiency is not that good.
- Need to tackle with the systematics of non-resonant component of $\rho$.
- Also check the sys. of $\rho$ mass dependence of amplitudes.
CPV in Decay

\[ B^- \rightarrow D^0_{CP} K^- \]

\[ D^0_{CP} : CP \text{ eigenstate. e.g. } K_S \pi^0, K^+ K^- \ldots \]

Both \( D^0 \) and \( \bar{D}^0 \) decay to a \( CP \) eigenstate.  
\[ \rightarrow 2 \text{ diagrams} \]

\[ a \equiv Amp(B^- \rightarrow D^0 K^-) \]
\[ \lambda_c \equiv V_{cb}V_{us}^* \]
Color-favored
\[ (a_1 + a_2 \sim 1.24) \]

\[ \bar{a} \equiv Amp(B^+ \rightarrow \bar{D}^0 K^+) \]
\[ \bar{b} \equiv Amp(B^+ \rightarrow D^0 K^+) \]

\[ \bar{a} = a^* \quad \bar{b} = b^* \]

\[ (\lambda_c : \lambda_u \sim 1 : 0.4) \]
Strong final-state-interaction phase: $b$ relative to $a$: $e^{i\delta}$ ($\delta$ could be complex)

Phase convention: $a = a^*$

$$D_{1,2} = \frac{1}{\sqrt{2}} (D^0 \pm \bar{D}^0) \quad (CP \pm),$$

$$A(B^- \to D_1K^-) = \frac{1}{\sqrt{2}} (a + b e^{i\delta})$$

$$A(B^+ \to D_1K^+) = \frac{1}{\sqrt{2}} (a^* + b^* e^{i\delta})$$

$$\Gamma(B^- \to D_1K^-) \neq \Gamma(B^+ \to D_1K^+): \text{direct CPV}$$
\( CP \) asymmetry expected:

\[
\begin{align*}
a_{cp} & \equiv \frac{\Gamma[B^+ \to D^0_{CP}K^-] - \Gamma[B^- \to D^0_{CP}K^+]}{\Gamma[B^- \to D^0_{CP}K^-] + \Gamma[B^+ \to D^0_{CP}K^+]} \\
\frac{|b|}{|a|} & \sim \left( \text{color factor} \right) \left( \text{CKM factor} \right) \sim 0.08 \\
\frac{a_2}{a_1 + a_2} & \sim 0.2 \quad \frac{\lambda_u}{\lambda_c} \sim 0.4
\end{align*}
\]

\( \rightarrow a_{cp} \) is of order 10%.

Relevant \( D^0 \) decay modes:

| \( CP \) eigenstates | \( K_S \pi^0 \) & 1.06 ± 0.11\% | \( CP^- \) | \( K_S \rho^0 \) & 0.60 ± 0.09\% | \( CP^- \) | \( K_S \phi \) & 0.84 ± 0.10\% | \( CP^- \) | \( K^+ K^- \) & 0.43 ± 0.03\% | \( CP^+ \) | \( \pi^+ \pi^- \) & 0.15 ± 0.01\% | \( CP^+ \) |
|---------------------|------------------|-----------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| calibration         | \( K^- \pi^+ \)   & 3.83 ± 0.12\%    |                  |

\( D^0 \) decay FSI phase does not contribute.

\( \rightarrow \) can be combined.
Classification of $\bar{B}^0 \to D K$

$\lambda_c$ \hspace{2cm} $\lambda_u$

\begin{align*}
\lambda_c &= V_{cb} V_{cs}^* , \quad \lambda_u = V_{ub} V_{us}^* .
\end{align*}

\begin{align*}
Amp(\bar{B}^0 \to D^+ K^-) &= \lambda_c T_c \\
Amp(\bar{B}^0 \to D^0 \bar{K}^0) &= \lambda_c C_c \\
Amp(\bar{B}^0 \to \bar{D}^0 \bar{K}^0) &= \lambda_u C_u \\
Amp(\bar{B}^0 \to D_s^- \pi^+) &= \lambda_u T_u \quad (4)
\end{align*}

$T$: tree, $C$: color-suppressed

($T, C$: depends on $b \to c$ or $b \to u$)
Classification of $B^- \rightarrow DK$

\[
Amp(B^- \rightarrow D^0 K^-) = \lambda_c T_c + \lambda_c C_c \quad (5a)
\]
\[
Amp(B^- \rightarrow \overline{D}^0 K^-) = \lambda_u C_u + \lambda_u A \quad (5b)
\]
\[
Amp(B^- \rightarrow D^- \overline{K}^0) = \lambda_u A \quad (5c)
\]
\[
Amp(B^- \rightarrow D^-_s \pi^0) = \frac{1}{\sqrt{2}} \lambda_u T_u \quad (5d)
\]
$B \rightarrow DK$ Modes

Final state: one charm, one strange.

- **No penguin contaminations**

  Penguin should have even number of charms. (True for charged and neutral $B$)

- **Neutral $B$ has no annihilations**

  Annihilations should have even number of stranges.

- **All tree diagrams (no complications by loops)**
Final-state Rescatterings

Final-state rescattering can occur:

\[ \bar{B}^0 \rightarrow D^+ K^- (T_c) \rightarrow D^0 \bar{K}^0 (C_c) \]
\[ \bar{B}^0 \rightarrow D_s^- \pi^+ (T_u) \rightarrow \bar{D}^0 \bar{K}^0 (C_u) \]

We define \( T_c, C_c, T_u, C_u \) by (4) including rescattering effects.

Then, is (5a) still true?

\[
\text{Amp}(B^- \rightarrow D^0 K^-) = \lambda_c T_c + \lambda_c C_c \\
= \text{Amp}(\bar{B}^0 \rightarrow D^+ K^-) + \text{Amp}(\bar{B}^0 \rightarrow D^0 \bar{K}^0)
\]

which is nothing but the isospin relation for \( H_{\text{eff}} \) having \(|1/2, -1/2\rangle\) structure:

(good to all orders as long as \( m_u = m_d \))
**Final-state Rescatterings - annihilation**

Final-state $D^- \bar{K}^0$ can be reached by

$$B^- \rightarrow D_s^- \pi^0 \rightarrow D^- \bar{K}^0$$

This is a ‘long-distance’ annihilation:

We thus define $A$ by

$$Amp(B^- \rightarrow D^- \bar{K}^0) = \lambda_u A \quad (5c)$$

including the rescattering effect.

Then, the annihilation in $B^- \rightarrow \bar{D}^0 K^-$ (5b) has exactly the same rescattering contribution:
Gronau-London-Wyler (GLW) method

\[ a \equiv A(B^- \to D^0 K^-) = \lambda_c(T_c + C_c) \]
\[ b \equiv A(B^- \to \bar{D}^0 K^-) = \lambda_u(C_u + A) \]

Measure |a|, |b|, \( A(B^- \to D_1 K^-) \), and \( A(B^+ \to D_1 K^+) \).
Reconstruct the two triangles \( \rightarrow \gamma \).

**Problem:**

How to measure \( B = \text{Amp}(B^- \to \bar{D}^0 K^-) \)?

\( B^- \xrightarrow{b} \bar{D}^0 K^- \) but also \( B^- \xrightarrow{a} D^0 K^- \)
\( \leftrightarrow K^+\pi^- \) \( \leftrightarrow K^+\pi^- \) \((DCSD)\)

The ratio of the two amplitudes \( (r_{DCSD}) \):

\[ r_{DCSD} = \frac{A}{B} \frac{\text{Amp}(D^0 \to K^+\pi^-)}{\text{Amp}(D^0 \to K^-\pi^+)} \sim 1 \]
\[ \sim \frac{1}{0.08} 0.088 \pm 0.020 \] \((CLEO \ 94)\)

Phase of \( r_{DCSD} \) not known \( \rightarrow \) difficult to measure \( |b| \).
(Difficult to detect \( D^0 \to X_s^-\ell^+\nu \))
The interference of DCSD and B-amplitude causes CP asymmetry of order unity in the wrong-sign $K\pi$ modes:

**ADS method to extract $\phi_3/\gamma$**

Measure $B^- \rightarrow DK^-$ in two decay modes of $D$: wrong-sign flavor-specific modes or $CP$ eigenstates, say $K^+\pi^-$ and $K_S\pi^0$ (and their conjugate modes).

\[
\Gamma[B^- \rightarrow (K^+\pi^-)K^-] \quad \Gamma[B^+ \rightarrow (K^-\pi^+)K^+] \\
\Gamma[B^- \rightarrow (K_S\pi^0)K^-] \quad \Gamma[B^+ \rightarrow (K_S\pi^0)K^+]
\]

Assume we know $|A|$ and $D$ branching fractions $\rightarrow$ 4 unknowns:

\[\phi_3, \quad \delta_{K^-\pi^+}, \quad \delta_{K_S\pi^0}, \quad \frac{|B|}{|A|}\]

$\rightarrow$ can be solved.

**Statistics:** Possible at B-factories
(300 fb$^{-1}$ needed for $\sigma_{\phi_3} \sim 0.3$ rad.)
Avoid using wrong-sign $B^+ \to D^0 K^+$

External input (experiment, theory):

\[ r = \left| \frac{B}{A} \right| = \left| \frac{\bar{B}}{\bar{A}} \right| \sim 0.08 \]

Measure

\[
\begin{align*}
\Gamma(B^- \to D_1 K^-) &= 1 + r^2 + 2r \cos(\phi_3 + \delta) \\
\Gamma(B^- \to D_2 K^-) &= 1 + r^2 - 2r \cos(\phi_3 + \delta) \\
\Gamma(B^+ \to D_1 K^+) &= 1 + r^2 + 2r \cos(\phi_3 - \delta) \\
\Gamma(B^+ \to D_2 K^+) &= 1 + r^2 - 2r \cos(\phi_3 - \delta)
\end{align*}
\]

in unit of $\Gamma(B^- \to D^0 K^-)$.

\[ \rightarrow \text{fit for } \phi_3 \text{ and } \delta. \]

Ambiguity: the equations are symmetric under

\[
\begin{align*}
\phi_3 &\rightarrow n\pi + \delta \quad \text{or} \quad \phi_3 &\rightarrow n\pi - \delta \\
\delta &\rightarrow -n\pi + \gamma \quad \text{or} \quad \delta &\rightarrow n\pi - \phi_3
\end{align*}
\]

\( n : \text{integer} \)
Fit result for $\phi_3$ and $\delta$

Input:

$\phi_3 = 1.8, \delta = 0.4$

$\sigma(\Gamma's) = 10\%$ (100 events each)

(300fb$^{-1}$)
Using \( B \rightarrow K\pi, \pi\pi \)

Tree-penguin interference
→ large direct \( CP \) asymmetries expected.

For example: \( B^- \rightarrow K^-\pi^0 \)

Interference → asymmetry \( B^- \rightarrow K^-\pi^0 \) vs \( B^+ \rightarrow K^+\pi^0 \)
(infromation on \( \arg V_{ub} = -\phi_3/\gamma \).)

Need to remove unknown strong FSI phase.
One historical method:
• Charged $B$ modes $\rightarrow$ self-tagging.

• SU(3) breaking effect are reasonably under control. Complication by EW penguins which breaks the isospin.

• Requires substantial development in theory.
  $\rightarrow$ QCD factorization formalism:

Probably the way to approach is to take theorist's predictions of branching ratios (ratios of branching ratios) for various modes and perform a global fit.
**Summary**

- Test of SM involves sizes as well as phases of CKM elements.
  → Enough efforts needed for measurements of $|V_{ij}|$’s.

- Lepton asymmetry ($CPV$ in mixing) sensitivity is already $\sigma_\delta \sim 0.1$. It is quite possible that non-zero $\delta$ is measured soon.

- $\beta/\phi_1$: in good shape both theoretically and experimentally.
  $\sigma_{\sin 2\beta} \sim 0.1$ with 150 fb$^{-1}$ (in a few years).

- $\alpha/\phi_2$: $\pi^+\pi^-$ mode - $\sigma_{\sin 2\alpha} \sim 3\sigma_{\sin 2\beta}$ (stat only)

- $\gamma/\phi_3$: $DK$, $D^*\pi$, $D^*\rho$ have similar sensitivities.
  $\sigma_{\gamma/\phi_3} \sim 20^\circ$ at 300 fb$^{-1}$ each.
  $K\pi$, $\pi\pi$ have more statistical power, but requires substantial theoretical development.