A Note on ϕ_3 determination from $B \to DK$ and $2\pi_1 + \phi_3$ determination from $B \to D\pi$

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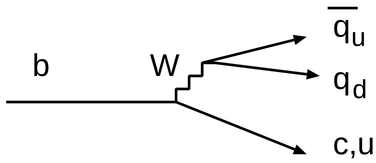
1 Effective Hamiltonian

Weak decays occur at the tree level through the graph like shown below;



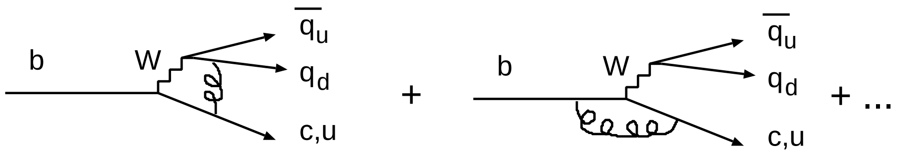
The W boson is so heavy that we can write down 4-Fermi effective Hamiltonian.

$$\begin{aligned} \mathcal{H}_{eff}^{tree} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ij}{}^* \overline{c} \gamma_\mu (1-\gamma_5) b \, \overline{q_j} \gamma^\mu (1-\gamma_5) q_i \\ &+ \frac{G_F}{\sqrt{2}} V_{ub} V_{ij}{}^* \overline{u} \gamma_\mu (1-\gamma_5) b \, \overline{q_j} \gamma^\mu (1-\gamma_5) q_i + \text{ (h.c)}. \end{aligned} \tag{1}$$

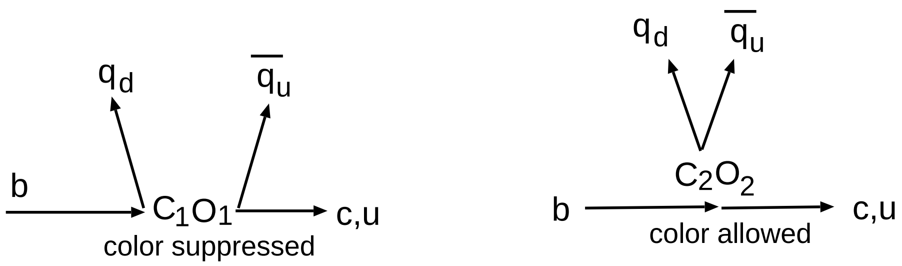
At the B meson mass scale we have to include QCD corrections using renormalization group equations[1]. Then the effective Hamiltonian becomes as follows,

$$\begin{aligned} \mathcal{H}_{eff} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ij}{}^* [C_2 \, \overline{c} \gamma_\mu (1-\gamma_5) b \, \overline{d_j} \gamma^\mu (1-\gamma_5) u_i + C_1 \, \overline{d_j} \gamma_\mu (1-\gamma_5) b \, \overline{c} \gamma^\mu (1-\gamma_5) u_i] \\ &+ \frac{G_F}{\sqrt{2}} V_{ub} V_{ij}{}^* [C_2 \, \overline{u} \gamma_\mu (1-\gamma_5) b \, \overline{d_j} \gamma^\mu (1-\gamma_5) u_i + C_1 \, \overline{d_j} \gamma_\mu (1-\gamma_5) b \, \overline{u} \gamma^\mu (1-\gamma_5) u_i] \\ &+ \text{(penguin contributions)} \\ &+ \text{(h.c)} \\ &\equiv \frac{G_F}{\sqrt{2}} V_{cb} V_{ij}{}^* [C_1 O_1^c + C_2 O_2^c] + \frac{G_F}{\sqrt{2}} V_{ub} V_{ij}{}^* [C_1 O_1^u + C_2 O_2^u] + \cdots , \end{aligned} \tag{2}$$

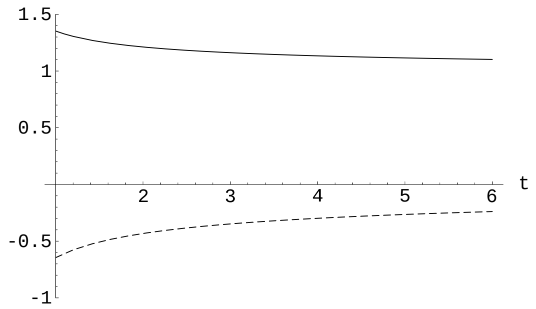
where C_i's are the Wilson coefficients calculated in perturbative QCD. The color quantum nnmber is assigned so that the first and second (third and fourth) quark fields form a color singlet. Below we use this effective Hamiltonian to estimate B decay amplitudes.



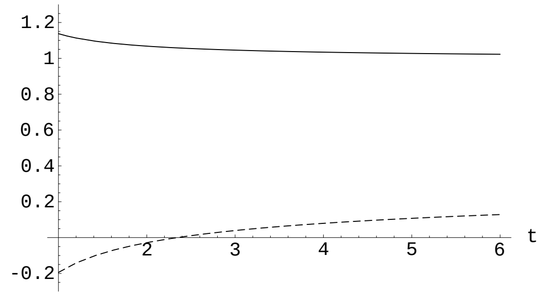
$\Downarrow$



The decays occured through O_1 and O_2 usually correspond to "color suppressed decay" and "color allowed decay", respectively. For the u_i and $\overline{d_j}$ in eq.(2) to form a meson the color quantum number of u_i and d_j should be the same. So that the factor $1/N_C$ (N_c:the number of colors) appears in color suppressed decays in this case. The Wilson coefficients appear in the calculation of decay amplitudes in the combination of $C_2 + C_1/N_C$ or $C_1 + C_2/N_C$. The Wilson coefficients are calculated at the one-loop level as follows;



C_1 (dashed) and C_2 (solid) at the energy scale t (GeV).

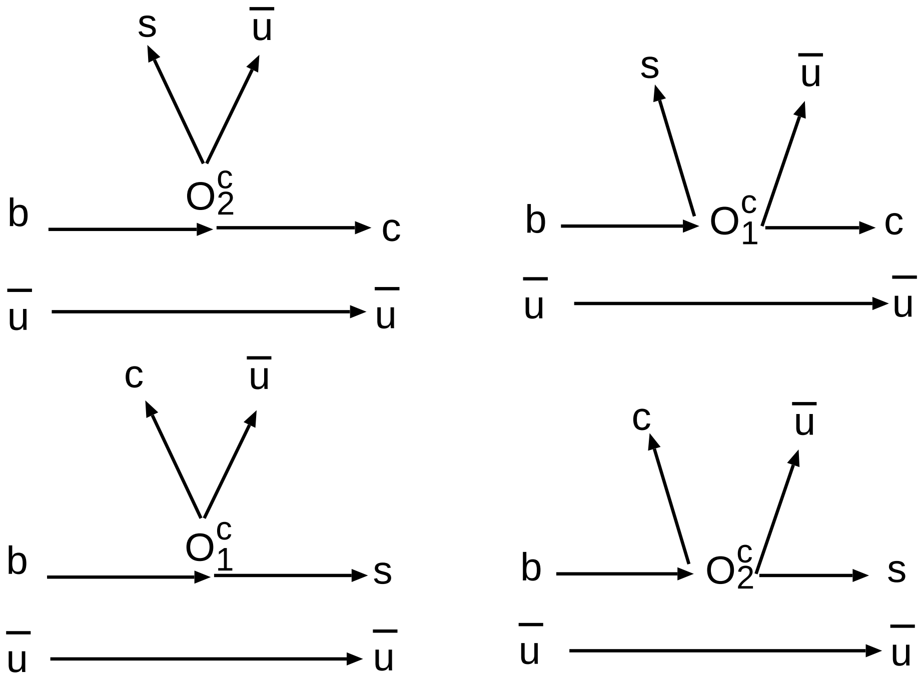


$C_1 + C_2/N_C$ (dashed) and $C_2 + C_1/N_C$ (solid) at the energy scale t (GeV). $N_C = 3$ is the number of color.

2 $B \to DK$ decays and ϕ_3

In the decays, $B \to \{D^0, \overline{D^0}\}K$, where D^0 and $\overline{D^0}$ decay into a common final state ($KK, K^+\pi^-$, ...), the interference between $b \to c$ and $b \to u$ amplitudess leads to CP asymetry related to ϕ_3. The detailed arguments on the methods and theoretical errors are given in ref.[2] and references therein. Below we draw diagrams of $B \to DK$ decays (Gluons are not shown) and discuss model independent etsimation of amplitude ratios.

2.1 $B^- \to D^0 K^-$



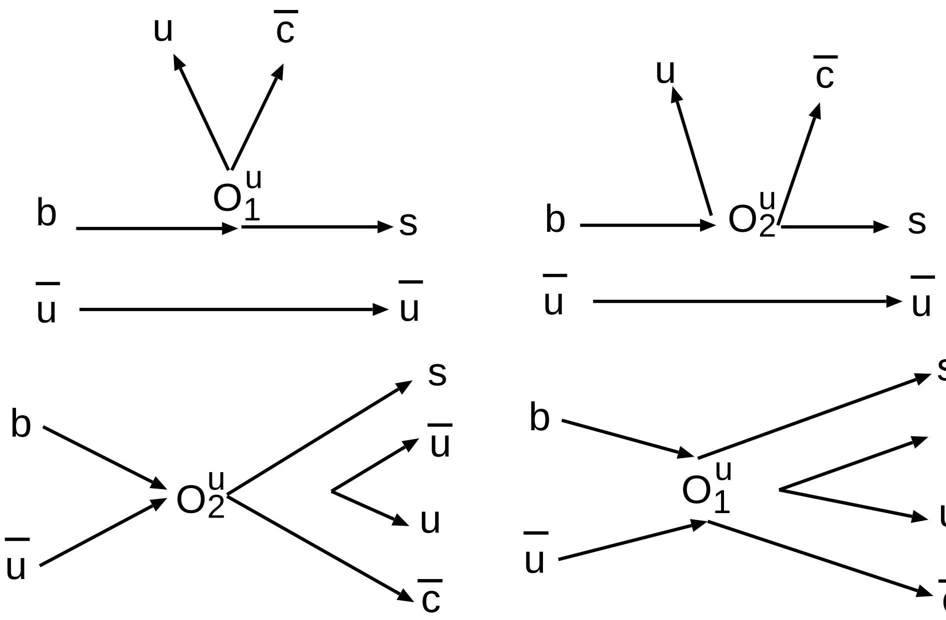
$$A(B^- \to D^0 K^-) = V_{cb} V_{us}^* \left[\langle C_2 + \frac{C_1}{N_C} \rangle A^H + \langle C_1 + \frac{C_2}{N_C} \rangle A^L \right] , \tag{3}$$

where $A^{H(L)}$ represnts the amplitude corrsponds to the upper (lower) diagram up to KM and Wilson coefficients factors. The notation $\langle \cdots \rangle$ means the energy scale of the factor inside the braket is convoluted in the calculation. $A^{H(L)}$ becomes the heavy to heavy (light) form factor times a meson decay constant in the naive factorizatio approximation;

$$\langle D^0 K^- | (\bar{c}b)_{V-A} (\bar{s}u)_{V-A} | B^- \rangle \Rightarrow \langle K^- | (\bar{s}u)_{V-A} | 0 \rangle \langle D^0 | (\bar{c}b)_{V-A} | B^- \rangle \ . \tag{4}$$

(We do not take this approximation in order for the analysis to be as model independent as possible.)

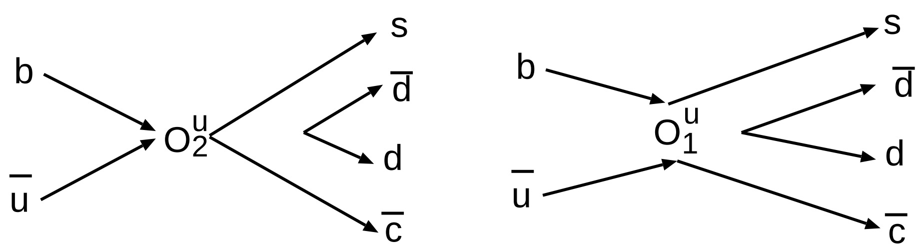
2.2 $B^- \to \overline{D^0} K^-$



$$A(B^- \to \overline{D^0} K^-) = V_{ub} V_{cs}^* \left[\langle C_1 + \frac{C_2}{N_C} \rangle \bar{A}^L + \langle C_2 + \frac{C_1}{N_C} \rangle A^{ann} \right] \ , \tag{5}$$

where A^{ann} corresponds to the annihilation amplitude. We distinguish A_L and $\bar{A}_L$ depending on whether c or $\bar{c}$ quark appears since they may be different if we include non-factorizable contributions.

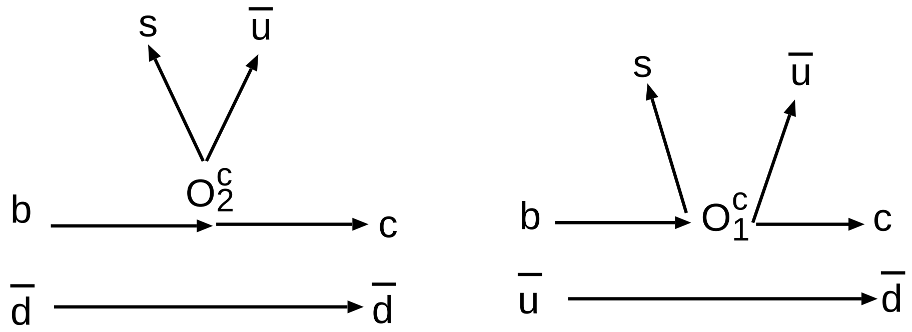
2.3 $B^- \to D^- \overline{K^0}$



$$A(B^- \to D^- \overline{K^0}) = V_{ub} V_{cs}^* \left[\langle C_2 + \frac{C_1}{N_C} \rangle A^{ann} \right] \tag{6}$$

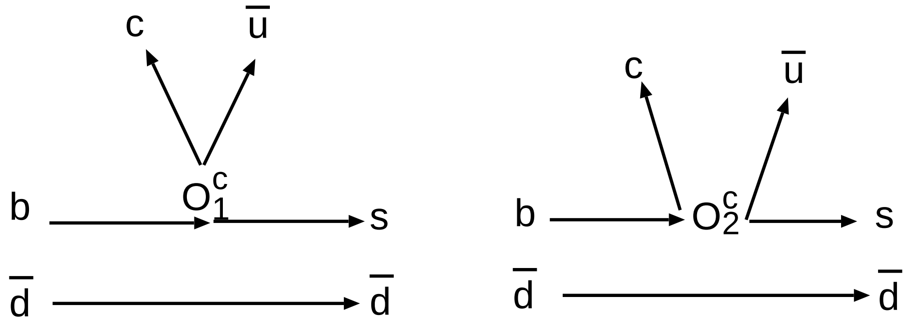
Note that this amplitude is same as the annihilation part of $A(B^- \to \overline{D^0} K^-)$ due to isospin symmetry up to overall phase. Thus if we can measure the branching ratios of both $B^- \to \overline{D^0} K^-$ and $B^- \to D^- \overline{K^0}$, we can constarin the magnutude of the annihilation part. ($B^- \to D^- K^0$ is not allowed since the final state, $(\bar{c}d)$ $(\bar{s}d)$, cannot be realized unless some new physics contributes.)

2.4 $\overline{B^0} \to D^+ K^-$



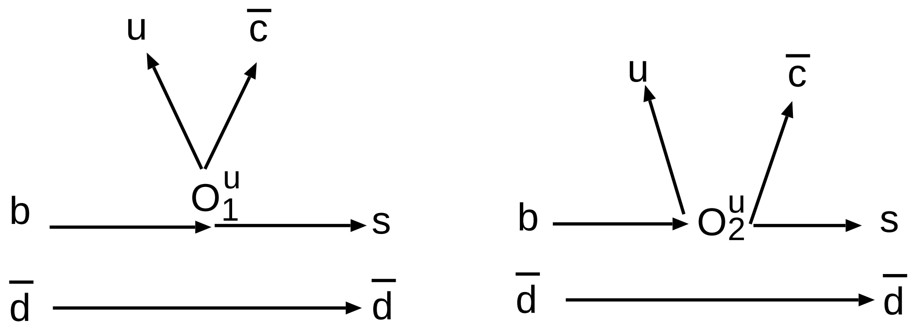
$$A(\overline{B^0} \to D^+ K^-) = V_{cb} V_{us}^* \left[\langle C_2 + \frac{C_1}{N_C} \rangle A^H \right] , \tag{7}$$

2.5 $\overline{B^0} \to D^0 \overline{K^0}$



$$A(\overline{B^0} \to D^0 \overline{K^0}) = V_{cb} V_{us}^* \left[\langle C_1 + \frac{C_2}{N_C} \rangle A^L \right] , \tag{8}$$

2.6 $\overline{B^0} \to \overline{D^0}\ \overline{K^0}$



$$A(\overline{B^0} \to \overline{D^0}\ \overline{K^0}) = V_{ub} V_{cs}^* \left[\langle C_1 + \frac{C_2}{N_C} \rangle \bar{A}^L \right] , \tag{9}$$

2.7 Model independent estimation of the amplitude ratios

With eqs.(3) and (8) we obtain

$$\frac{\mathrm{Br}(\overline{B^0} \to D^0 \overline{K^0})}{\mathrm{Br}(B^- \to D^0 K^-)} = \left| \frac{\xi_{HL}}{1+\xi_{HL}} \right|^2 \quad \text{with} \quad \xi_{HL} = \frac{\langle C_1 + \frac{C_2}{N_C} \rangle A^L}{\langle C_2 + \frac{C_1}{N_C} \rangle A^H}. \tag{10}$$

From Belle date (Belle prerpint 2003-3[3] and Gershon's talk at CKM workshop)

$$\mathrm{Br}(B^- \to D^0 K^-) = (0.077 \pm 0.0005 \pm 0.0006) \mathrm{Br}(B^- \to D^0 \pi^-) \simeq 4 \times 10^{-4}, \tag{11}$$

$$\mathrm{Br}(\overline{B^0} \to D^0 \overline{K^0}) = (5.0 \pm 1.3 \pm 0.6) \times 10^{-5} \simeq 5 \times 10^{-5}, \tag{12}$$

we obtain

$$0.2 < |\xi_{HL}| < 0.6 \; . \tag{13}$$

This value is estimated to be much more small if we put the values of Wilson coefficients and form factors assuming naive factorization. This show that non-factorizable contribution in $\langle C_1 + \frac{C_2}{N_C}\rangle A^L$ cannot be neglected, which gives relatively large $\mathrm{Br}(\overline{B^0} \to D^0 \overline{K^0})$. This is similar to the case of large $\mathrm{Br}(\overline{B^0} \to D^0 \pi^0)$[4].

We also need the amplitude $\langle C_2 + \frac{C_1}{N_C}\rangle A^{ann}$, which can be obtained if we can measure the branching ratios of both $B^- \to \overline{D^0} K^-$ (or $\overline{B^0} \to \overline{D^0}\,\overline{K^0}$ and $B^- \to D^- \overline{K^0}$ as stated in Sec.2.3. With eqs.(5), (6) and (9) we obtain

$$\frac{\mathrm{Br}(B^- \to D^- \overline{K^0})}{\mathrm{Br}(B^- \to \overline{D^0} K^-)} = \left| \frac{\xi_{La}}{1 + \xi_{La}} \right|^2 \; , \quad \frac{\mathrm{Br}(B^- \to D^- \overline{K^0})}{\mathrm{Br}(\overline{B^0} \to \overline{D^0}\,\overline{K^0})} = |\xi_{La}|^2 \; , \tag{14}$$

where

$$\xi_{La} = \frac{\langle C_2 + \frac{C_1}{N_C}\rangle A^{ann}}{\langle C_1 + \frac{C_2}{N_C}\rangle \bar{A}^L} . \tag{15}$$

We can also check if $A_L = \bar{A}^L$ or not.

$$\frac{\mathrm{Br}(\overline{B^0} \to \overline{D^0}\;\overline{K^0})}{\mathrm{Br}(\overline{B^0} \to D^0 \overline{K^0})} = \left| \frac{V_{ub}V_{cs}}{V_{cb}V_{us}} \frac{\langle C_1 + \frac{C_2}{N_C}\rangle \bar{A}^L}{\langle C_1 + \frac{C_2}{N_C}\rangle A^L} \right|^2 \equiv \left| \frac{V_{ub}V_{cs}}{V_{cb}V_{us}} \right|^2 |\xi_{\bar{L}L}|^2 \tag{16}$$

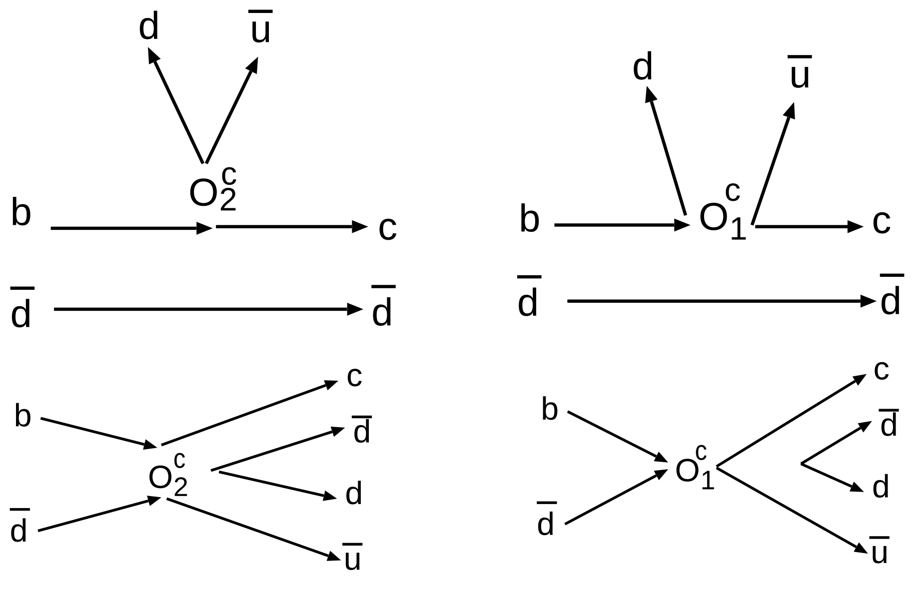
However, the data is not available yet, so we refer a theoretical value based on the preliminary results by pQCD calculation[5].

$$\xi_{La} = 1.1 e^{110^\circ i} \; , \quad \xi_{\bar{L}L} = 1.0 e^{-156^\circ i} \; . \tag{17}$$

3 $B^0(\overline{B^0}) \to D\pi$ decays and $2\phi_1 + \phi_3$

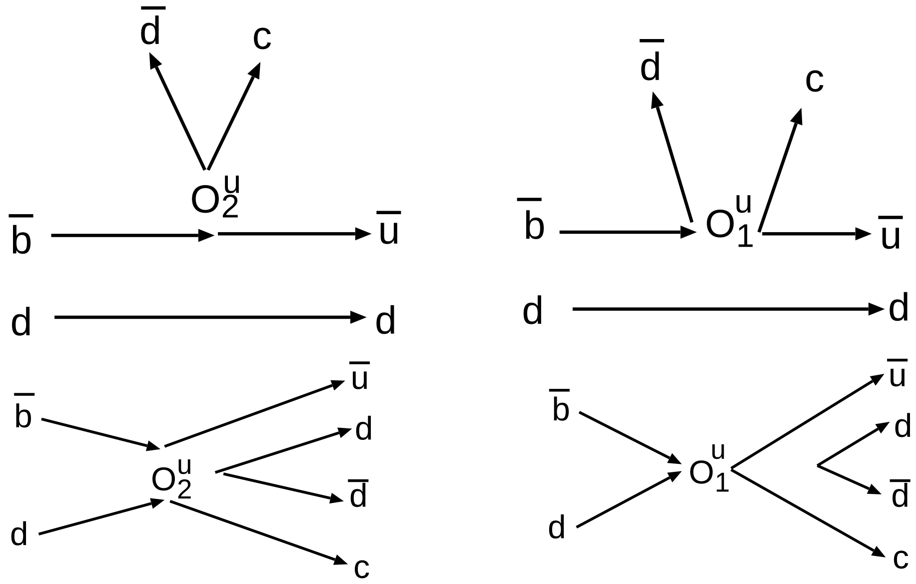
$\overline{B^0}$ can decay into both $D^+\pi^-$ and $D^-\pi^+$ though the latter decay is doubly CKM suppressed. The interference between $\overline{B^0} \to D^+\pi^-$ and $\overline{B^0} \to B^0 \to D^+\pi^-$ cause CP syammetry related to $2\phi_1 + \phi_3$. Below we draw diagrams of $B^0(\overline{B^0}) \to D^+\pi^-$ decays (Gluons are not shown) and discuss model independent etsimation of amplitude ratios.

3.1 $\overline{B^0} \to D^+\pi^-$



$$A(\overline{B^0} \to D^+\pi^-) = V_{cb}V_{ud}^* \left[\langle C_2 + \frac{C_1}{N_C} \rangle A^H + \langle C_1 + \frac{C_2}{N_C} \rangle A^{ann} \right] . \tag{18}$$

3.2 $B^0 \to D^+\pi^-$

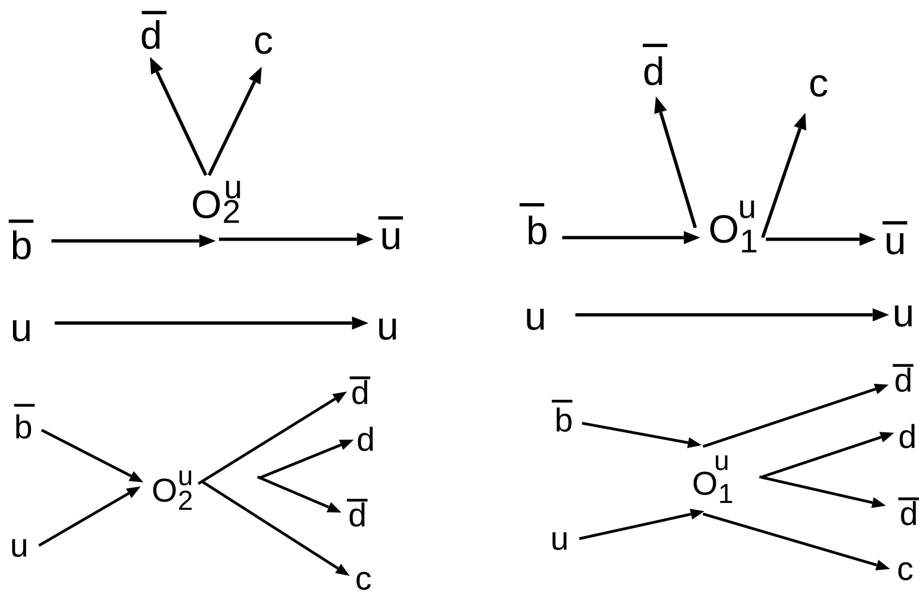


$$A(B^0 \to D^+\pi^-) = V_{ub}^* V_{cd} \left[\langle C_2 + \frac{C_1}{N_C} \rangle \bar{A}^L + \langle C_1 + \frac{C_2}{N_C} \rangle \bar{A}^{ann} \right] , \tag{19}$$

We distinguish A^{ann} and $\bar{A}^{ann}$ depending on whether it comes from b of $\bar{b}$ since they may be different if we include non-factorizable contributions.

We need tha ratio $A(B^0 \to D^+\pi^-)/A(\overline{B^0} \to D^+\pi^-)$ to extarct $2\phi_1 + \phi_3$. But it is hard to obtain it directly from exp. data. So let us look at other decays to extarct $A(\overline{B^0} \to D^+\pi^-)$.

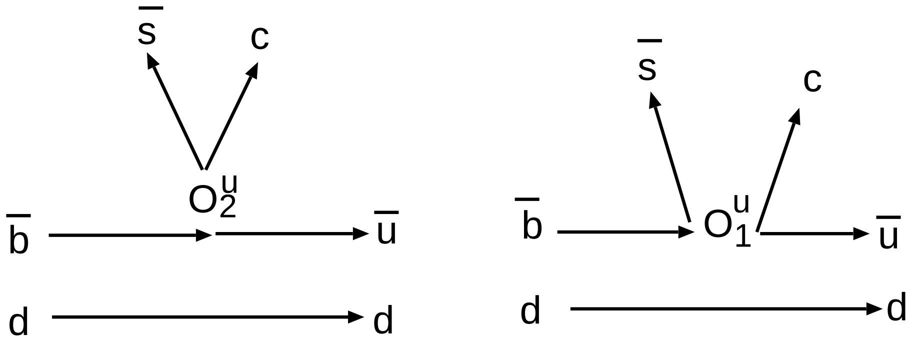
3.3 $B^+ \to D^+\pi^0$



$$A(B^+ \to D^+\pi^0) = V_{ub}^* V_{cd} \left[\langle C_2 + \frac{C_1}{N_C} \rangle \bar{A}^L + \langle C_2 + \frac{C_1}{N_C} \rangle \bar{A}^{ann} \right] . \tag{20}$$

If the annihilation contributions can be neglected, this give the same amplitude with $A(B^0 \to D^+\pi^-)$.

3.4 $B^0 \to D_s^+ \pi^-$

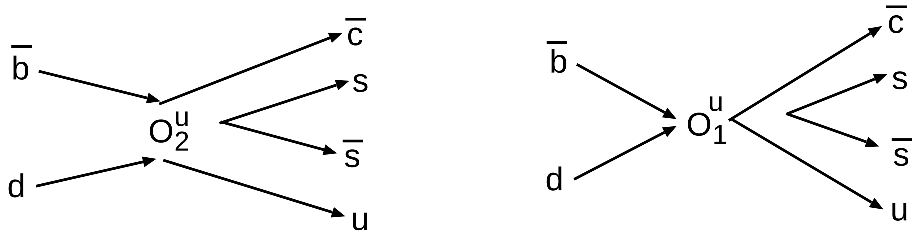


$$A(B^0 \to D_s^+ \pi^-) = V_{ub}^* V_{cs} \left[\langle C_2 + \frac{C_1}{N_C} \rangle \bar{A}^L \right] \ . \tag{21}$$

If the annihilation contributions in $A(B^0 \to D^+ \pi^-)$ can be neglected, this give the same amplitude up to KM factor. Belle group has already got the data of this decay[6];

$$\mathrm{Br}(B^0 \to D_s^- K^+) = (2.4 \pm 1.0 \pm 0.7) \times 10^{-5}. \tag{22}$$

3.5 $B^0 \to D_s^- K^+$



$$A(B^0 \to D_s^- K^+) = V_{cb}^* V_{ud} \left[\langle C_1 + \frac{C_2}{N_C} \rangle A^{ann} \right] \ . \tag{23}$$

This is a purely annihilation decay. Belle group has already got the data[6];

$$\mathrm{Br}(B^0 \to D_s^- K^+) = (4.6 \pm 1.2 \pm 1.3) \times 10^{-5}. \tag{24}$$

3.6 Estimation of annihilation contribution

With the results of preceding subsections we can estimate the ratio of the annihilation contribution to spectator contribution. Eqs.(21) to (24) gives

$$\left| \frac{\langle C_1 + \frac{C_2}{N_C} \rangle A^{ann}}{\langle C_2 + \frac{C_1}{N_C} \rangle \bar{A}^L} \right| \simeq \left| \frac{V_{ub} V_{cs}}{V_{cb} V_{ud}} \right| \sqrt{\frac{\mathrm{Br}(B^0 \to D_s^- K^+)}{\mathrm{Br}(B^0 \to D_s^+ \pi^-)}} \sim 0.1 \tag{25}$$

Then we can approximate $|A(B^0 \to D^+ \pi^-)|$ by $|A(B^0 \to D_s^+ \pi^-)|$ up to the KM factor in 10 % error.

References

[1] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev.Mod.Phys **68-4**, 1125 (1996).

[2] D. Atwood and A. Soni, hep-ph/0212071 (2002).

[3] BELLE group, hep-ex/0304032 (2003).

[4] Y.Y. Keum, T. Kurimoto, H-n. Li, C.D. Lu and A.I. Sanda, hep-ph/0305335 (2003).

[5] H. Hayakawa, T. Kurimoto M. Nagashima and N. Yoshikawa, in preparation.

[6] Belle group, PRL **89**, 231804 (2002)