

dE/dx PARTICLE IDENTIFICATION FOR COLLIDER DETECTORS

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We review some basic features of dE/dx particle identification that are relevant to high energy physics tracking devices. Gas-based drift chambers as well as silicon trackers are discussed.

1 Model of ionization

One of the most successful models of ionization is the photo-absorption ionization model¹. The model assumes that the process of interest is dominated by single-photon exchange which allows one to relate the ionization process to the photo-absorption process for which the data is more readily available. Also the model is restricted to non-relativistic recoil electron, thus it does not include high-energy knock-on electrons (δ rays). In this model, the single collision cross section can be written as

$$\frac{d\sigma}{dT} = \frac{\alpha}{\beta^2\pi} \left[\frac{\sigma_\gamma}{TZ} \ln \frac{2m_e\beta^2/T}{\sqrt{(1-\beta^2\epsilon_1)^2 + \beta^4\epsilon_2^2}} + \underbrace{\frac{\Theta}{n_e} \left(\beta^2 - \frac{\epsilon_1}{|\epsilon|^2} \right)}_{\text{Cerenkov}} + \underbrace{\frac{1}{T^2} \int_0^T dT' \frac{\sigma_\gamma}{Z}}_{\text{Rutherford}} \right], \quad (1)$$

where the first term results in the logarithmic rise of ionization, the second term is the Cerenkov term, and the last term is the Rutherford scattering off a quasi-free electron. The parameters are:

T	energy loss per collision	$\sigma_\gamma(T)$	photo abs. cross section
m_e	electron mass	n_e	electron density
α	1/137	β	velocity of projectile
Z	atomic number of material	$\epsilon_1 + i\epsilon_2$	$\equiv \epsilon$: dielectric constant
Θ	$= \arg(1 - \epsilon^*\beta^2)$		

The logarithmic rise can be seen in the first term by taking the limit of $\beta \rightarrow 1$ and low density $\epsilon \equiv \epsilon_1 + i\epsilon_2 \rightarrow 1$:

$$\ln \frac{1}{\sqrt{(1 - \beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2}} \rightarrow \ln \frac{1}{1 - \beta^2} = \ln \gamma^2. \quad (2)$$

On the other hand, at large photon energy T (far above poles), the dielectric constant of material is given by the plasma energy $\hbar\omega_p$:

$$\epsilon \sim 1 - \left(\frac{\hbar\omega_p}{T} \right)^2, \quad (3)$$

where

$$\hbar\omega_p (\text{eV}) = 28.8 \sqrt{\frac{Z}{A} \rho (\text{g/cm}^3)}. \quad (4)$$

For a molecule, Z and A refer to those of the whole molecule; the plasma energy is proportional to the square root of the electron number density. Then, for $\beta \rightarrow 1$, we have a plateau at

$$\ln \frac{1}{\sqrt{(1 - \beta^2 \epsilon_1)^2 + \beta^4 \epsilon_2^2}} \rightarrow \ln \frac{T^2}{(\hbar\omega_p)^2} \quad (5)$$

which the logarithmic rise (2) would cross at

$$\gamma = \frac{T}{\hbar\omega_p}. \quad (6)$$

This is for a given energy of the exchanged photon. In practice, the saturation energy is given by replacing the recoil energy T by the effective excitation energy I :

$$\gamma_{\text{sat}} \sim \frac{I}{\hbar\omega_p}. \quad (7)$$

For elements, we have approximately²

$$I(\text{eV}) \sim 13.5Z \quad (Z \leq 14), \quad I(\text{eV}) \sim 10Z \quad (Z > 14), \quad (8)$$

which is good to 20% except for $I(\text{H}) = 19.2$ eV and $I(\text{He}) = 41.8$ eV. For compound molecules or gas mixtures, a good approximation is the Bragg's additivity

$$\log I = \frac{\sum_i n_i \log I_i}{\sum_i n_i}, \quad (9)$$

gas	I (eV)	$\hbar\omega_p$ (eV)	γ_{sat}	$p_{\text{sat}}^{\pi/K}$ (GeV/c)
He	41.8	0.27	154	21/76
Ar	188	0.82	230	32/115
Xe	482	1.41	341	48/170
CH ₄	41.7	0.61	68.4	10/34
C ₂ H ₆	45.4	0.82	55.3	8/28
C ₃ H ₈	47.1	0.96	49.1	7/24
C ₄ H ₁₀	48.3	1.14	42.4	6/21

Table 1:

where n_i is the total number of electrons per molecule of a given element i . For hydrocarbons C_kH_l , for example, $n_C = 6k$ and $n_H = l$, and gives around 45 ± 5 eV for wide range of values for k and l . For gas mixtures, we have $n_i = Z_i f_i$ where Z_i is the total number of electrons per i -th molecule and f_i is the fraction by volume. Even though the effective excitation energy in principle should depend on the type of chemical bond, the above formula was found to work for most cases.² The heavier the element, and the lower the density, the higher is the saturation point γ_{sat} . Table 1 gives the saturation point for some gases as well as the saturation momentum for π (P_{sat}^π) and K (P_{sat}^K). For a given type of gas, the $K\pi$ separation starts to degrade as the momentum goes higher than P_{sat}^π and becomes useless around P_{sat}^K .

2 Mean dE/dx vs $\beta\gamma$

When averaged over the recoil energy, (1) eventually leads to the (truncation-improved) Bethe-Bloch formula:³

$$\frac{dE}{dx} \propto \frac{1}{\beta^2} \left(\frac{1}{2} \log \frac{2m_e\beta^2\gamma^2 T_0}{I^2} - \frac{\beta^2}{2} \left(1 + \frac{T_0}{T_{\text{max}}} \right) - \frac{\delta}{2} \right) \quad (10)$$

where $T_0 = \min(T_{\text{cut}}, T_{\text{max}})$, T_{cut} is the effective cutoff for the knock-on electron energy, and T_{max} is its kinematic limit:

$$T_{\text{max}} = \frac{2(\gamma\beta)^2 m_e}{1 + x^2 + 2\gamma x}, \quad (11)$$

where $x = m_e/M$ with M being the mass of the projectile. We have $T_{\text{max}} \sim P$ for $\gamma x \gg 1$, or the recoil energy can be as large as the energy of projectile itself.

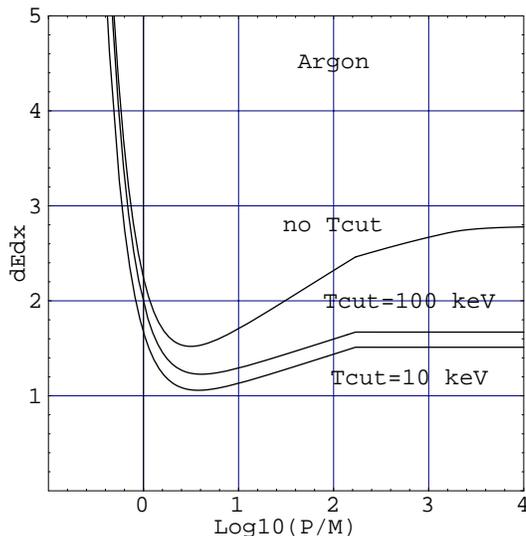


Figure 1: The ionization energy deposit dE/dx (MeV/g cm^{-2}) for argon plotted as a function of $\beta\gamma$ of projectile. Three curves corresponds to no T_{cut} , $T_{\text{cut}} = 100$ keV, and $T_{\text{cut}} = 10$ keV.

Such large recoil electrons should clearly not be included in the evaluation of dE/dx measured along the projectile since they will form separate tracks. The truncation T_{cut} in the expression above is designed to take care of such effect. Typical value for T_{cut} is 10 to 100 keV depending on the selection criteria used for drift chamber hits. A knock-on electron of 1 MeV will curl up with $r = 1.5$ mm in a 2 Tesla field, and will probably be rejected by tracking code. Also, usually a fixed fraction of high-side (or sometimes also low-side) tail is rejected in forming the dE/dx of a track (the truncated mean method) which also leads to an effective T_{cut} . Figure 1 plots (10) for argon with no T_{cut} , $T_{\text{cut}} = 100$ keV, and $T_{\text{cut}} = 10$ keV. The absolute value of the total relativistic rise is reduced by about factor of two upon applying T_{cut} , but is relatively insensitive to the values of T_{cut} in the range of 10-100 keV.

3 Resolution

Empirically, the resolution of gas-sampling device can be approximated by⁴

$$\frac{\sigma}{\mu} = 0.41n^{-0.43}(xP)^{-0.32}, \quad (12)$$

det.	n	$x(\text{cm})$	P	exp.	meas.
Belle	52	1.5	1 atm	6.6%	5.1% (μ)
CLEO2	51	1.4	1 atm	6.4%	5.7% (μ)
Aleph	344	0.36	1 atm	4.6%	4.5% (e)
TPC/PEP	180	0.5	8.5 atm	2.8%	2.5%
OPAL	159	0.5	4 atm	3.0%	3.1% (μ)
MKII/SLC	72	0.833	1 atm	6.9%	7.0% (e)

Table 2: Measured resolutions are compared to the expected dE/dx resolutions that are estimated by the empirical formula given in the text.

where n is the number of samples, x the sampling thickness in cm, and P the pressure in atm. This is obtained by a fit to actual resolutions measured, and includes optimizations of resolution using such techniques as the truncated mean method. Another method is to employ a maximum likelihood fit to all the pulse-heights along a track. Such technique is known to result in similar, if slightly better, resolutions. Table 3 shows a comparison of some resolutions measured by recent experiments and those ‘expected’ from the formula above. The formula works well except for CLEO2 and Belle for which the gas contains a large fraction (50%) of hydrocarbon (ethane).^a Hydrocarbons typically have better resolutions for a given sampling condition n, x and P since they have relatively small values of mean ionization energy with respect to the number of electrons per molecule resulting in a larger total number of ionization.

If each sampling is independent and there are no other sources of error such as electrical noises, then the resolution would scale as $n^{-0.5}$. Since the power on n is larger than that on x , for a fixed total length one obtains a better resolution for a finer sampling. At some point, however, the number of primary ionization which is approximately given by^b

$$n_p \sim 1.5Z/\text{cm} \quad (Z : \text{whole molecule}) \quad (13)$$

becomes of order unity and there will be no gain in resolution thereafter. Typically, the critical sampling size is about a few mm.

We also notice in Table 3 that higher pressure is effective in improving the resolution. As we have seen in (6) and (4), however, the saturation energy

^aA better fit can be obtained by replacing (xP) with $6.83n_e x(\text{cm})P(\text{atm})/I(\text{eV})$ where n_e is the number of electron per molecule. See Reference 1.

^bThis formula is usually good to 20%. Some exceptions are 5.2 for H₂, 5.9 for He, and 44 for Xe.

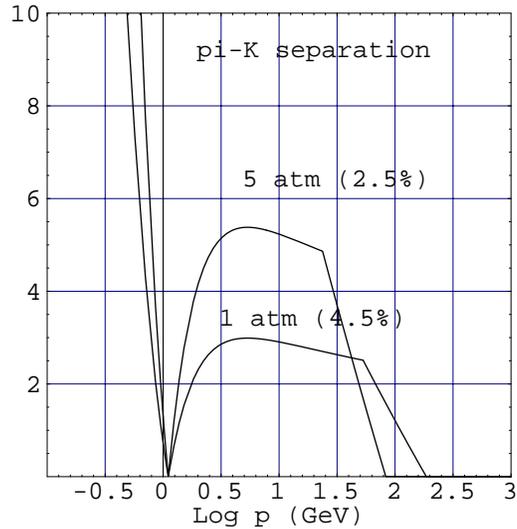


Figure 2: K/π separation (number of sigma) for argon as functions of $\log P(\text{GeV}/c)$. Two curves correspond to 1 atm and 5 atm.

is proportional to $1/\sqrt{\rho}$ or equivalently to $1/\sqrt{P}$ and thus dE/dx saturates at lower energies for higher pressures. Also, a high-pressure chamber adds problems to design and fabrication of the tracking device, and also adds more materials compared to non-pressurized chambers. To study particle type separation, we take two reference points for argon: 4.5% resolution at 1 atm and 2.5% at 5 atm. These are quite realistic values. The resulting K/π separation defined as $(dE/dx|_{\pi} - dE/dx|_K)/\sigma_{dE/dx}$ is shown in Figure 2 for the two cases. For 1 atm, 2σ separation can be achieved for $p < 0.8 \text{ GeV}/c$ and $1.75 < p < 65 \text{ GeV}/c$. On the other hand for 5 atm, 2σ separation can be achieved for $p < 0.9 \text{ GeV}/c$ and $1.25 < p < 50 \text{ GeV}/c$, and 4σ separation can be achieved for $p < 0.6 \text{ GeV}/c$ and $1.75 < p < 30 \text{ GeV}/c$.

4 Silicon trackers

Silicon has $\rho = 2.33 \text{ gr}/\text{cm}^3$ and $I = 173 \text{ eV}$. As a result, the saturation occurs at a low value of $\gamma_{\text{sat}} = 5.6$ which can be compared to the minimum dE/dx location of $\gamma \sim 4$. There is essentially no logarithmic rise which is a general feature of solid or liquid, and thus K/π separation in the logarithmic rise region is not practical. It can, however, be used below the minimum ionization region.

Achievable resolution is limited by the number of sampling even though the total ionization energy in 5 layers of 0.3 mm thick silicon is about the same as that in 1.5 m of argon gas. Since each sampling is ‘thick’, the fluctuation in the Landau tail degrades the resolution. A Babar study⁵ based on the Vavilov model of ionization showed that 10.4 % resolution can be obtained by 5 layers of silicon strip detectors (0.3 mm thick) for pions at 450 MeV/c. Also an ALICE study based on 2 layers of silicon strip detector and 2 layers of silicon drift detector resulted in 10.6 % resolution for kaons at 0.98 GeV/c. In both cases, the numbers correspond to the truncated mean method where two highest pulse heights were discarded. Such resolution can give 2σ separation for $p < 0.65$ GeV/c, and quite adequate for search of new heavy charged particles such as stau in the gauge-mediated SUSY breaking models. It should be noted, however, that in order to fully utilize the low energy region where dE/dx rapidly rise as $1/\beta^2$, the dynamic range of the readout system should cover up to around 20 times the minimum value of dE/dx .

5 Summary

The logarithmic rise of dE/dx and the saturation energy of the rise are well understood; in particular, the saturation point is higher for heavier element and pressurization results in a lower saturation. The measured dE/dx resolutions can be well approximated by an empirical formula which can be used as a guide to select the type of gas. The resolution is usually better for gases with higher content of hydrocarbons, but they typically have lower saturation energies. The larger is the number of sampling, the better is the resolution, but up to the sampling size of a few mm. For silicon trackers, the logarithmic rise is essentially absent, and thus only useful region is below the minimum ionizing point. Still, the achievable resolution is about 10% which is adequate for detection of new heavy particles such as stau in the gauge-mediated SUSY breaking models.

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