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Lepton CP Asymmetries in B Decays

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Abstract

In the decay of $\Upsilon(4S)$, the decay time distribution of $\Upsilon(4S) \to f + X$, where f is a final state that B^0 or $\overline{B}^0$ can decay to, is the sum of the decay time distributions of $B^0 \to f$ and $\overline{B}^0 \to f$. Using this general rule, we estimate the sensitivity of single lepton CP violation measurements with respect to that of traditional di-lepton measurements. We find that the sensitivity of the single lepton method is comparable to or better than that of the di-lepton method. The two data samples are largely statistically independent, so that they can be combined to improve sensitivity. The advantage of the single lepton measurement increases for large mixings, which suggests that the single lepton method holds promise for B_s. We also discuss lepton asymmetry measurements on the Z^0 peak and in hadron colliders. The achievable sensitivity with the currently available data is already in the range relevant to standard model predictions.

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1 Introduction

One of the ways CP violation can manifest itself is in the particle-antiparticle imbalance in mass eigenstates of neutral meson systems. In the case of the neutral kaon, it can be measured as the asymmetry between $K_L \to \pi^- \ell^+ \nu$ and $K_L \to \pi^+ \ell^- \nu$ [1, 2]:

$$\delta_K \equiv \frac{Br(K_L \to \pi^- \ell^+ \nu) - Br(K_L \to \pi^+ \ell^- \nu)}{Br(K_L \to \pi^- \ell^+ \nu) + Br(K_L \to \pi^+ \ell^- \nu)} = (3.27 \pm 0.12) \times 10^{-3} , \tag{1}$$

which indicates that K_L contains more K^0 than $\overline{K}^0$ (assuming $\Delta S = \Delta Q$). If CPT is conserved, K_S has the same asymmetry δ_K with the same sign; thus there is no need to specify which of the two mass eigenstates is being considered.

For the neutral B meson system, one can similarly define the asymmetry δ as

$$\delta \equiv \frac{|\langle B^0 | B_{a,b} \rangle|^2 - |\langle \overline{B}^0 | B_{a,b} \rangle|^2}{|\langle B^0 | B_{a,b} \rangle|^2 + |\langle \overline{B}^0 | B_{a,b} \rangle|^2} , \tag{2}$$

where B_a and B_b are the two mass eigenstates which, as in the case of the kaon, have the same asymmetry (assuming CPT). In practice, however, it is experimentally difficult to isolate B_a or B_b. The traditional method is to measure the same-sign di-lepton asymmetry in $\Upsilon(4S) \to B^0 \overline{B}^0$ [3, 4]:

$$A_{\ell\ell} \equiv \frac{N(\ell^+\ell^+) - N(\ell^-\ell^-)}{N(\ell^+\ell^+) + N(\ell^-\ell^-)} \sim 2\delta . \tag{3}$$

There is in principle no dilution due to $\Upsilon(4S) \to B^+ B^-$ since the same-sign di-lepton events are caused by mixing of the neutral B meson (assuming $\Delta B = \Delta Q$).

Within the framework of the standard model, the short distance calculation gives [5, 6, 7]

$$A_{\ell\ell} \sim -4\pi \frac{m_c^2}{m_t^2} \Im \left(\frac{V_{cb} V_{cd}^*}{V_{tb} V_{td}^*} \right) , \tag{4}$$

which is around 10^{-3}. Including long distance effects, Altomari *et al.* estimated $A_{\ell\ell}$ to be in the range 10^{-3} to 10^{-2} [8]. The uncertainty is primarily due to hadronic intermediate states, and even the sign cannot be reliably predicted. As a consequence, a measurement of CP violation in the semileptonic asymmetry does not lead to the determination of basic CP violating parameters in the standard model. Outside of the standard model, however, the asymmetry can be larger, and thus an experimental value of $|\delta|$ above $\sim 10^{-2}$ would signal new physics [9, 10]. The current experimental number is not very recent or precise: $A_{\ell\ell} = 0.031 \pm 0.096 \pm 0.032$ [11]. For B_s, the short distance prediction of $A_{\ell\ell}$ is obtained by replacing d by s in (4), and it is even smaller than for B^0.

The CP asymmetry in single lepton sample had been suggested as a possible observable to search for CP violation in the case when the mixing is small [7, 12]. The logic was that if the mixing is small, then the statistics of the di-lepton events will decrease, making the di-lepton method impractical. After the observation of substantial mixing in the neutral B meson system [13], however, the single lepton method has not received much attention. In this note, we point out that the advantage of the single lepton method over the di-lepton method actually increases for large mixings, and that, on $\Upsilon(4S)$, the single lepton method has a comparable or better sensitivity than the di-lepton method. This is so in spite of the fact that in the single lepton measurement, one usually cannot distinguish charged and neutral B mesons. We begin by briefly reviewing the phenomenological background, and then move on to estimate experimental sensitivities. In the appendix, we present a general rule that relates the inclusive decay time distribution on $\Upsilon(4S)$ to those of B^0 and $\overline{B}^0$ tagged at $t=0$, as well as decay rate formulas without assuming CPT invariance.

2 Phenomenology

The mass eigenstates can be written in terms of B^0 and $\overline{B}^0$ as

$$\begin{cases} B_a = pB^0 + q\overline{B}^0 & (\text{mass: } m_a,\ \text{decay rate: } \gamma_a) \\ B_b = p'B^0 - q'\overline{B}^0 & (\text{mass: } m_b,\ \text{decay rate: } \gamma_b) \end{cases}, \tag{5}$$

where the normalization is

$$|p|^2 + |q|^2 = 1\ , \qquad |p'|^2 + |q'|^2 = 1\ . \tag{6}$$

If CPT is a good symmetry, we have

$$p' = p\ , \quad q' = q \quad (CPT)\ . \tag{7}$$

In the following we assume CPT invariance. CPT symmetry also effectively allows us to take [14]

$$|Amp(B^0 \to \ell^+)| = |Amp(\overline{B}^0 \to \ell^-)| \equiv A_0 \quad (CPT)\ . \tag{8}$$

Furthermore, we will assume $\Delta B = \Delta Q$ [15]. The probability that a pure B^0 or $\overline{B}^0$ at $t=0$ decays to $\ell^\pm$ at time t is then given by (see Appendix)

$$\begin{aligned} \Gamma_{B^0\to\ell^+}(t) &= \Gamma_{\overline{B}^0\to\ell^-}(t) = \frac{A_0^2}{2} e^{-\gamma_+ t}\left[\cosh\gamma_- t + \cos\delta m t\right]\ , \\ \Gamma_{\overline{B}^0\to\ell^+}(t) &= \frac{|p|^2}{|q|^2}\frac{A_0^2}{2} e^{-\gamma_+ t}\left[\cosh\gamma_- t - \cos\delta m t\right]\ , \\ \Gamma_{B^0\to\ell^-}(t) &= \frac{|q|^2}{|p|^2}\frac{A_0^2}{2} e^{-\gamma_+ t}\left[\cosh\gamma_- t - \cos\delta m t\right]\ , \end{aligned} \tag{9}$$

where

$$\delta m \equiv m_a - m_b \ , \quad \gamma_\pm \equiv \frac{\gamma_a \pm \gamma_b}{2} \ . \tag{10}$$

The short distance calculation predicts $\gamma_a \sim \gamma_b$ [12]. There is, however, no stringent experimental limit, and we will allow for the possibility that the difference is sizable. Also, the following expressions are applicable to B_s mesons which is expected to have a sizable decay rate difference. The fraction of B^0 or $\overline{B}^0$ at $t = 0$ eventually decaying to $\ell^\pm$, which we denote as $Br(B^0(\overline{B}^0) \to \ell^\pm)$, is obtained by integrating the above expressions:

$$\begin{aligned} Br(B^0 \to \ell^+) &= Br(\overline{B}^0 \to \ell^-) = \frac{b_{sl}}{1-y^2}(1-\chi) \ , \\ Br(\overline{B}^0 \to \ell^+) &= \frac{b_{sl}}{1-y^2}\frac{|p|^2}{|q|^2}\chi \ , \\ Br(B^0 \to \ell^-) &= \frac{b_{sl}}{1-y^2}\frac{|q|^2}{|p|^2}\chi \ , \end{aligned} \tag{11}$$

where χ is the standard mixing parameter [16] defined by

$$\chi \equiv \frac{1}{2}\frac{x^2+y^2}{1+x^2} = 0.175 \pm 0.016 \ [1] \ , \tag{12}$$

$$x \equiv \frac{\delta m}{\gamma_+} \ , \quad y \equiv \frac{\gamma_-}{\gamma_+} \ , \tag{13}$$

and b_{sl} is the 'normalized' semileptonic branching fraction which reduces to the experimental semileptonic branching fraction in the limit of $\gamma_a = \gamma_b$ and CP symmetry:

$$b_{sl} \equiv \frac{A_0^2}{\gamma_+} \xrightarrow{\gamma_a=\gamma_b, CP} 2Br(B \to X\ell\nu) = 2 \times (0.1043 \pm 0.0024) \ [1] \ , \tag{14}$$

where the factor 2 comes from the fact that there are electron and muon modes. From the first line of (11), we see that there is no asymmetry in the 'right-sign' lepton branching fractions. The particle-antiparticle imbalance in $B_{a,b}$ (namely, $|p|^2 \neq |q|^2$) shows up only in the 'wrong-sign' decays:

$$\frac{Br(\overline{B}^0 \to \ell^+) - Br(B^0 \to \ell^-)}{Br(\overline{B}^0 \to \ell^+) + Br(B^0 \to \ell^-)} = \frac{|p|^4 - |q|^4}{|p|^4 + |q|^4} = \frac{2\delta}{1+\delta^2} \ , \tag{15}$$

with

$$\delta = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} \ , \tag{16}$$

which follows from definition (2). Experimentally, measurement of this asymmetry requires flavor tagging at $t=0$.

In the decay $\Upsilon(4S) \to B^0\overline{B}^0$, one needs to take into account the quantum correlations arising from the fact that the two mesons are in a coherent $L=1$ state. We consider the case when one side decays to ℓ^+ and the other side decays to ℓ^- (and all other charge combinations). The decay time variable accessible in an asymmetric B-factory is the time difference of decays:

$$t_- \equiv t_1 - t_2 \; . \tag{17}$$

In terms of t_-, the decay time distributions are (see Appendix)

$$\begin{aligned}
\Gamma_{\Upsilon(4S)\to\ell^+\ell^-}(t_-) &= \Gamma_{\Upsilon(4S)\to\ell^-\ell^+}(t_-) \\
&= \frac{A_0^4}{8\gamma_+} e^{-\gamma_+|t_-|}\left[\cosh\gamma_- t_- + \cos\delta m t_-\right] \; , \\
\Gamma_{\Upsilon(4S)\to\ell^+\ell^+}(t_-) &= \frac{|p|^2}{|q|^2}\frac{A_0^4}{8\gamma_+} e^{-\gamma_+|t_-|}\left[\cosh\gamma_- t_- - \cos\delta m t_-\right] \; , \\
\Gamma_{\Upsilon(4S)\to\ell^-\ell^-}(t_-) &= \frac{|q|^2}{|p|^2}\frac{A_0^4}{8\gamma_+} e^{-\gamma_+|t_-|}\left[\cosh\gamma_- t_- - \cos\delta m t_-\right] \; .
\end{aligned} \tag{18}$$

Note that the $\ell^+\ell^+$ and $\ell^-\ell^-$ distributions have exactly the same t_- dependence, and thus the asymmetry between them does not depend on t_-. We thus integrate (18) to obtain the fraction of $\Upsilon(4S) \to B^0\overline{B}^0$ decays that eventually result in a lepton pair $\ell^\pm\ell^\pm$, which we denote by $Br(\Upsilon(4S) \to \ell^\pm\ell^\pm)$:

$$\begin{aligned}
Br(\Upsilon(4S) \to \ell^+\ell^-) &= Br(\Upsilon(4S) \to \ell^-\ell^+) = \frac{1}{2}\frac{b_{sl}^2}{1-y^2}(1-\chi) \; , \\
Br(\Upsilon(4S) \to \ell^+\ell^+) &= \frac{1}{2}\frac{b_{sl}^2}{1-y^2}\frac{|p|^2}{|q|^2}\chi \; , \\
Br(\Upsilon(4S) \to \ell^-\ell^-) &= \frac{1}{2}\frac{b_{sl}^2}{1-y^2}\frac{|q|^2}{|p|^2}\chi \; .
\end{aligned} \tag{19}$$

The common factor $b_{sl}^2/1-y^2$ can be written as

$$\frac{b_{sl}^2}{1-y^2} = Br(B_a \to \ell^\pm)\, Br(B_b \to \ell^\pm) \; . \tag{20}$$

Then, in the absence of CP violation (namely, $|p|^2 = |q|^2$), the total yield of di-lepton events becomes $Br(B_a \to \ell^\pm)Br(B_b \to \ell^\pm)$, as expected from the simple picture $\Upsilon(4S) \to B_aB_b$. The asymmetry between $(\ell^+\ell^+)$ and $(\ell^-\ell^-)$ is the same as that of the 'wrong-sign' leptons discussed above:

$$A_{\ell\ell} \equiv \frac{Br(\Upsilon(4S) \to \ell^+\ell^+) - Br(\Upsilon(4S) \to \ell^-\ell^-)}{Br(\Upsilon(4S) \to \ell^+\ell^+) + Br(\Upsilon(4S) \to \ell^-\ell^-)} = \frac{2\delta}{1+\delta^2} \; . \tag{21}$$

No flavor tagging is required for this measurement.

In order to study the single lepton asymmetry on $\Upsilon(4S)$, one has to take into account the cases where one side decays semileptonically and the other side decays to anything. To do so, we use the following general rule (see Appendix):

$$\Gamma_{\Upsilon(4S)\to f}(t) = 2\sum_{f_1}\int_0^\infty \Gamma_{\Upsilon(4S)\to f_1 f}(t_1,t)\,dt_1 = \Gamma_{B^0\to f}(t) + \Gamma_{\overline{B}^0\to f}(t)\ , \tag{22}$$

where $\Gamma_{\Upsilon(4S)\to f}(t)$ is the probability density that one finds a given final state f decaying at time t in the process $\Upsilon(4S)\to B^0\overline{B}^0$, $\Gamma_{\Upsilon(4S)\to f_1 f_2}(t_1,t_2)$ is the probability density that one side of $\Upsilon(4S)\to B^0\overline{B}^0$ decays to final state f_1 at time t_1 and the other side decays to f_2 at time t_2, and $\Gamma_{B^0(\overline{B}^0)\to f}(t)$ is the probability density that a pure $B^0(\overline{B}^0)$ at $t=0$ decays to final state f at time t. The factor 2 in (22) accounts for the fact that the final state f can come from either side of the $\Upsilon(4S)$ decay. These functions are related to the branching fractions discussed above by

$$Br(\Upsilon(4S)\to f_1 f_2) = \int_0^\infty\int_0^\infty \Gamma_{\Upsilon(4S)\to f_1 f_2}(t_1,t_2)\,dt_1 dt_2\ , \tag{23}$$

$$Br(B^0(\overline{B}^0)\to f) = \int_0^\infty \Gamma_{B^0(\overline{B}^0)\to f}(t)\,dt\ . \tag{24}$$

The relation (22) is a consequence of quantum mechanics and conservation of probability, and is valid even when CPT is violated. The same relation holds for $e^+e^-\to V\to K^0\overline{K}^0$, $D^0\overline{D}^0$, and $B_s\overline{B}_s$, where V is a vector state or a virtual photon.

Let us define the inclusive quantity on $\Upsilon(4S)$ by

$$N(\Upsilon(4S)\to f) \equiv \int_0^\infty \Gamma_{\Upsilon(4S)\to f}(t)\,dt\ , \tag{25}$$

which is the total expected number of final state f in one $\Upsilon(4S)\to B^0\overline{B}^0$ decay. The normalizations are given by (see Appendix)

$$\sum_{f_1,f_2} Br(\Upsilon(4S)\to f_1 f_2) = 1\ , \tag{26}$$

$$\sum_f Br(B^0(\overline{B}^0)\to f) = 1\ , \tag{27}$$

$$\sum_f N(\Upsilon(4S)\to f) = 2\ . \tag{28}$$

The last normalization reflects the fact that there are two B mesons per $\Upsilon(4S)$ decay.

From (11),(22),(24), and (25), one then obtains the inclusive lepton yields on $\Upsilon(4S)$:

$$N(\Upsilon(4S) \to \ell^+) = \frac{b_{sl}}{1-y^2}\left[1 + \left(\frac{|p|^2}{|q|^2} - 1\right)\chi\right] ,$$
$$N(\Upsilon(4S) \to \ell^-) = \frac{b_{sl}}{1-y^2}\left[1 + \left(\frac{|q|^2}{|p|^2} - 1\right)\chi\right] . \tag{29}$$

One sees that when $|p|^2 \neq |q|^2$ there is an asymmetry. In practice, however, leptons from $B^\pm$ are difficult to reject, and the resulting dilution needs to be taken into account.

3 Experimental Sensitivities

We will now estimate the sensitivities to δ of single and di-lepton asymmetry measurements. We assume that the lepton detection efficiency ϵ_ℓ for each lepton is the same in the single and di-lepton cases, and that they are uncorrelated in the latter. Also we assume $\delta \ll 1$ for the expressions of asymmetries below. In estimating statistics, we further assume $\gamma_a \sim \gamma_b$ (or equivalently $y \ll 1$). If we have N_0 $\Upsilon(4S) \to B^0\overline{B}^0$ decays, then from (19) the total number of same sign di-lepton events detected is $N_0\, b_{sl}^2\, \chi\, \epsilon_\ell^2$. Using (21), the error in δ is then

$$\sigma_\delta(\ell\ell) = \frac{1}{2}\frac{1}{\sqrt{N_0\, b_{sl}^2\, \chi\, \epsilon_\ell^2}} . \tag{30}$$

The single lepton asymmetry on $\Upsilon(4S)$ can be obtained from (29):

$$A_\ell(\Upsilon(4S)) \equiv D\,\frac{N(\Upsilon(4S) \to \ell^+) - N(\Upsilon(4S) \to \ell^-)}{N(\Upsilon(4S) \to \ell^+) + N(\Upsilon(4S) \to \ell^-)} = 2D\,\chi\,\delta , \tag{31}$$

where D is the dilution factor due to charged B mesons, and is equal to the fraction of leptons coming from neutral B mesons. Other dilution effects such as those due to misidentified leptons or leptons from charmed hadrons could also be absorbed into D. Assuming that there are the same number of leptons from charged B's as from neutral B's, we take $D = 1/2$. The total number of single lepton events detected is $N_0\, 4b_{sl}\, \epsilon_\ell$; thus the sensitivity to δ of the single lepton measurement is

$$\sigma_\delta(\ell) = \frac{1}{\chi}\frac{1}{\sqrt{N_0\, 4b_{sl}\, \epsilon_\ell}} . \tag{32}$$

The ratio of sensitivities of single to di-lepton measurements is then

$$\frac{\sigma_\delta(\ell)}{\sigma_\delta(\ell\ell)} = \sqrt{\frac{b_{sl}\,\epsilon_\ell}{\chi}} . \tag{33}$$

We see that the larger the mixing, the more advantageous the single lepton method becomes. This may be counter-intuitive, but can be understood as follows: as χ increases, the statistics goes up linearly for the di-lepton sample while its asymmetry stays the same. For the single lepton sample, the statistics stays the same while the asymmetry goes up linearly, which is equivalent to statistics increasing quadratically for a fixed asymmetry.

A typical value for ϵ_ℓ is 0.5. Together with the experimental values for b_{sl} and χ, the ratio above is 0.78. Namely, the single lepton measurement has a sensitivity comparable to or better than that of the di-lepton measurement. Note also that the single and di-lepton datasets are largely statistically independent (only about 10% of the single lepton events are also in the di-lepton dataset). The two measurements can thus be combined to improve overall sensitivity. For example, the current CLEO data corresponds to $N_0 \sim 2 \times 10^6$. This gives $\sigma_\delta(\ell) = 0.6\%$ and $\sigma_\delta(\ell\ell) = 0.8\%$ with the combined sensitivity of 0.5% which is already in the range relevant to standard model predictions.

When B^0's and $\overline{B}^0$'s are generated incoherently (e.g. on Z^0 or in $p\bar{p}$ collisions), one cannot perform the correlated di-lepton analysis. However, one can still perform single lepton asymmetry measurements. The discussion below applies also to B_s mesons. When an equal number of B^0 and $\overline{B}^0$ are generated, the decay time distribution of ℓ^+ can be compared to that of ℓ^- without tagging the flavor of the parent B meson. Using (9), the time dependent single lepton asymmetry is then

$$A_\ell(t) \equiv D\, \frac{\Gamma_{B^0,\overline{B}^0 \to \ell^+}(t) - \Gamma_{B^0,\overline{B}^0 \to \ell^-}(t)}{\Gamma_{B^0,\overline{B}^0 \to \ell^+}(t) + \Gamma_{B^0,\overline{B}^0 \to \ell^-}(t)} = D\,\delta \left(1 - \frac{\cos \delta m t}{\cosh \gamma_- t} \right) , \qquad (34)$$

where D is the dilution factor due to $B^\pm$'s as before. In this case, D could in principle be a function of decay time (e.g., if the lifetimes of neutral and charged B mesons are different). Because of the relation (22), the time dependent asymmetry for the single lepton measurement on $\Upsilon(4S)$ is also given by (34). We see that the asymmetry starts out as zero at $t = 0$ and reaches the first maximum at around $\delta m t \sim \pi$ (about 4 times the b lifetime). If we simply count the number of leptons without measuring the decay time, then the asymmetry becomes the same as that on $\Upsilon(4S)$:

$$A_\ell \equiv D\, \frac{Br(B^0, \overline{B}^0 \to \ell^+) - Br(B^0, \overline{B}^0 \to \ell^-)}{Br(B^0, \overline{B}^0 \to \ell^+) + Br(B^0, \overline{B}^0 \to \ell^-)} = 2D\,\chi\,\delta \; . \qquad (35)$$

where we have used (11). The factor D again includes the dilution due to $B^\pm$'s. This observable (with $D = 1$) was first proposed by Hagelin [7] as a measure of observability of CP violation in $B^0 - \overline{B}^0$ mixing, since it contains both the mixing parameter χ and the CP violation parameter δ.

The currently available statistics on Z^0 is smaller than that of CLEO; the $p\bar{p}$ collider at Fermilab, however, may be able to obtain a better sensitivity. In actual data analysis, the decay time is often required to be larger than a given threshold in order to reject non-B background. Such a requirement, however, should not sacrifice sensitivity significantly since the asymmetry at short decay time is small as seen in (34). Also, there is a possibility that vertexing allows separation of neutral B's from charged B's by counting the total charge emerging from a given vertex, which would substantially improve sensitivity. In addition, flavor tagging by leptons, jet charge, or associated pion production [17] may allow for measurement of the flavor-tagged asymmetry (15). The vertexing technique may become useful also at asymmetric B-factories.

In conclusion, we have studied the sensitivity of single lepton CP asymmetry relative to that of the traditional di-lepton asymmetry on $\Upsilon(4S)$. We find that the single lepton sensitivity is comparable to or better than that of the di-lepton analysis. The achievable sensitivities on $\Upsilon(4S)$ and in $p\bar{p}$ collisions with currently available datasets are already close to the predictions of the standard model. The single lepton method also holds promise for measurement of the leptonic CP asymmetry of B_s. In the near future (namely, at the B-factories, HERA-B, and the upgraded $p\bar{p}$ collider), it is quite possible that CP violation will be observed in the semileptonic modes.

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Appendix

This section is based on quantum mechanics, conservation of probability, and the Weisskopf-Wigner formalism [18]. We will not assume CPT invariance unless otherwise stated. No further approximations are made. Solving (5) for B^0 and $\overline{B}^0$, we obtain

$$\begin{cases} B^0 = c\,(q'B_a + qB_b) \\ \overline{B}^0 = c\,(p'B_a - pB_b) \end{cases} , \tag{A.1}$$

with

$$c \equiv \frac{1}{p'q + pq'} . \tag{A.2}$$

The time evolution of the mass eigenstates are given by

$$B_a \to e^{-(\gamma_a/2+im_a)t} B_a \ , \qquad B_b \to e^{-(\gamma_b/2+im_b)t} B_b \ . \tag{A.3}$$

If we have a pure B^0 or $\overline{B}^0$ at $t=0$, the decay time distributions to a final state f are then

$$\begin{aligned} \Gamma_{B^0\to f}(t) &= |c|^2 \left[|q' a_f|^2 e^{-\gamma_a t} + |q b_f|^2 e^{-\gamma_b t} + 2\Re((q' a_f)^*(q b_f) e^{-(\gamma_+ - i\delta m)t}) \right] , \\ \Gamma_{\overline{B}^0\to f}(t) &= |c|^2 \left[|p' a_f|^2 e^{-\gamma_a t} + |p b_f|^2 e^{-\gamma_b t} - 2\Re((p' a_f)^*(p b_f) e^{-(\gamma_+ - i\delta m)t}) \right] , \end{aligned} \tag{A.4}$$

where

$$a_f \equiv Amp(B_a \to f) \ , \quad b_f \equiv Amp(B_b \to f) \ . \tag{A.5}$$

The parameters $\gamma_\pm$ and δm are defined in the main text. Note that we would have $|c|^2 = 1$ if CPT and CP were conserved. The normalization of the decay amplitudes is such that

$$\sum_f \frac{|a_f|^2}{\gamma_a} = 1 \ , \quad \sum_f \frac{|b_f|^2}{\gamma_b} = 1 \ . \tag{A.6}$$

Integrating (A.4) over time gives the fraction of a pure B^0 or $\overline{B}^0$ at $t=0$ eventually decaying to a final state f:

$$\begin{aligned} Br(B^0 \to f) &= |c|^2 \left[|q'|^2 \frac{|a_f|^2}{\gamma_a} + |q|^2 \frac{|b_f|^2}{\gamma_b} + 2\Re\left(q'^* q \frac{a_f^* b_f}{\gamma_+ - i\delta m} \right) \right] , \\ Br(\overline{B}^0 \to f) &= |c|^2 \left[|p'|^2 \frac{|a_f|^2}{\gamma_a} + |p|^2 \frac{|b_f|^2}{\gamma_b} - 2\Re\left(p'^* p \frac{a_f^* b_f}{\gamma_+ - i\delta m} \right) \right] . \end{aligned} \tag{A.7}$$

The normalization (27) can be obtained by summing the above equations over f and using the Bell-Steinberger relation[20]

$$\frac{\sum_f a_f^* b_f}{\gamma_+ - i\delta m} = \langle B_a | B_b \rangle \ \ (= p' p^* - q' q^*) \ , \tag{A.8}$$

which expresses the conservation of probability.

On $\Upsilon(4S)$, the $B^0\overline{B}^0$ pair is created in the coherent $L=1$ state

$$\begin{aligned} &\frac{1}{\sqrt{2}} \left(|\overline{B}^0(1)\rangle |B^0(2)\rangle - |B^0(1)\rangle |\overline{B}^0(2)\rangle \right) \\ &\qquad = \frac{c}{\sqrt{2}} \left(|B_a(1)\rangle |B_b(2)\rangle - |B_b(1)\rangle |B_a(2)\rangle \right) \ , \end{aligned} \tag{A.9}$$

where the numbers 1 and 2 distinguish the sides; namely, they may be distinguished by the direction of the B meson in the $\Upsilon(4S)$ C.M. system: $\hat{k}$ or $-\hat{k}$. Then the probability density that the side 1 decays to final state f_1 at time t_1 and the side 2 decays to f_2 at time t_2 is given by

$$\begin{aligned}\Gamma_{\Upsilon(4S)\to f_1 f_2}(t_1,t_2) \;=\; & \frac{|c|^2}{2}\left[e^{-\gamma_a t_1-\gamma_b t_2}|a_{f_1}b_{f_2}|^2+e^{-\gamma_b t_1-\gamma_a t_2}|b_{f_1}a_{f_2}|^2\right. \\ & \left.-2\Re\left(e^{-(\gamma_+-i\delta m)t_1}e^{-(\gamma_++i\delta m)t_2}(a_{f_1}b_{f_2})^*(b_{f_1}a_{f_2})\right)\right], \end{aligned} \quad (A.10)$$

or equivalently,

$$\begin{aligned}&\Gamma_{\Upsilon(4S)\to f_1 f_2}(t_+,t_-) = \\ &\quad \frac{|c|^2}{4}e^{-\gamma_+ t_+}\left[e^{-\gamma_- t_-}|a_{f_1}b_{f_2}|^2+e^{\gamma_- t_-}|b_{f_1}a_{f_2}|^2-2\Re\left((a_{f_1}b_{f_2})^*(b_{f_1}a_{f_2})e^{i\delta m t_-}\right)\right],\end{aligned} \quad (A.11)$$

with

$$t_\pm \equiv t_1 \pm t_2 \;, \quad (A.12)$$

and we have used the relation $2dt_1dt_2 = dt_+dt_-$. Integrating (A.10) over t_2 and summing over all possible final states f_2, we obtain

$$\begin{aligned}&\Gamma_{\Upsilon(4S)\to f_1}(t_1) \equiv 2\sum_{f_2}\int_0^\infty \Gamma_{\Upsilon(4S)\to f_1 f_2}(t_1,t_2)dt_2 \\ &= |c|^2\left[|a_{f_1}|^2e^{-\gamma_a t_1}+|b_{f_1}|^2e^{-\gamma_b t_1}-2\Re\left(e^{-(\gamma_+-i\delta m)t_1}a^*_{f_1}b_{f_1}\frac{\sum_{f_2}b^*_{f_2}a_{f_2}}{\gamma_++i\delta m}\right)\right],\end{aligned} \quad (A.13)$$

where we have used (A.6), and the factor 2 arises from the fact that the given final state can come from either side. This together with (A.4) and the Bell-Steinberger relation (A.8) establishes the general rule (22). The normalization (26) and (28) then follows from (22) and (27).

Expressions for semileptonic decays are obtained by the substitutions

$$\begin{aligned}a_{\ell^+} &= pa_0 \;,\; b_{\ell^+} = p'a_0 \;, \\ a_{\ell^-} &= q\bar{a}_0 \;,\; b_{\ell^-} = -q'\bar{a}_0 \;,\end{aligned} \quad (A.14)$$

where we have used the assumption $\Delta B = \Delta Q$, and

$$a_0 \equiv Amp(B^0 \to \ell^+) \;, \quad \bar{a}_0 \equiv Amp(\bar{B}^0 \to \ell^-) \;. \quad (A.15)$$

Namely, (A.4) gives

$$\begin{aligned}\Gamma_{B^0\to\ell^+}(t) &= |c|^2|a_0|^2e^{-\gamma_+ t}\left[|pq'|^2e^{-\gamma_- t}+|p'q|^2e^{\gamma_- t}+2\Re\left((pq')^*(p'q)e^{i\delta mt}\right)\right], \\ \Gamma_{\overline{B}^0\to\ell^-}(t) &= |c|^2|\bar{a}_0|^2e^{-\gamma_+ t}\left[|p'q|^2e^{-\gamma_- t}+|pq'|^2e^{\gamma_- t}+2\Re\left((p'q)^*(pq')e^{i\delta mt}\right)\right], \\ \Gamma_{\overline{B}^0\to\ell^+}(t) &= 2|c|^2|a_0|^2|pp'|^2e^{-\gamma_+ t}\left[\cosh\gamma_- t-\cos\delta mt\right], \\ \Gamma_{B^0\to\ell^-}(t) &= 2|c|^2|\bar{a}_0|^2|qq'|^2e^{-\gamma_+ t}\left[\cosh\gamma_- t-\cos\delta mt\right].\end{aligned} \quad (A.16)$$

The CPT relations (7) and (8) then lead to (9).

On $\Upsilon(4S)$, (A.11) and (A.14) give the di-lepton decay distributions:

$$\begin{aligned}
&\Gamma_{\Upsilon(4S)\to\ell^+\ell^-}(t_+,t_-) = \\
&\qquad \frac{|c|^2}{4}|\bar{a}_0 a_0|^2 e^{-\gamma_+ t_+}\left[|pq'|^2 e^{-\gamma_- t_-} + |p'q|^2 e^{\gamma_- t_-} + 2\Re\left((pq')^*(p'q)e^{i\delta m t_-}\right)\right], \\
&\Gamma_{\Upsilon(4S)\to\ell^-\ell^+}(t_+,t_-) = \\
&\qquad \frac{|c|^2}{4}|\bar{a}_0 a_0|^2 e^{-\gamma_+ t_+}\left[|p'q|^2 e^{-\gamma_- t_-} + |pq'|^2 e^{\gamma_- t_-} + 2\Re\left((p'q)^*(pq')e^{i\delta m t_-}\right)\right], \\
&\Gamma_{\Upsilon(4S)\to\ell^+\ell^+}(t_+,t_-) = \frac{|c|^2}{2}|a_0|^4|pp'|^2 e^{-\gamma_+ t_+}\left[\cosh\gamma_- t_- - \cos\delta m t_-\right], \\
&\Gamma_{\Upsilon(4S)\to\ell^-\ell^-}(t_+,t_-) = \frac{|c|^2}{2}|\bar{a}_0|^4|qq'|^2 e^{-\gamma_+ t_+}\left[\cosh\gamma_- t_- - \cos\delta m t_-\right].
\end{aligned} \tag{A.17}$$

Note that the opposite-sign lepton rates satisfy the relation

$$\Gamma_{\Upsilon(4S)\to\ell^+\ell^-}(t_+,t_-) = \Gamma_{\Upsilon(4S)\to\ell^-\ell^+}(t_+,-t_-)\;, \tag{A.18}$$

which corresponds to re-labeling the final states. Under CPT symmetry this simplifies to

$$\begin{aligned}
\Gamma_{\Upsilon(4S)\to\ell^+\ell^-}(t_+,t_-) &= \Gamma_{\Upsilon(4S)\to\ell^-\ell^+}(t_+,t_-) \\
&= \frac{A_0^4}{8}e^{-\gamma_+ t_+}\left[\cosh\gamma_- t_- + \cos\delta m t_-\right], \\
\Gamma_{\Upsilon(4S)\to\ell^+\ell^+}(t_+,t_-) &= \frac{|p|^2}{|q|^2}\frac{A_0^4}{8}e^{-\gamma_+ t_+}\left[\cosh\gamma_- t_- - \cos\delta m t_-\right], \\
\Gamma_{\Upsilon(4S)\to\ell^-\ell^-}(t_+,t_-) &= \frac{|q|^2}{|p|^2}\frac{A_0^4}{8}e^{-\gamma_+ t_+}\left[\cosh\gamma_- t_- - \cos\delta m t_-\right].
\end{aligned} \tag{A.19}$$

Integrating over t_+ from $|t_-|$ to ∞ gives (18). Note that CPT invariance ensures that the decay distributions are symmetric under sign change of t_-. Thus, such asymmetry (for example, if the ℓ^+ side tends to decay earlier then the ℓ^- side in the $\ell^+\ell^-$ sample) is a signature of CPT violation [21]. This is in contrast to the case of lepton tagged CP eigenstates (e.g., ΨK_s [22, 23]) where the asymmetry with respect to t_- is possible under CPT invariance and signals CP violation.

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