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NuFlux Workshop
@University of Notre Dame

Efficient modeling of flux correlations for multi-point measurements at the J-PARC Neutrino Beamline

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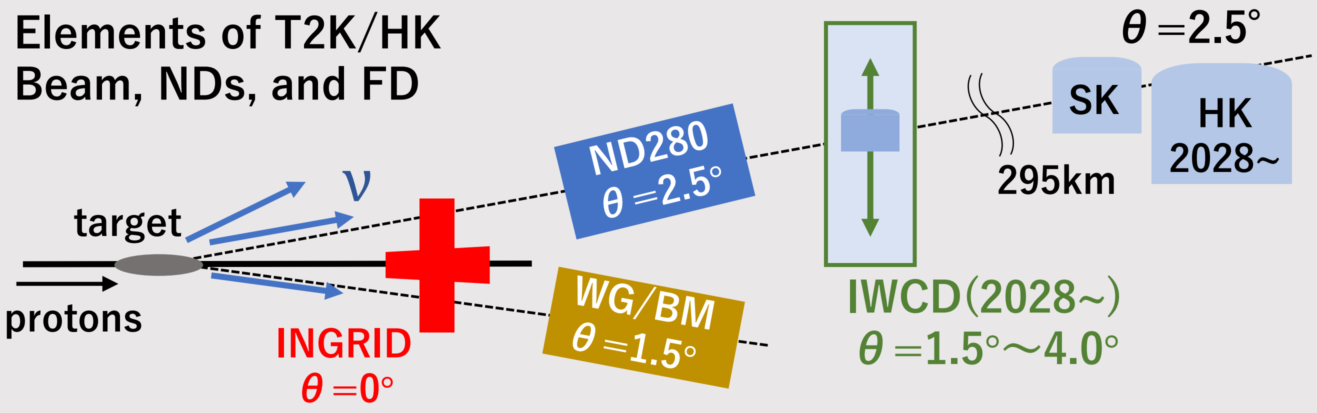
On behalf of T2K and Hyper-K Collaboration

T2K experiment and Hyper-Kamiokande experiment



SK→HK (×8 volume) + Beam enhance

→Huge enhance of statistics

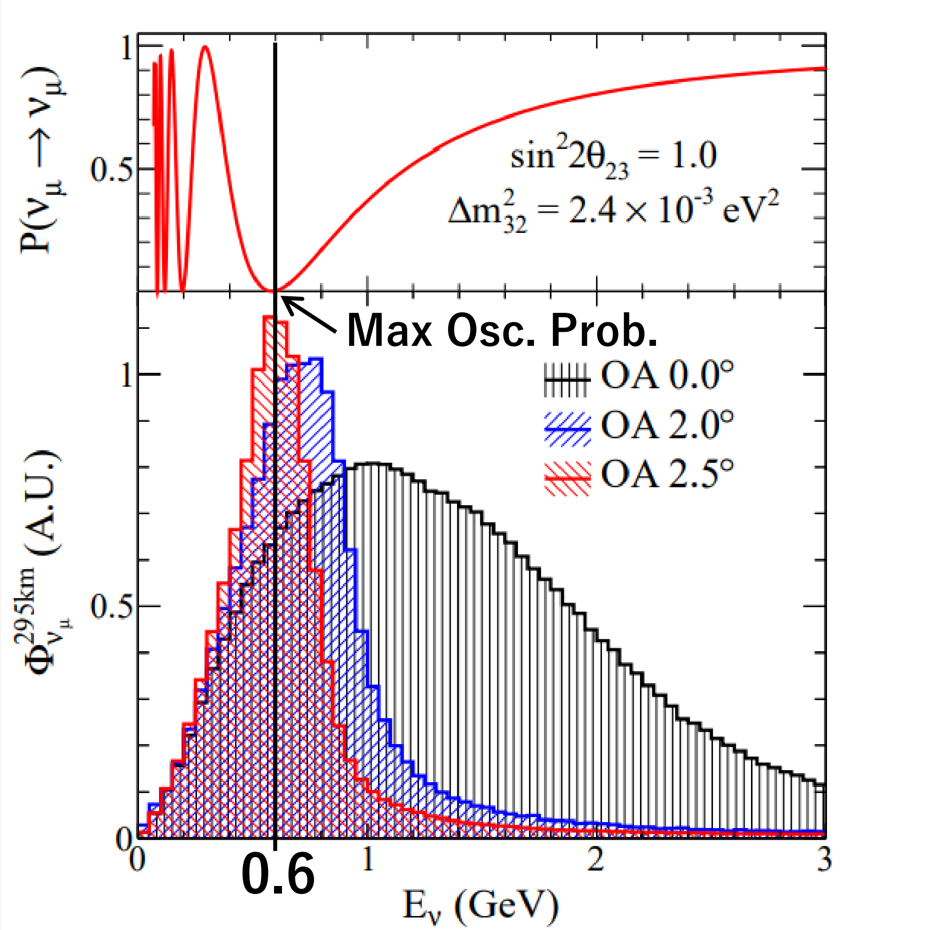
ν-nucleus X-sec will be crucial uncertainty

Need to know energy dependence of X-sec

→use ND280, WAGASCI/BabyMIND (WG/BM)

Construct IWCD for HK experiment

Use **Off-Axis neutrinos** and measure oscillation efficiently



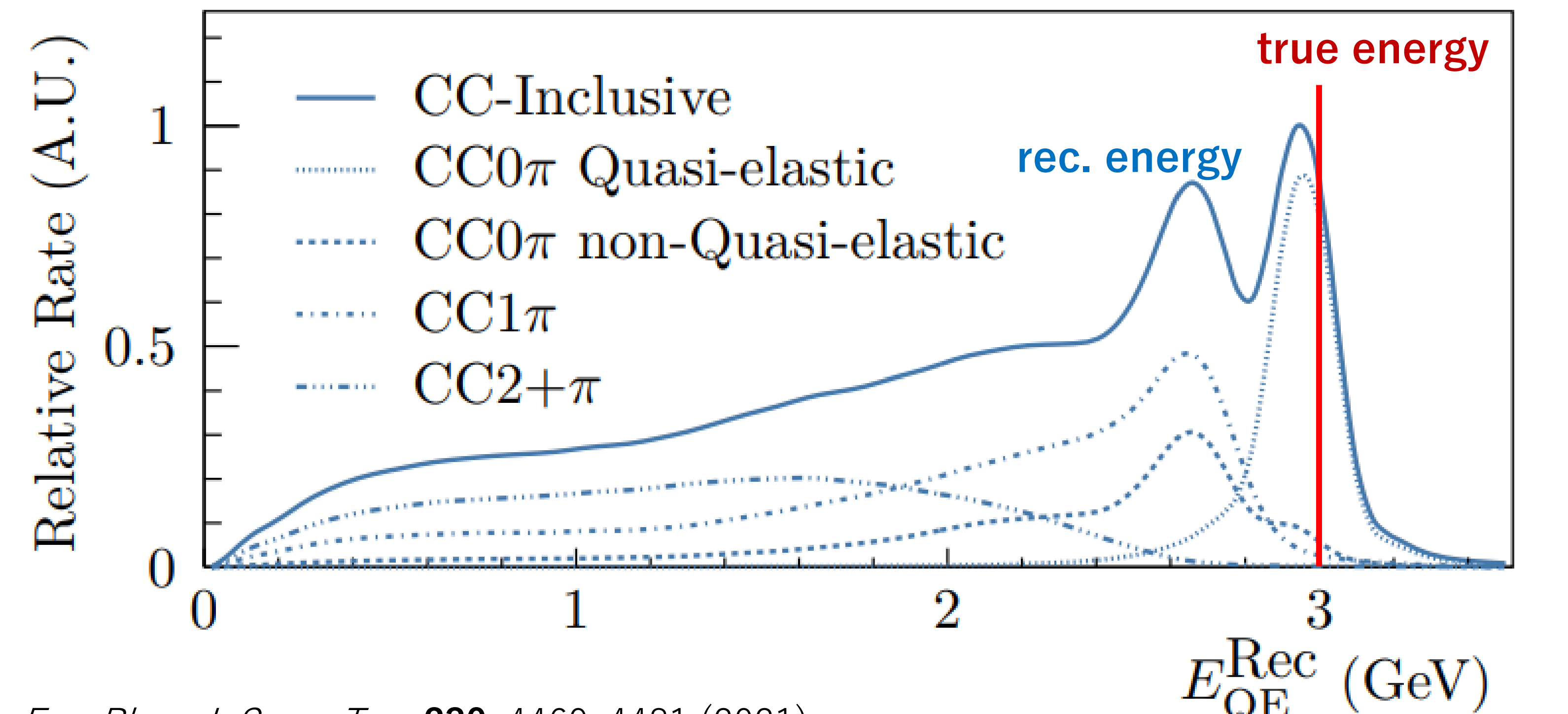
Phys. Rev. D 87, 012001

Energy Reconstruction and X-sec of Neutrinos

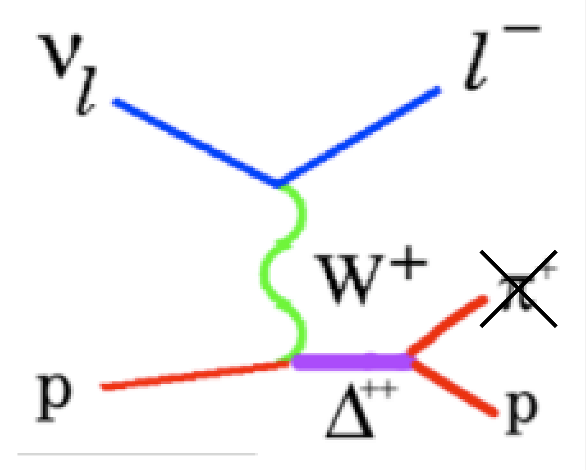
~~$P(E_\nu) \propto \Phi(E_\nu) \times \sigma(E_\nu)$~~ ← Not so easy in real measurement…

$$P(E_\nu^{\mathrm{Rec}}) \propto \int \Phi(E_\nu^{\mathrm{True}}) \times \boldsymbol{\sigma(E_\nu^{\mathrm{True}} \to E_\nu^{\mathrm{Rec}})} \mathrm{d}E_\nu^{\mathrm{True}}$$

Need to know X-sec as 2D function of $E_\nu^{\mathrm{True}}, E_\nu^{\mathrm{Rec}}$



Eur. Phys. J. Spec. Top. **230**, 4469–4481 (2021)



Cannot detect π in CC1π?
→Reconstructed as CCQE

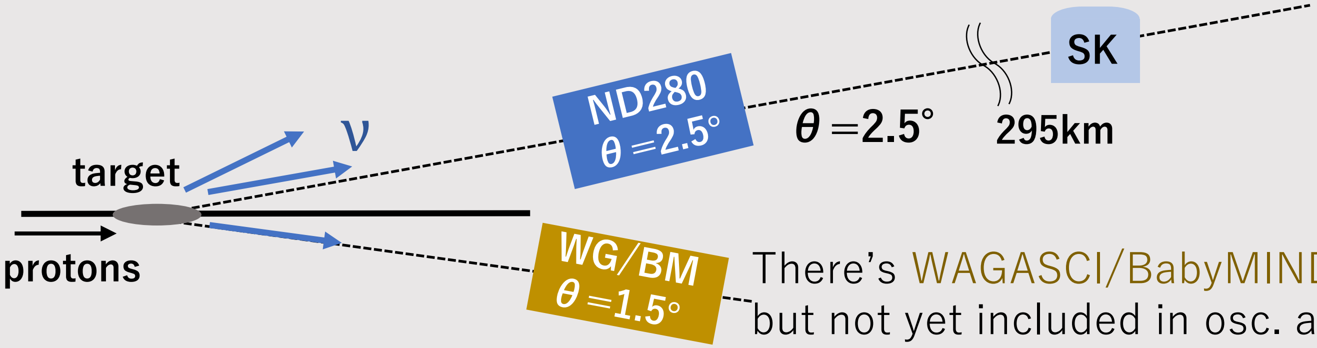
Cross-section measurement in T2K

Measured by ND280

$$\text{Event Rate}\,(p_l, \theta_l) \propto \int \Phi_{\text{ND280}}\left(E_\nu^{\text{True}}\right) \times \sigma\left(E_\nu^{\text{True}} \rightarrow (p_l, \theta_l)\right) \mathrm{d}E_\nu^{\text{True}}$$

Not enough to solve degeneracy of $\left(E_\nu^{\text{True}},\ E_\nu^{\text{Rec}}\right)$ 2D function

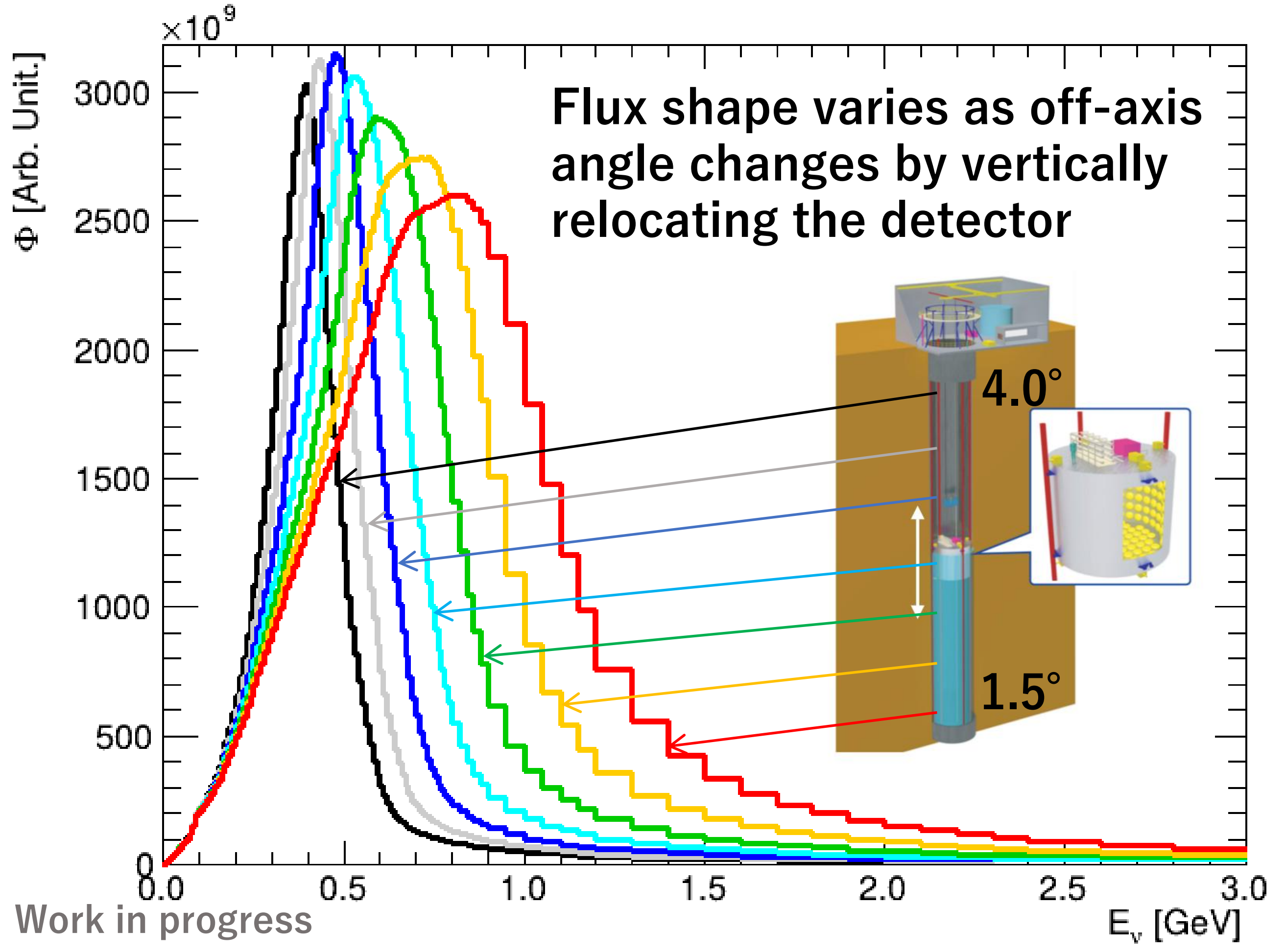
→Rely on model to solve it



There's WAGASCI/BabyMIND at 1.5°
but not yet included in osc. analysis

Cross-section measurement in Hyper-K

Addition of Intermediate Water Cherenkov Detector (IWCD)



$$\begin{cases} \int \Phi_{1.5^\circ}(E_\nu^{\mathrm{True}}) \times \sigma(E_\nu^{\mathrm{True}} \to E_\nu^{\mathrm{Rec}}) dE_\nu^{\mathrm{True}} \\ \int \Phi_{2.0^\circ}(E_\nu^{\mathrm{True}}) \times \sigma(E_\nu^{\mathrm{True}} \to E_\nu^{\mathrm{Rec}}) dE_\nu^{\mathrm{True}} \\ \vdots \end{cases}$$

→ Measurements with various flux conditions enables **model-independent understanding of X-sec**

Covariance Matrix for evaluation of systematic uncertainties

$$\boldsymbol{\sigma}(\boldsymbol{f}(\mathbf{x},\mathbf{y}))^2 \approx \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2 + 2\left(\frac{\partial f}{\partial x} \times \frac{\partial f}{\partial y}\right) \underline{\boldsymbol{\sigma_{xy}}}$$

Expressed as **Covariance Matrix or Correlation Matrix**

Syst. error in T2K:

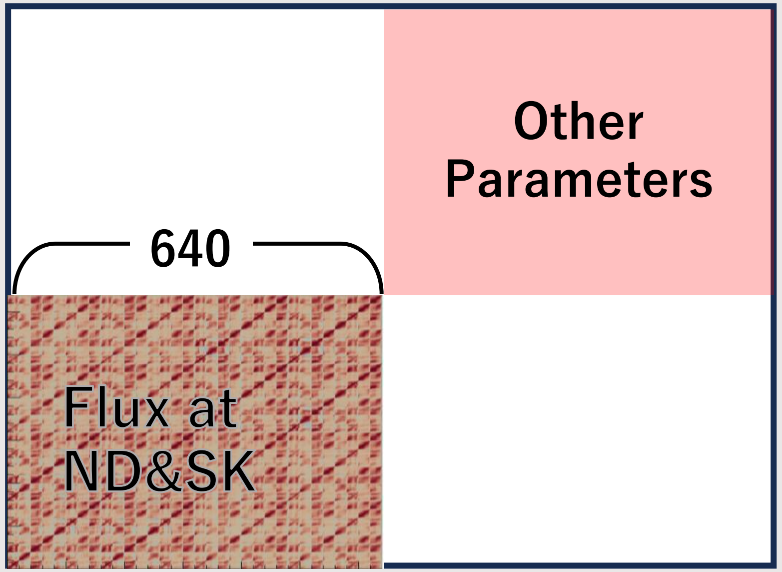
Flux intensity, Cross-section, Detector systematics

→**Correlation of flux intensity, especially among different positions, is most important** in this work

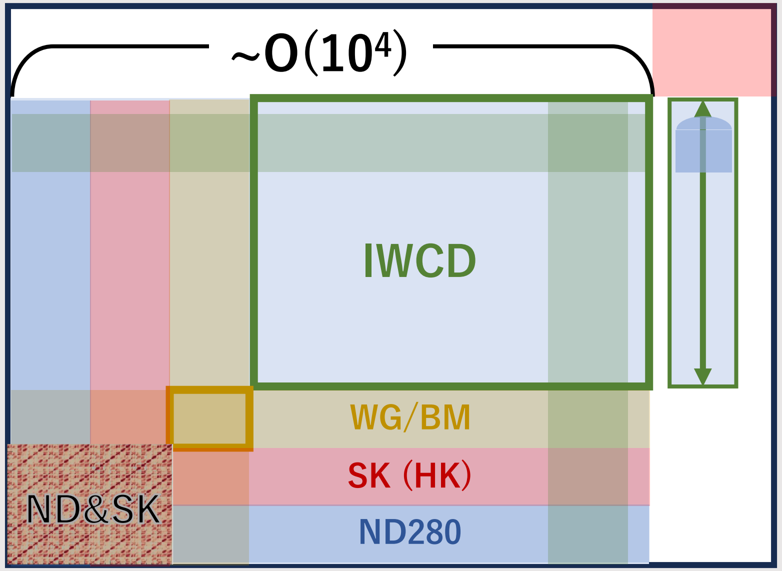
Severe Computational Cost in Hyper-K Experiment

Number of fit parameters will significantly increase due to **IWCD**

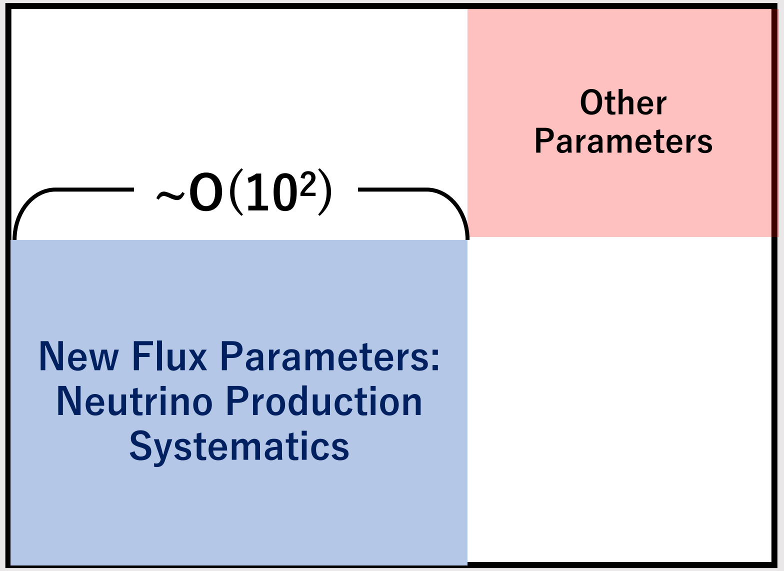
Reduce the number of parameters by **using systematic source parameters of neutrino production system**, instead of using neutrino flux intensity at each detection point



Current Matrix



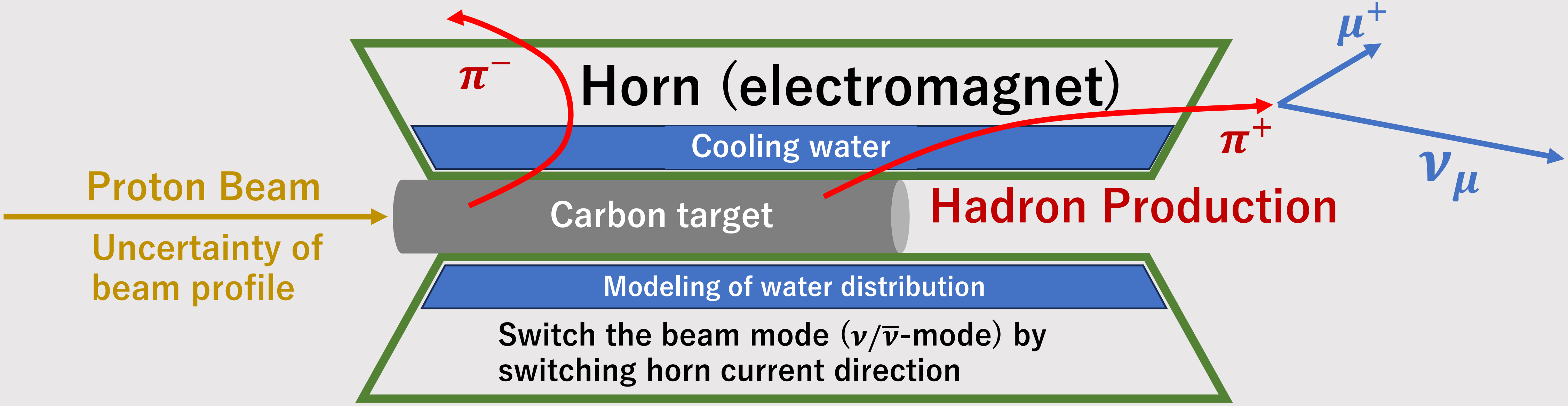
Matrix in HK era
but too large…



This work

Systematic Uncertainties of Neutrino Production System

30GeV protons strike a carbon target, which results in production of hadrons which decay into neutrinos

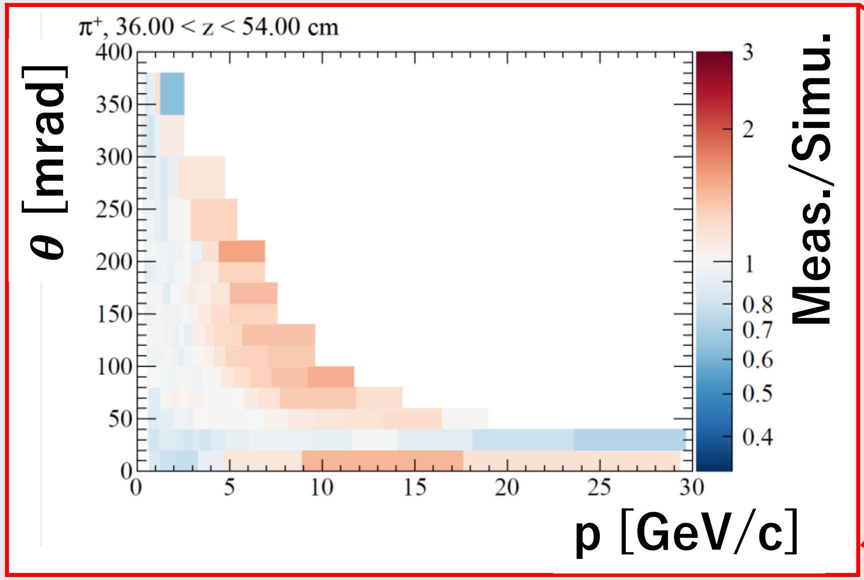


These parameters are independent from detection points

However, **hadron interactions** are described based on experimental data, which requires many parameters

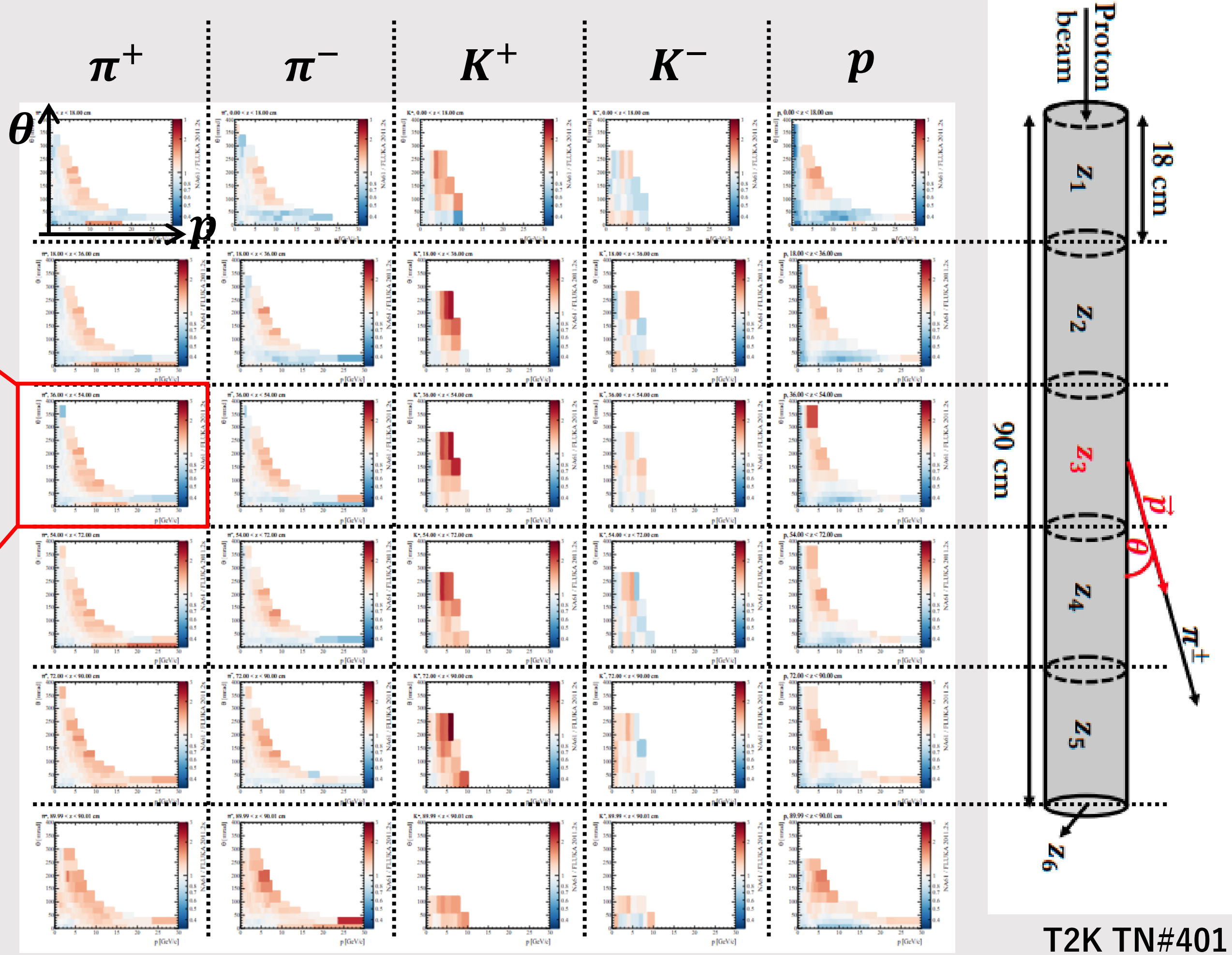
NA61/SHINE experiment

Measurement of hadron production from T2K "Replica" target



Ratio of production rate of model and real

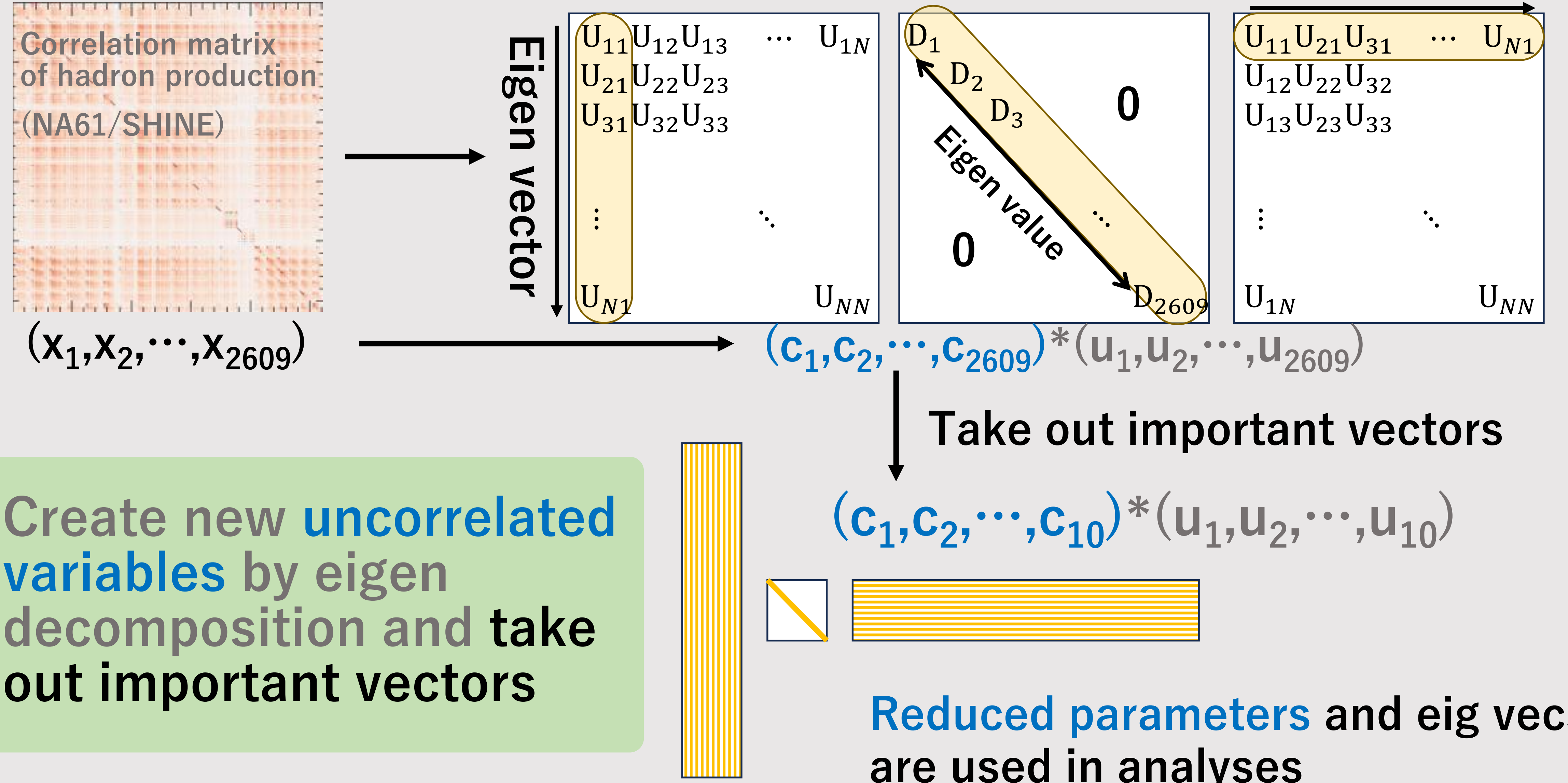
→"Tune" MC samples by this



T2K TN#401

↑ All of 2609 data points will be new parameters → **Too many parameters**

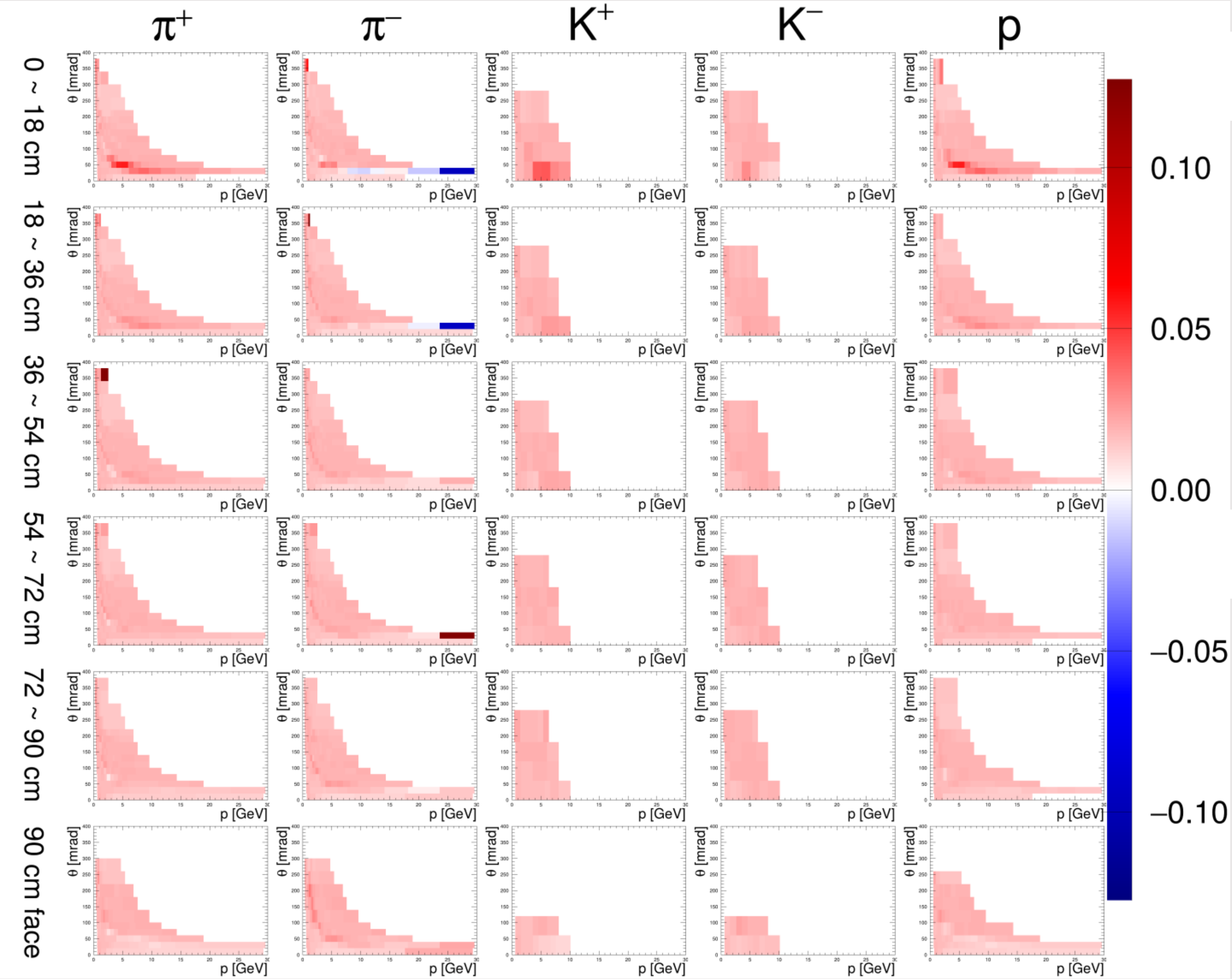
How to reduce number of NA61 replica parameters?



Create new **uncorrelated variables** by eigen decomposition and **take out important vectors**

Reduced parameters and eig vecs are used in analyses

Response of hadron phase space of eig vec #0



T2K work in progress

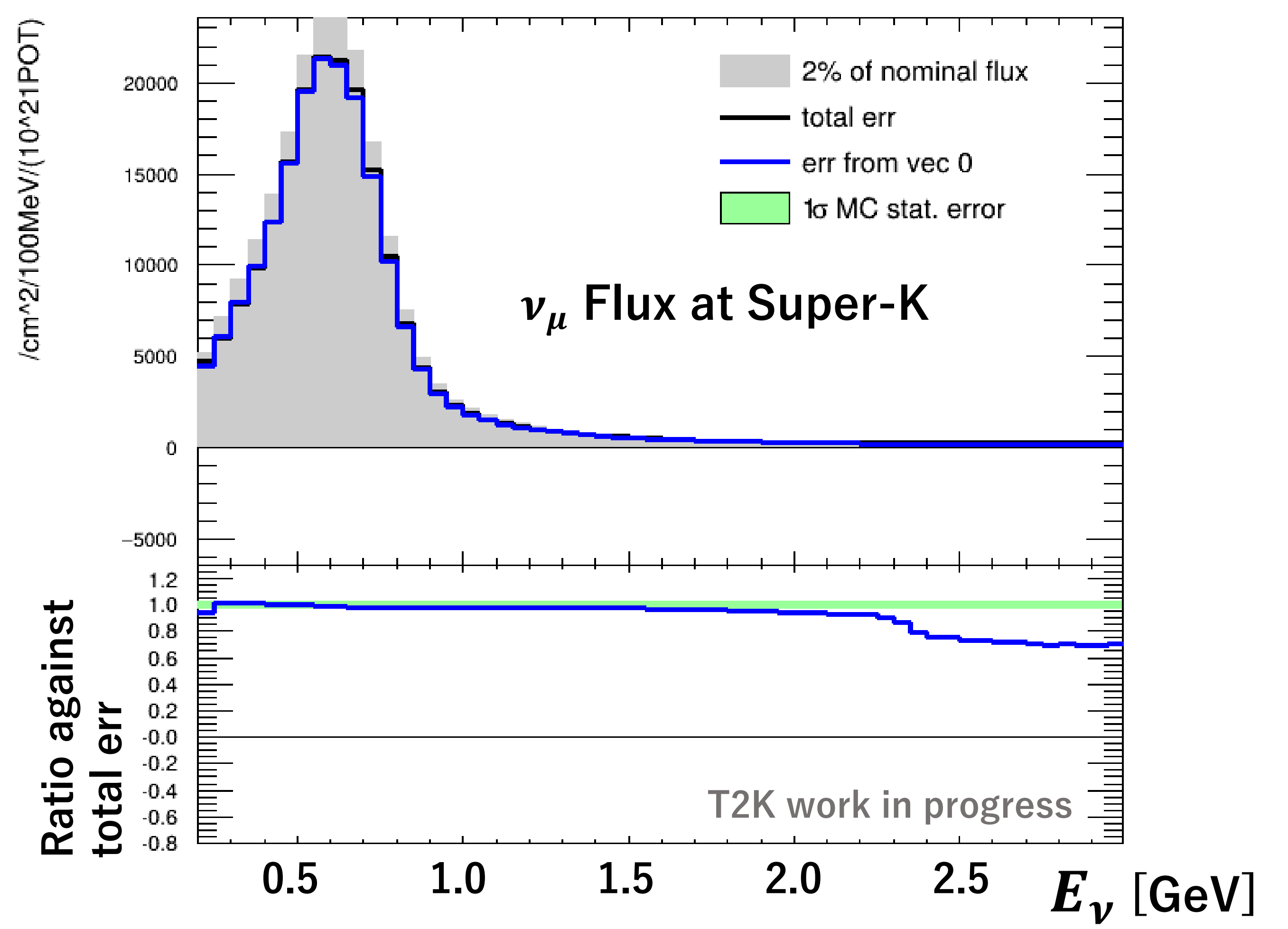
Hadron yields shift almost uniformly (~2% in 1 σ)

Corresponds to reconstruction bias in NA61/SHINE (2% over all the phase space)

Can expect that neutrino flux intensity also shift uniformly

Response of neutrino flux on new parameter: eig vec #0

Change in neutrino flux intensity when eig. vec. shifted 1 σ
Dominant error source for peak region



Gray: Flux intensity (scaled to 2%)
To show the scale of uncertainty

Black: Total Error from NA61 Replica
Evaluated by MC simulation

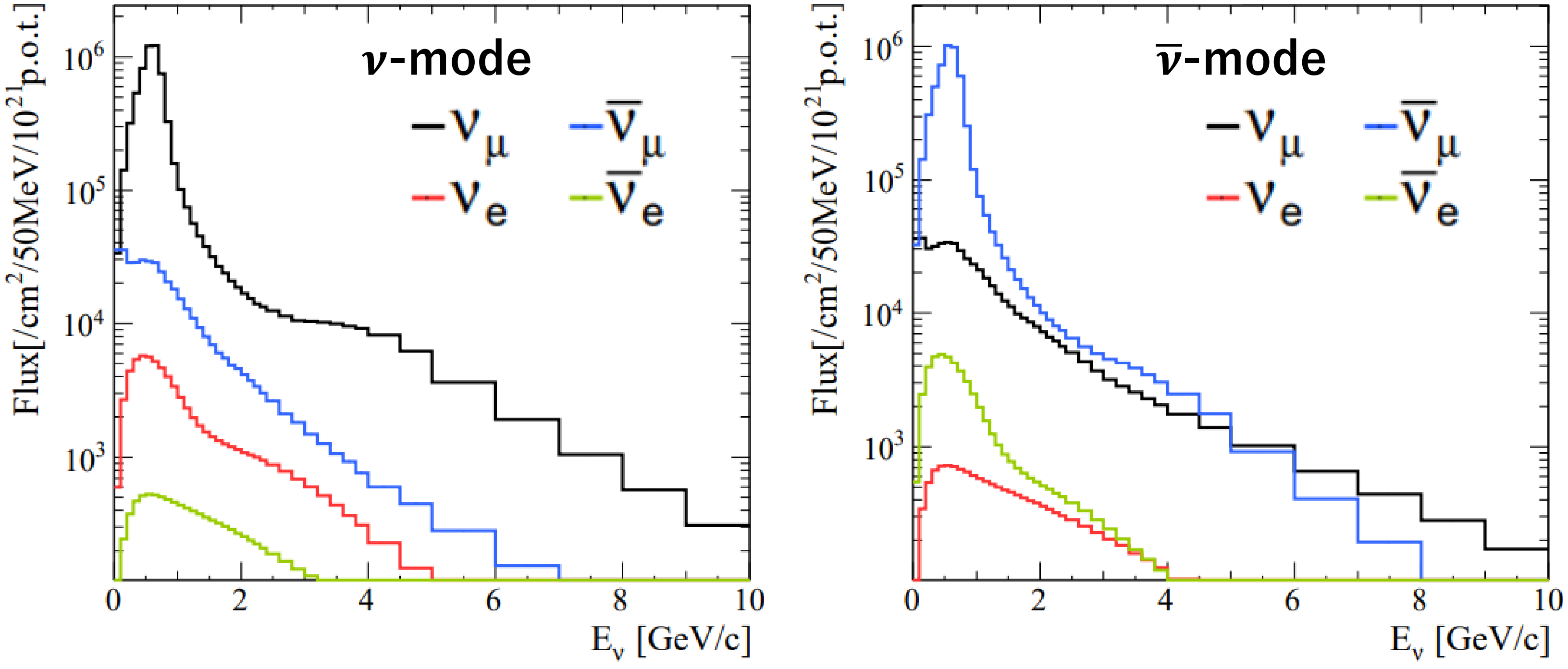
Blue: Response of flux (top)
Ratio against total error (bottom)

Green: Range to reach
1 σ range of total error due to MC statistical error

Flux intensity of each neutrino flavor

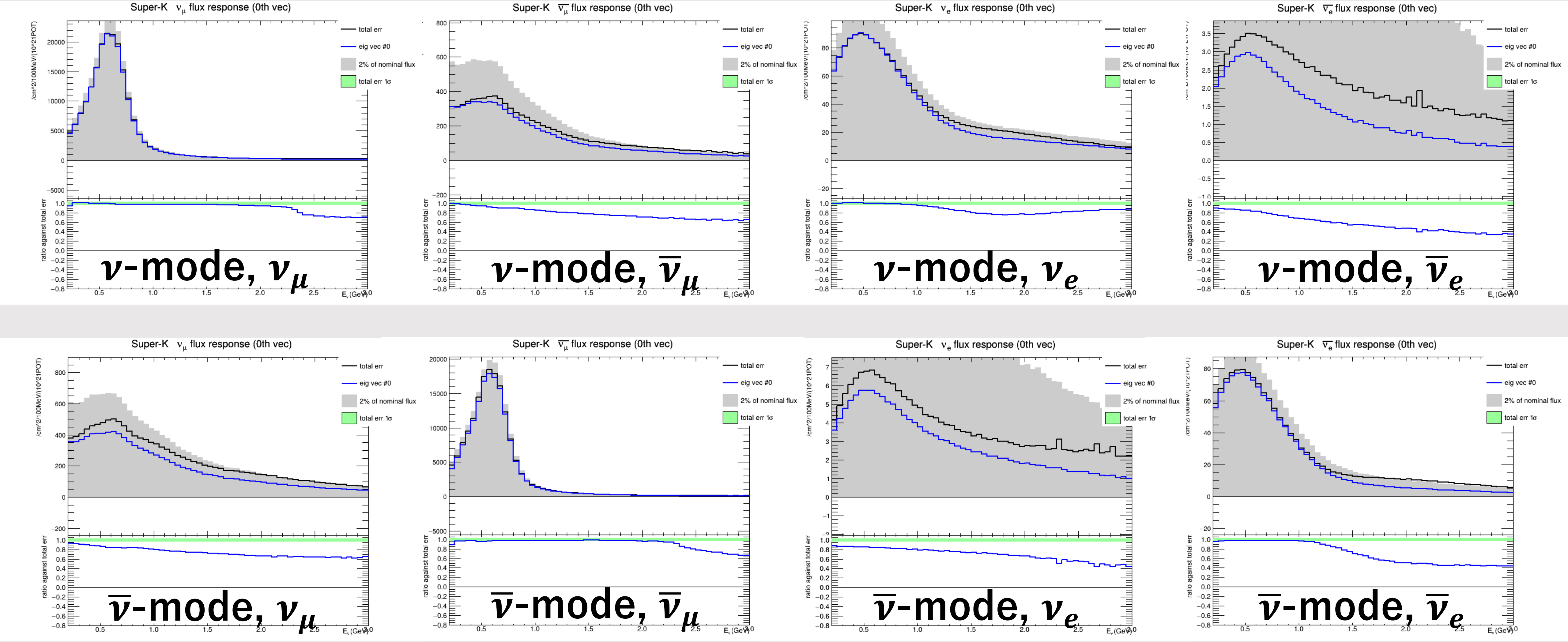
- ν_μ is the largest intensity at ν -mode, but the other flavors are produced from high momentum π^-, etc.
- The orders of intensity are different against each flavor
- Same for $\bar{\nu}$ -mode, thus need to consider 8 patterns of flux

Comparison of flux intensity of each flavor at Super-K



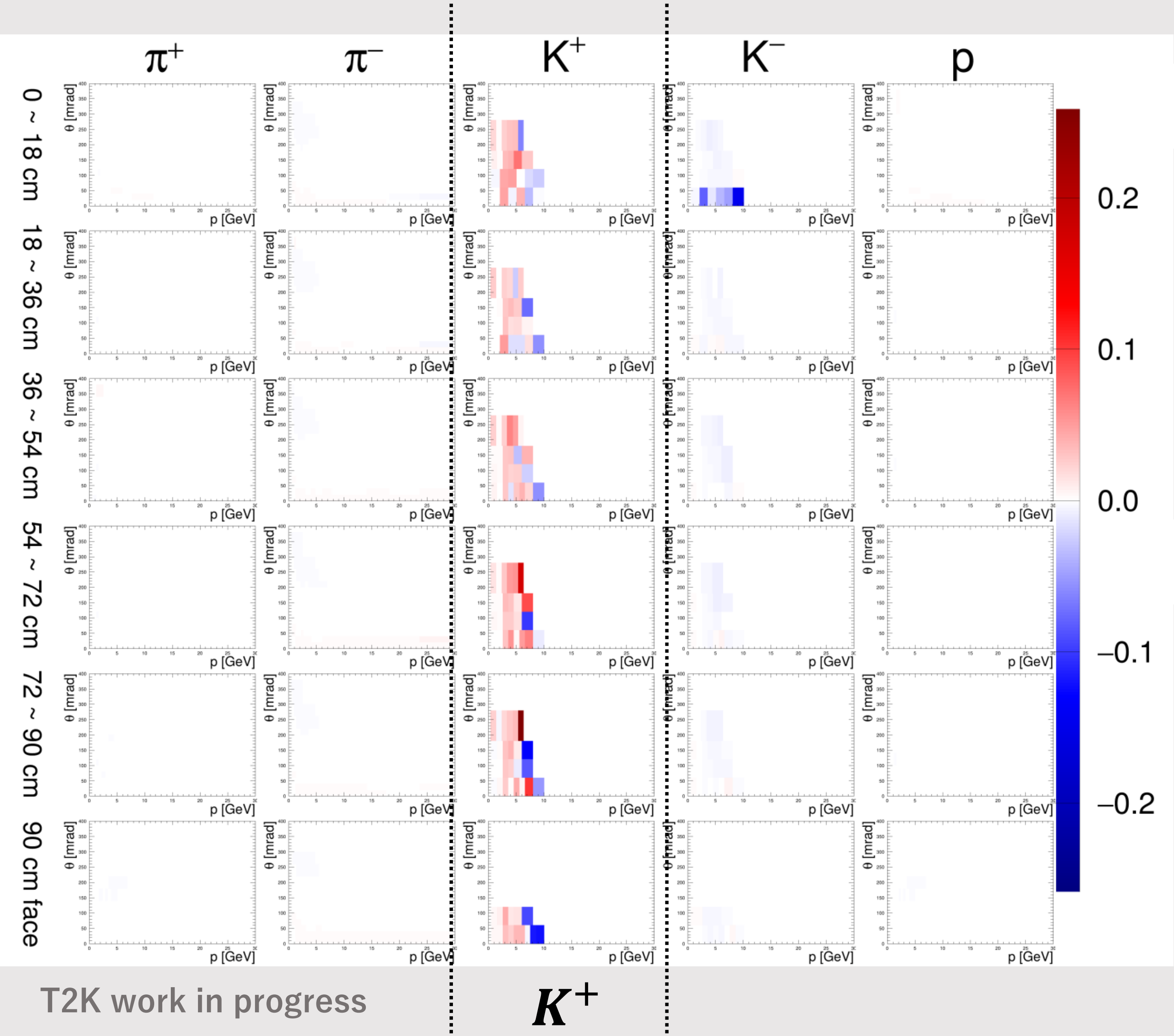
Response of neutrino flux on new parameter: eig vec #0

Response on large part of flux



T2K work in progress

Response of hadron phase space of eig vec #6



T2K work in progress

K^+

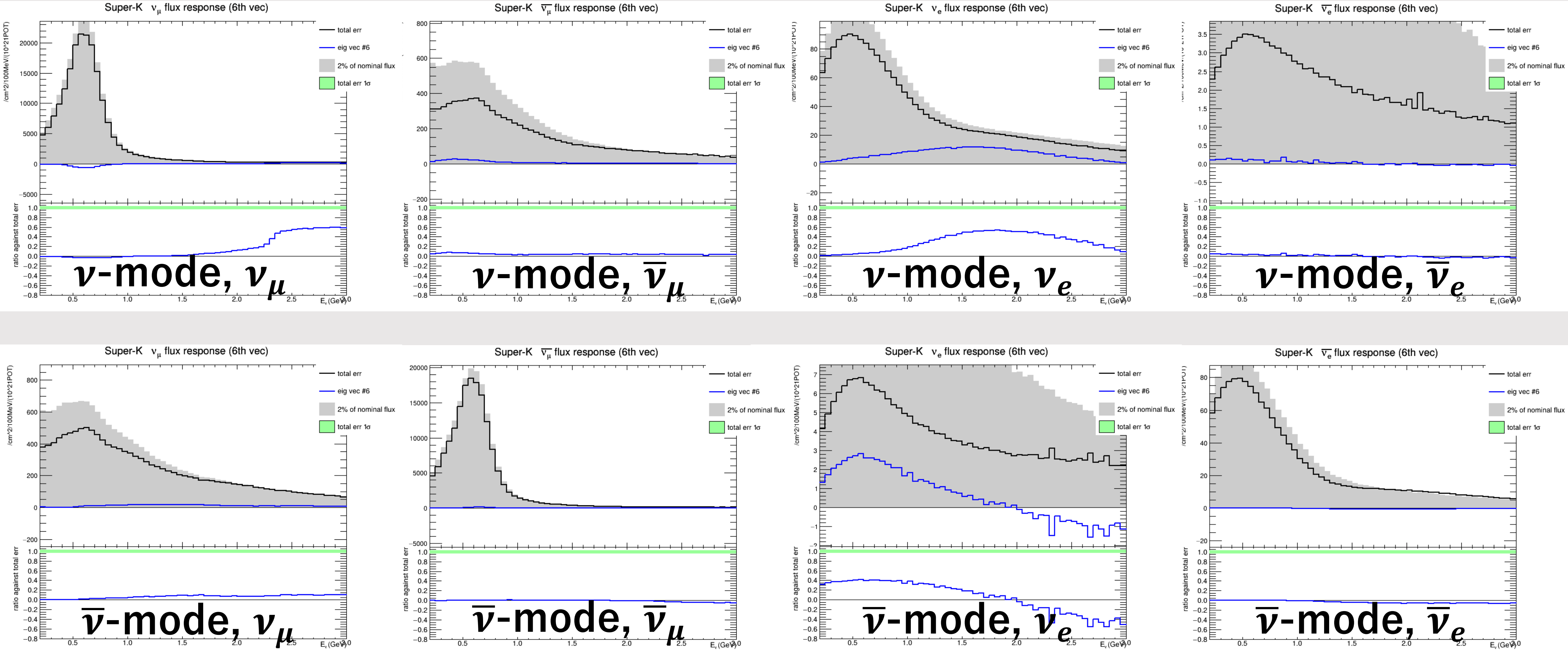
Error source that mainly shifts K^+ yields

Positive Kaons are one of main parent particles of ν_e, and high energy ν_μ

$$K^+ \rightarrow \begin{cases} \mu^+\nu_\mu \ 63.55\% \\ \pi^0\mu^+\nu_\mu \ 3.353\% \\ \pi^0 e^+\nu_e \ 5.07\% \end{cases}$$

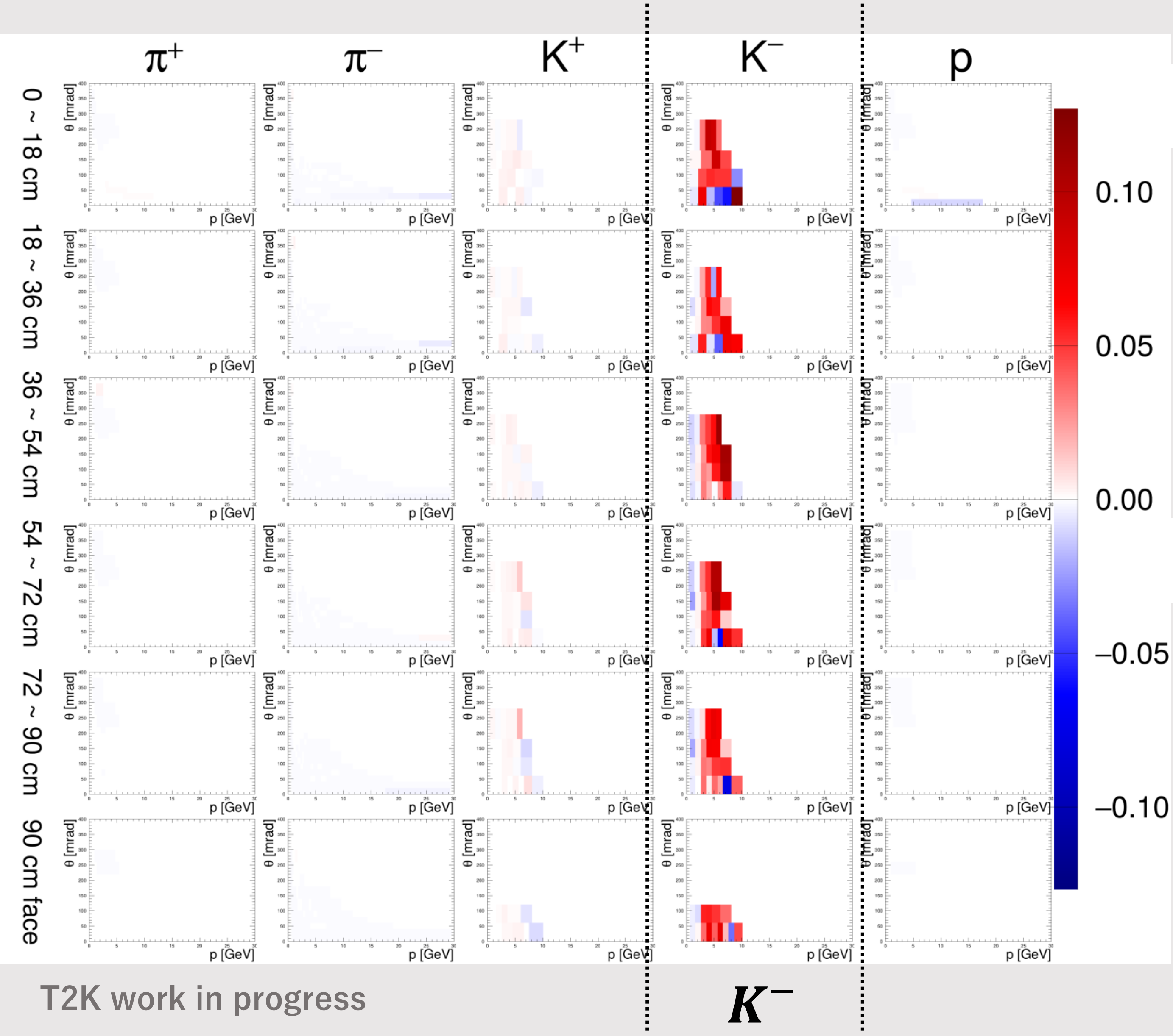
Response of neutrino flux on new parameter: eig vec #6

ν_e in both modes and high energy ν_μ in ν-mode



T2K work in progress

Response of hadron phase space of eig vec #7



T2K work in progress

K^-

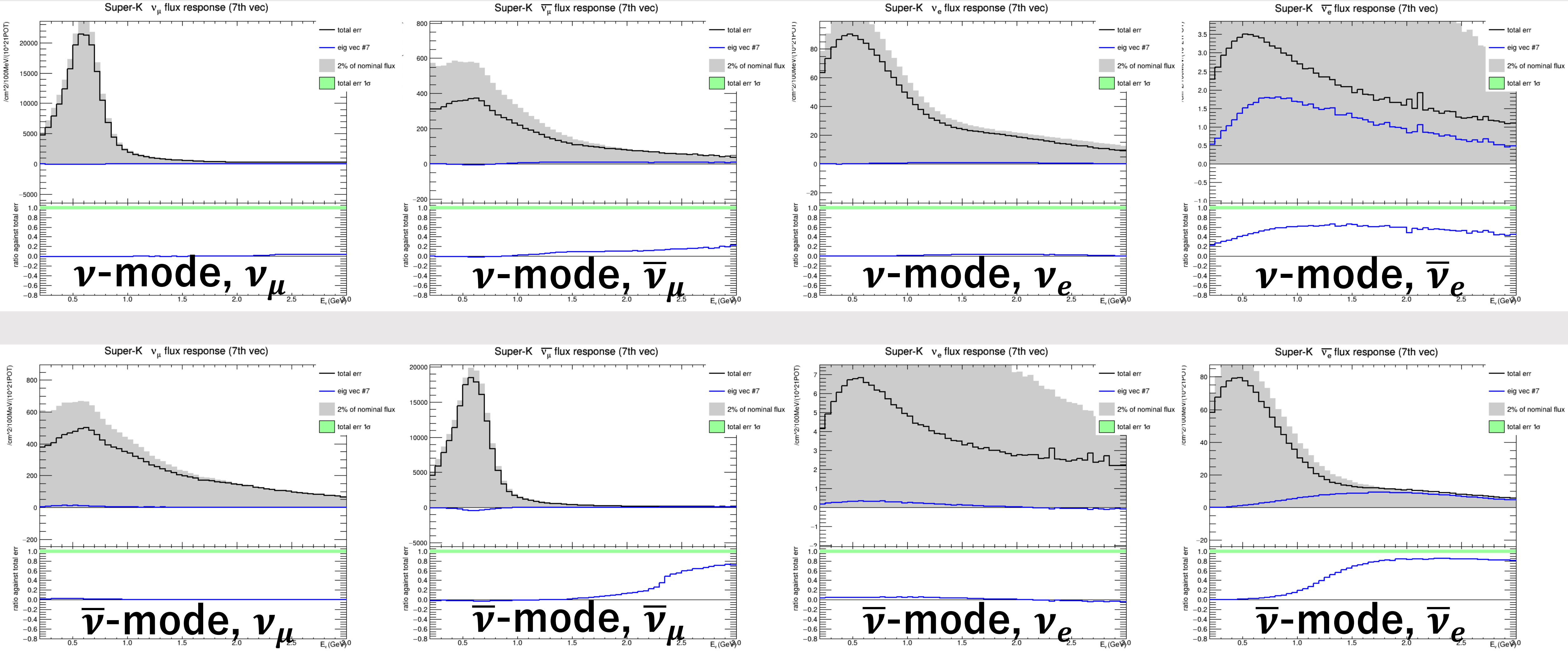
Error source that mainly shifts K^- yields

Negative Kaons are one of main parent particle of $\bar{\nu}_e$, and high energy $\bar{\nu}_\mu$

$$K^- \rightarrow \begin{cases} \mu^- \bar{\nu}_\mu \ 63.55\% \\ \pi^0 \mu^- \bar{\nu}_\mu 3.353\% \\ \pi^0 e^- \bar{\nu}_e 5.07\% \end{cases}$$

Response of neutrino flux on new parameter: eig vec #7

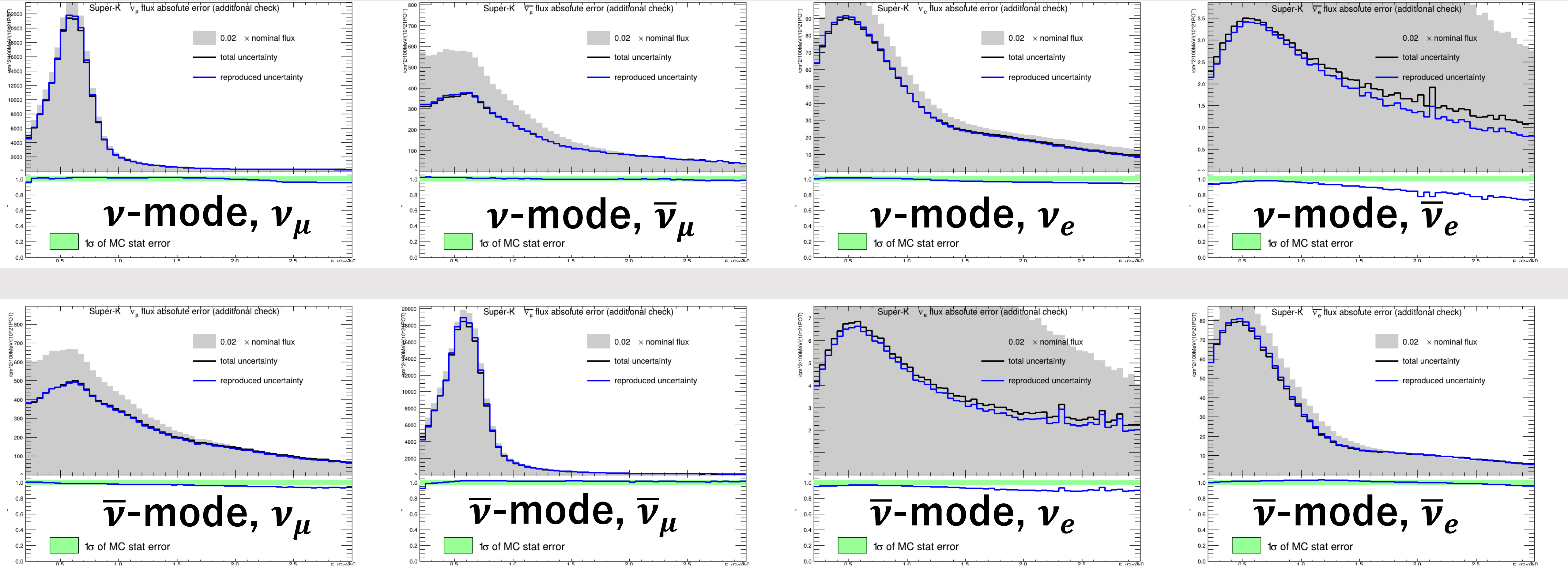
$\bar{\nu}_e$ in both modes and high energy $\bar{\nu}_\mu$ in $\bar{\nu}$-mode



T2K work in progress

Reproduction of flux uncertainty

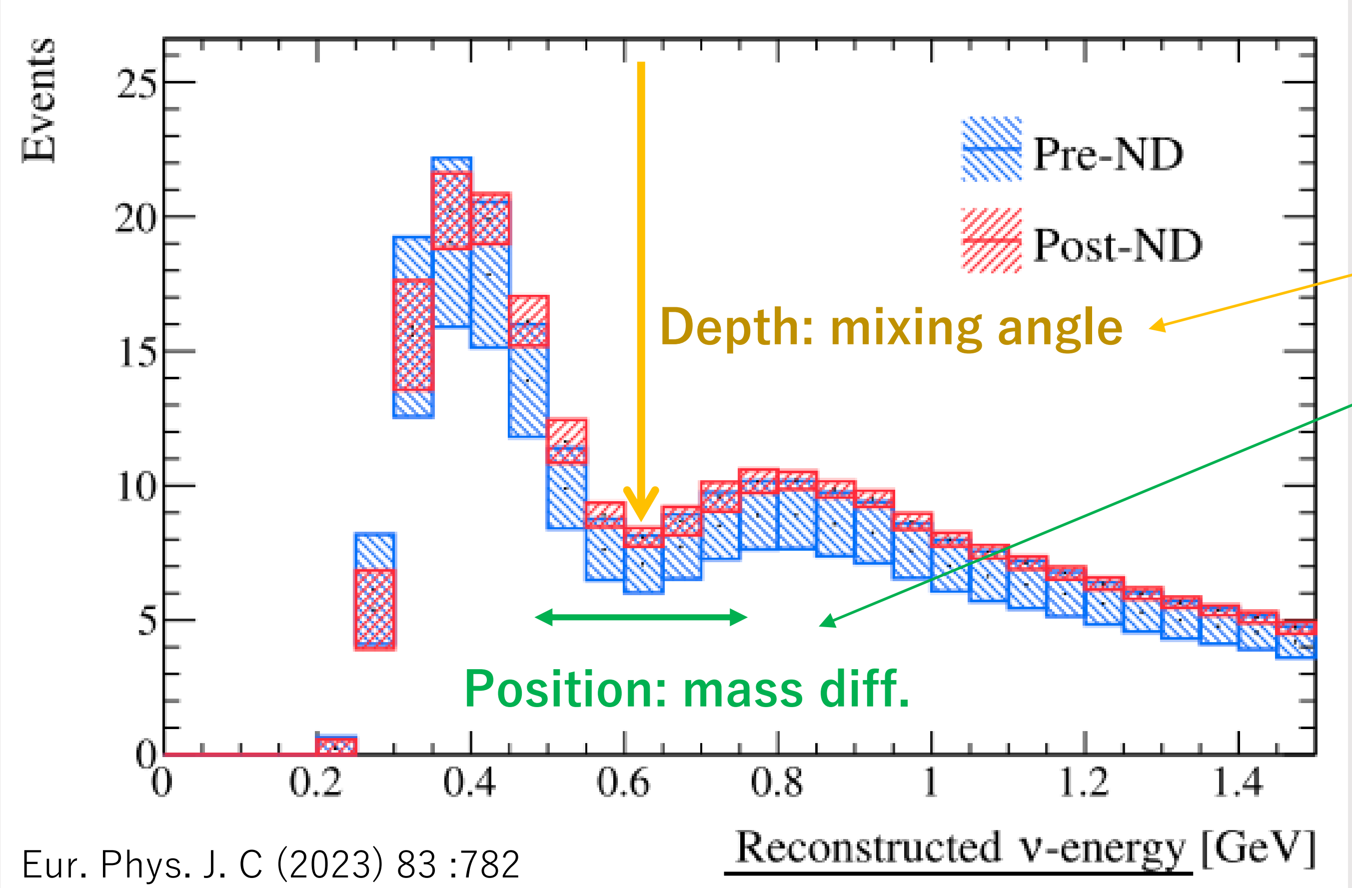
By using 11 parameters (corresponds to eig. vec. #0~#7, #200, #2501, #2539) instead of 2609, uncertainties are reproduced



T2K work in progress

Effect on neutrino oscillation analysis

Number of event at far detector ≈ Flux × X-sec × Osc. Prob.

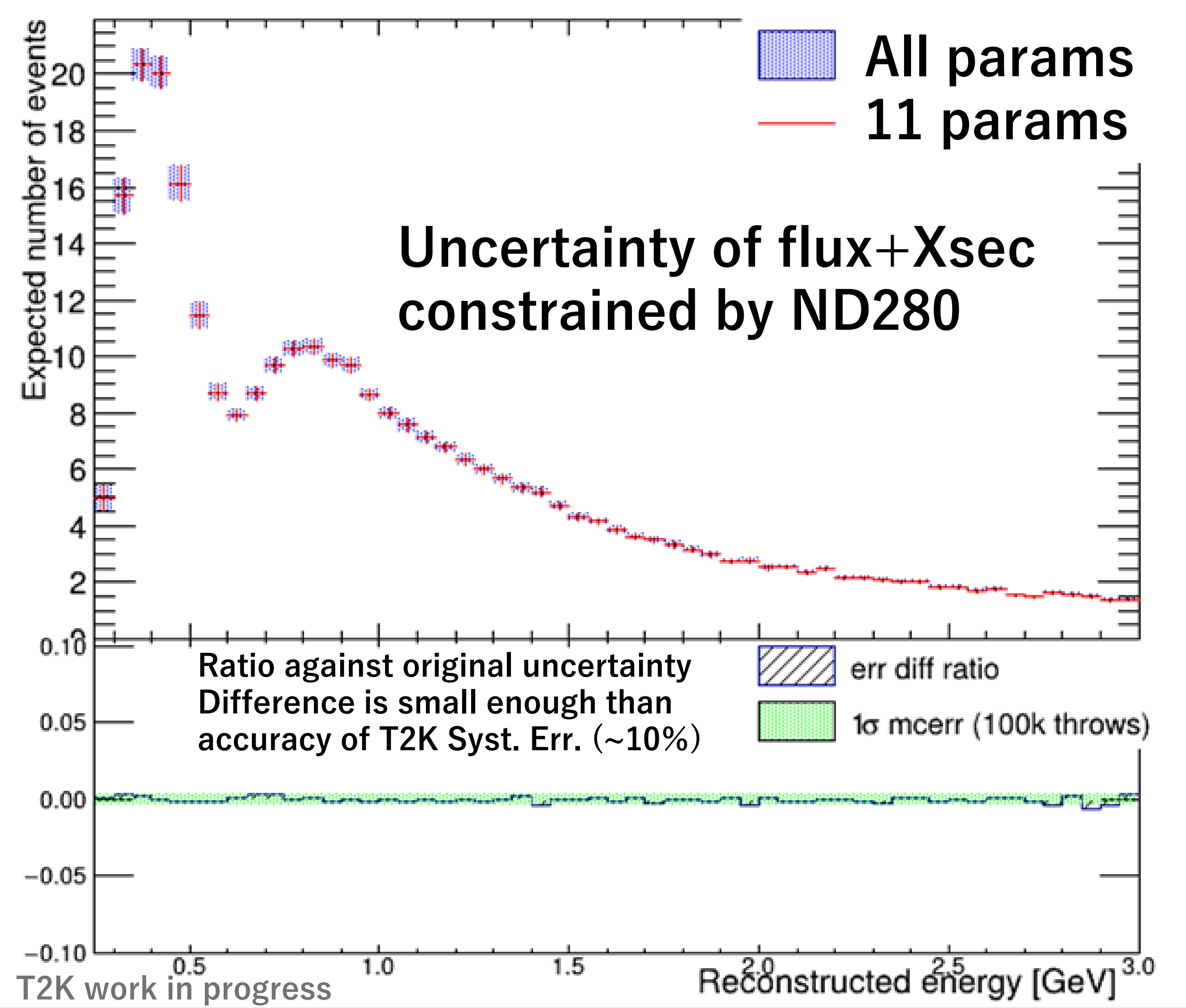


$$P_{\nu_\mu \to \nu_\mu} \approx 1 - \sin^2 2\theta_\nu \sin^2 \frac{\Delta m_\nu^2 L}{4E_\nu}$$

← Predict event distribution and total syst. uncertainty

ND measures Flux × X-sec
→ Reduce syst. uncertainty (Blue→Red)

Effect on constraint by ND280



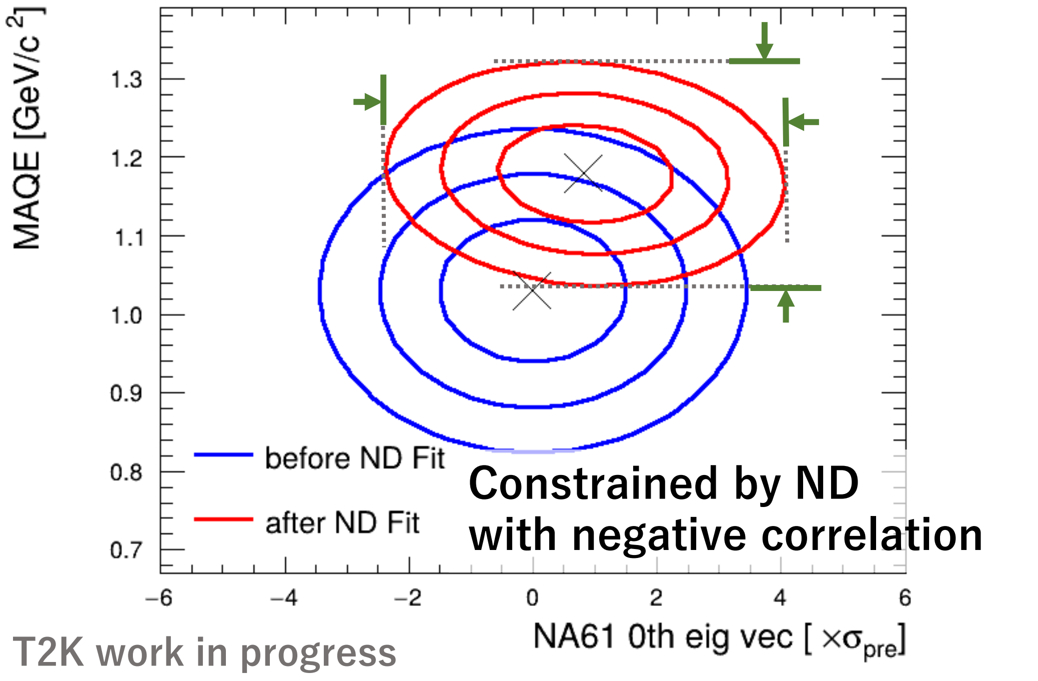
Comparison of systematic uncertainties calculated with all vs 11 NA61 Replica parameters

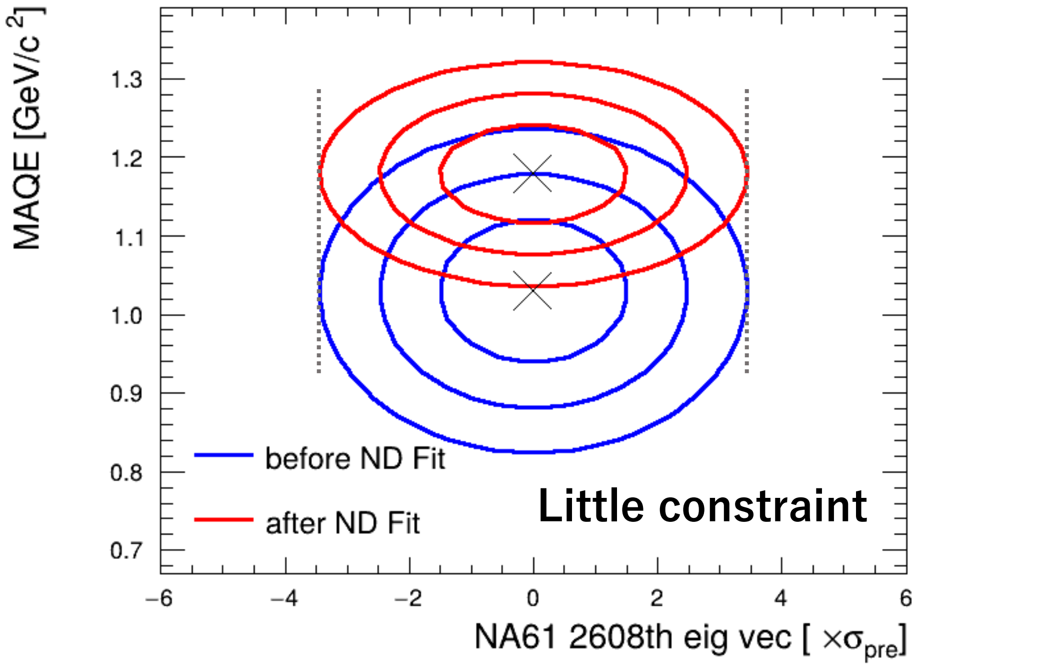
Uncertainty is reproduced **better than 1% accuracy**

ND constraint on new flux parameters

It allows analysis of ND constraint on underlying flux systematics (It used to be a black box for flux parameters)

2D contour plots of $\Delta\chi^2$ distribution between a flux parameter and a X-sec parameter (proton axial mass)
Not a new fit of ND data, but a recasting of the fit result

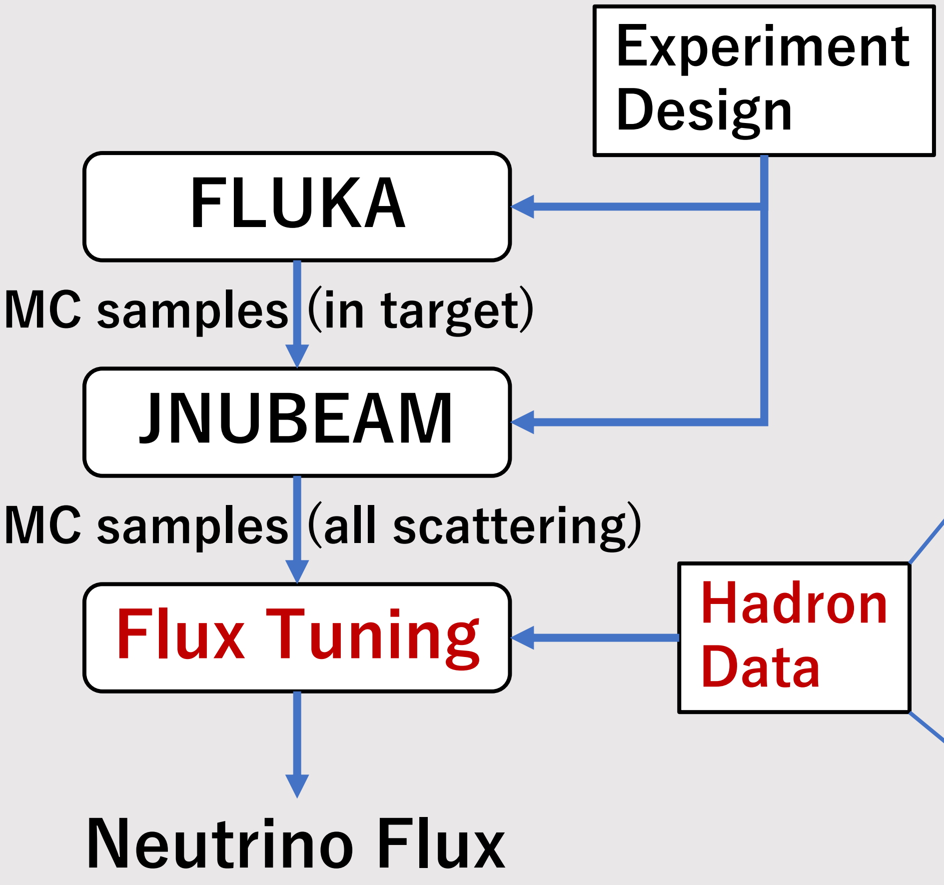




The parameters not chosen got little constraint → negligible

Next step: development for the other flux parameters

Recap of flux prediction in T2K experiment



← Includes non-hadronic systematic sources
No need to reduce these flux parameters

NA61/SHINE experiment
- Replica target → **Done**
- Thin target → **<u>Similar study is ongoing</u>**

Other experimental data
→ No need to reduce parameter

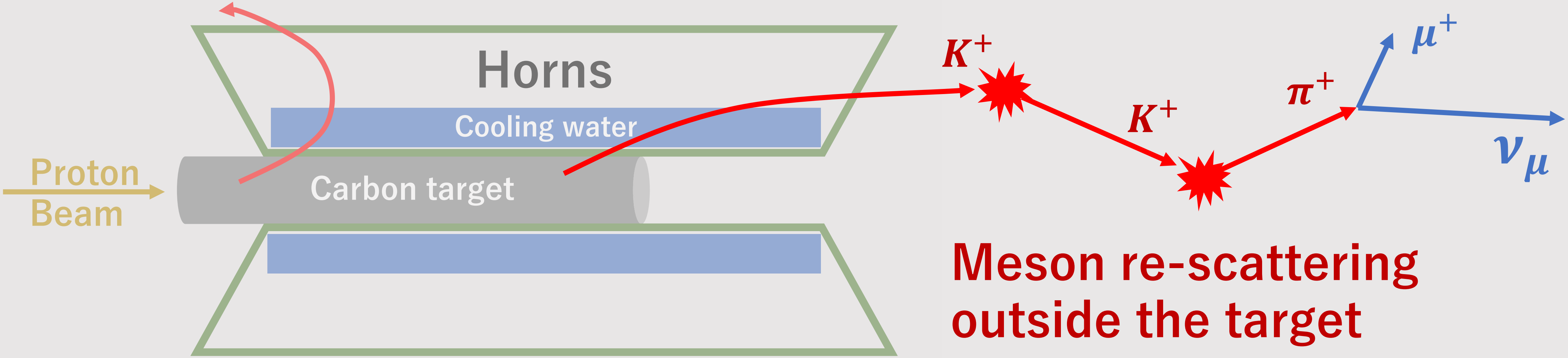
Unconstrained interaction
→ **<u>Done, from next page</u>**

Unconstrained interactions

Unconstrained hadron interactions which are not measured by any experiments are also complicated

These effects are **<u>not linear</u>** (Categorized as meson rescatter)

35 modes for 7 initial and 5 final particles are considered
$(\pi^{\pm}, K^{\pm}, K_L^0, p, n) \rightarrow (\pi^{\pm}, K^{\pm}, K_L^0)$



Unconstrained interactions

Phase space of each mode is divided into 6 regions in (x_F, p_T), thus there is 6×6 sized covariance matrix for each mode

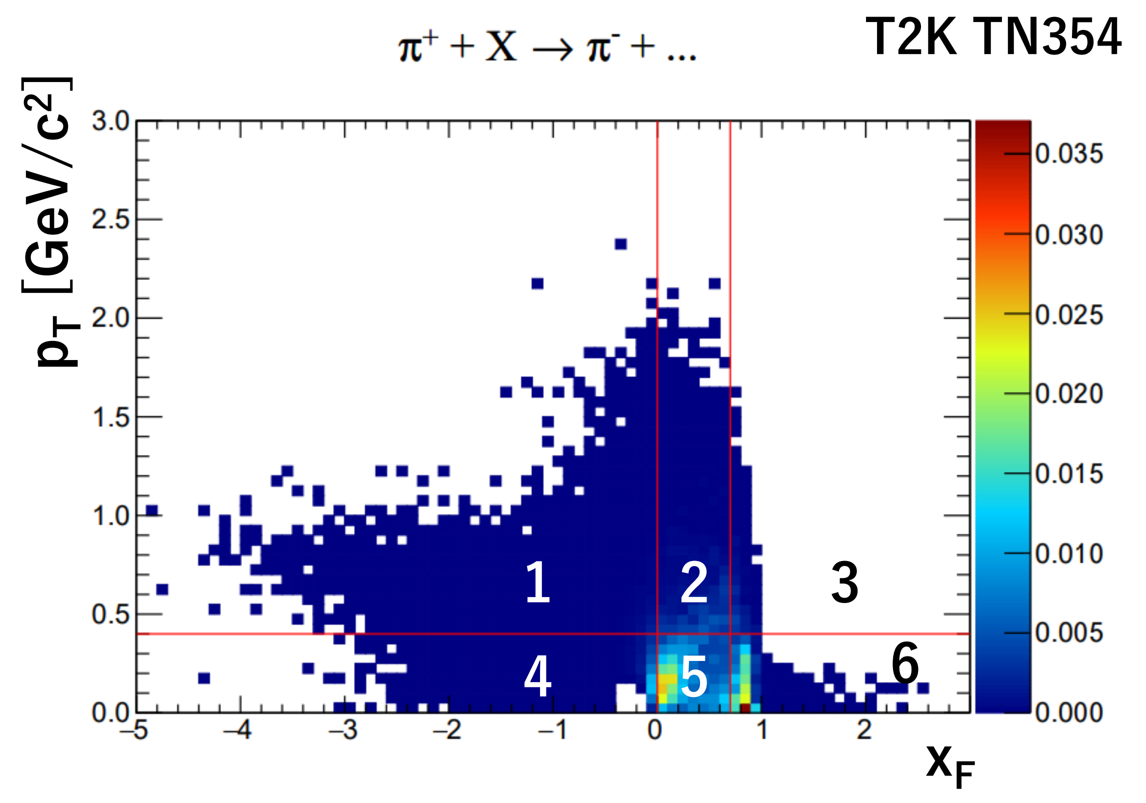
→50 % uncorrelated and 50 % correlated uncertainty is assigned based on MC physics list comparison

Error evaluation:

Calculate responses of flux on each eigen vector of the matrices

Both +/- 1σ shift for non-linearity

→ 35×6×2=420 parameters

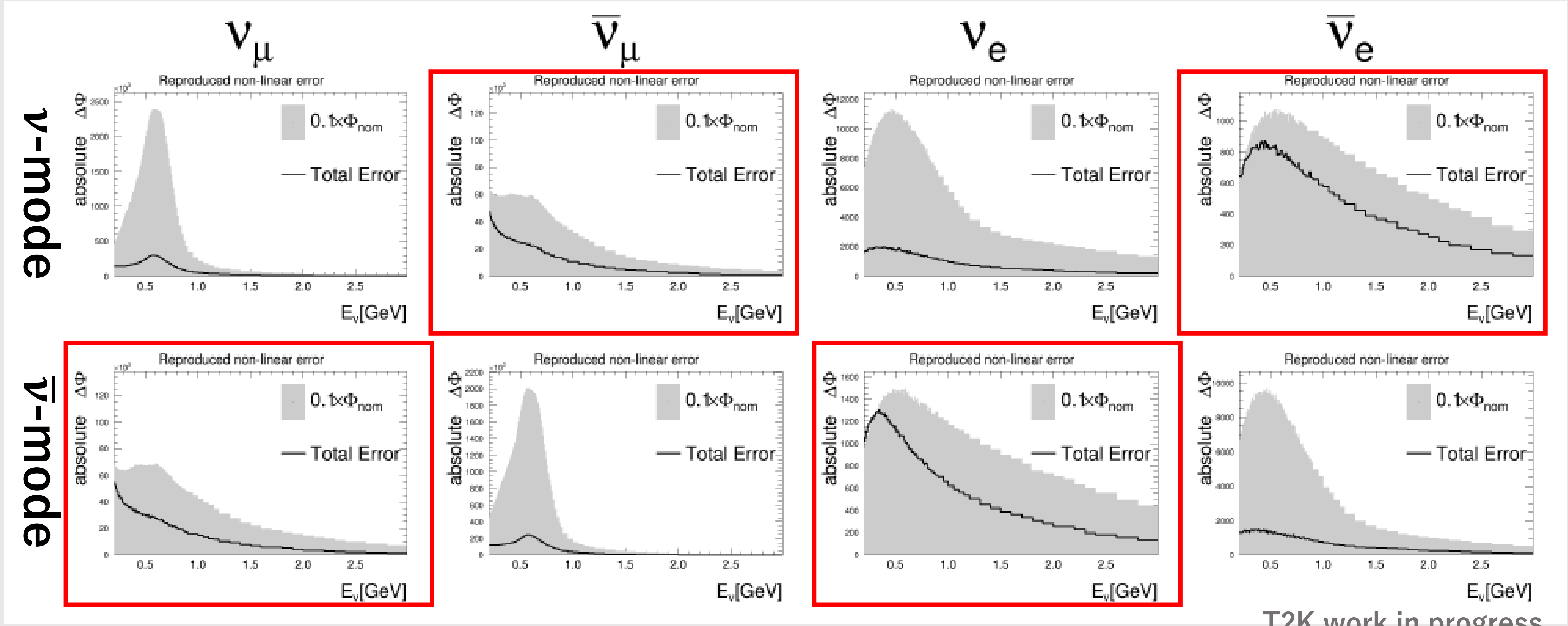


Total error from unconstrained interactions

Large error at wrong sign flux (~10% of nominal flux at max)

Gray: Nominal flux × 0.1

Black: Total uncertainty from unconstrained interactions



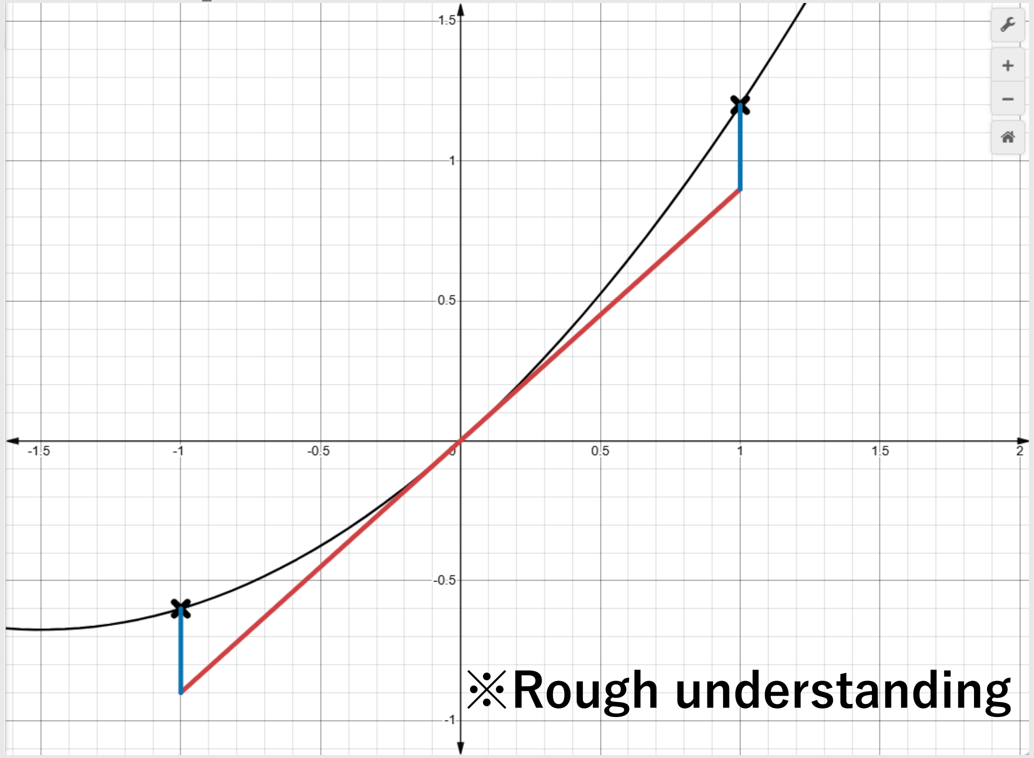
T2K work in progress

Linear / non-linear parameters

Separate ±1σ errors to symmetric and asymmetric elements

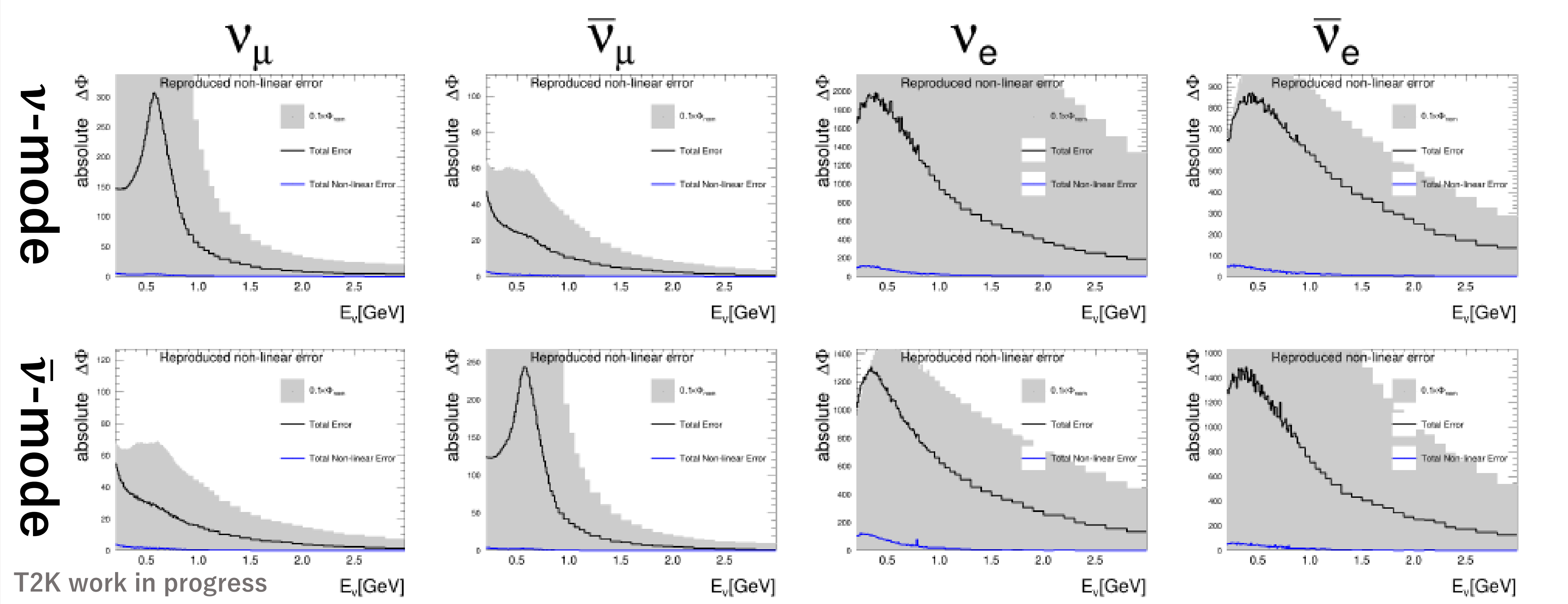
$$\Delta\Phi^2 = \frac{1}{2}(\Delta\Phi_+^2 + \Delta\Phi_-^2) = \left(\frac{1}{2}\Delta\Phi_+ - \frac{1}{2}\Delta\Phi_-\right)^2 + \left(\frac{1}{2}\Delta\Phi_+ + \frac{1}{2}\Delta\Phi_-\right)^2$$

They roughly corresponds linear and non-linear elements



Non-linearity

Black: total err
Blue: non-linear err (Small effect)

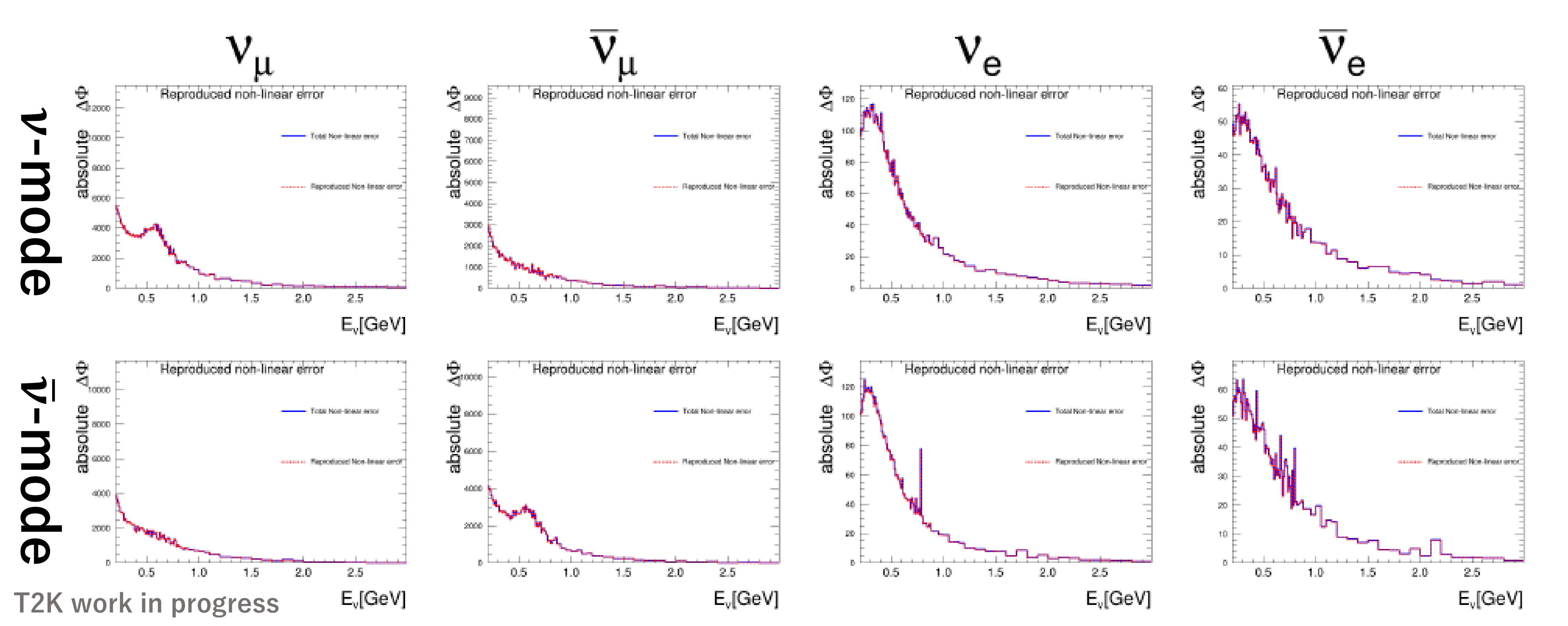


Optimize non-linearity

Non-linearity is reproduced with 5 parameters instead of 210

Blue: Total

Red: Reproduced



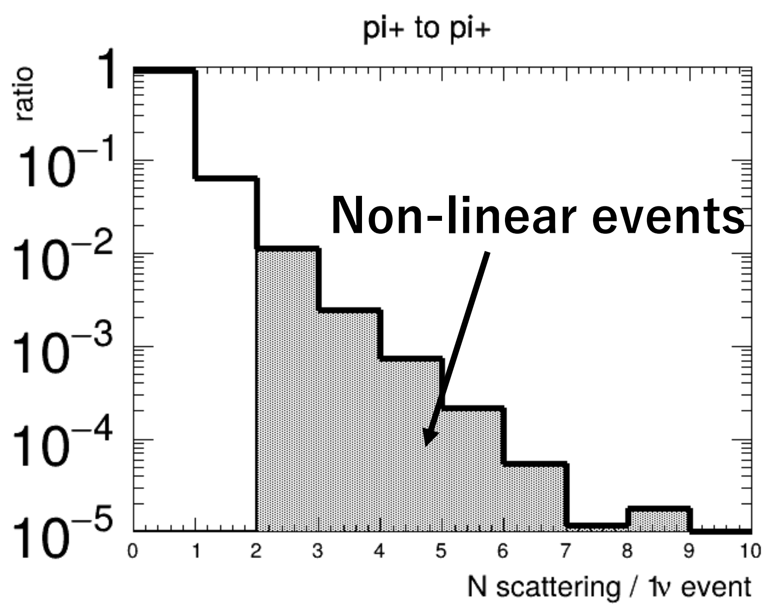
How many multi scattering occur?

Non-linear response occurs by ν events which experienced same scattering mode for multiple times in single ν event

Linear: $\mathrm{p} \to \mathrm{K}^+ \to \pi^+ \to \pi^+$

Non-linear: $\pi^+ \to \pi^+ \to \pi^+$

Checked distribution of each scattering per single ν events

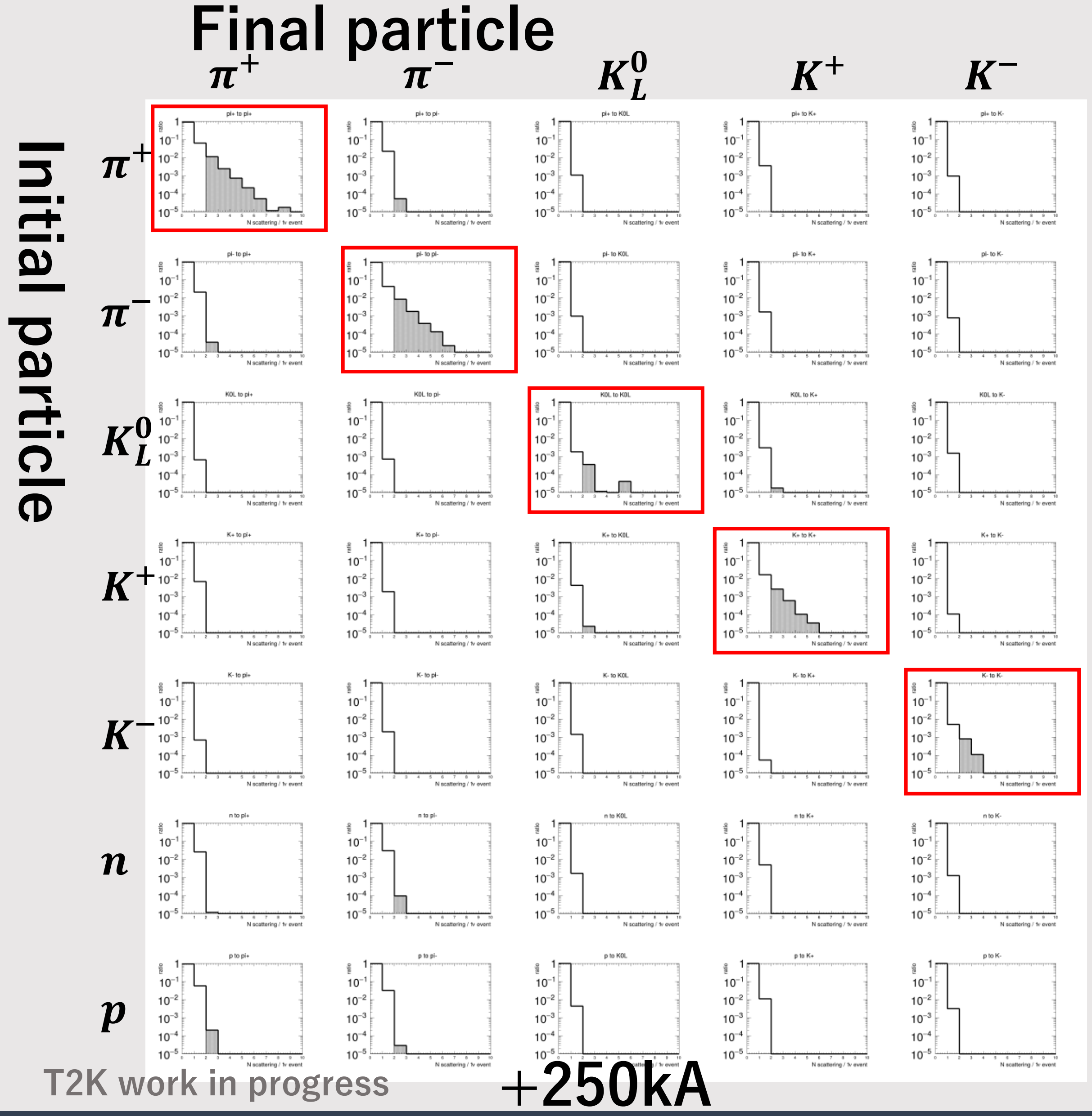


T2K work in progress

Event distribution which same scattering mode occurred N times in single ν event in JNUBEAM

Ratio of non-linear events was ~1% for $\pi^+ \to \pi^+$ mode

Investigation about multiple scattering from same mode



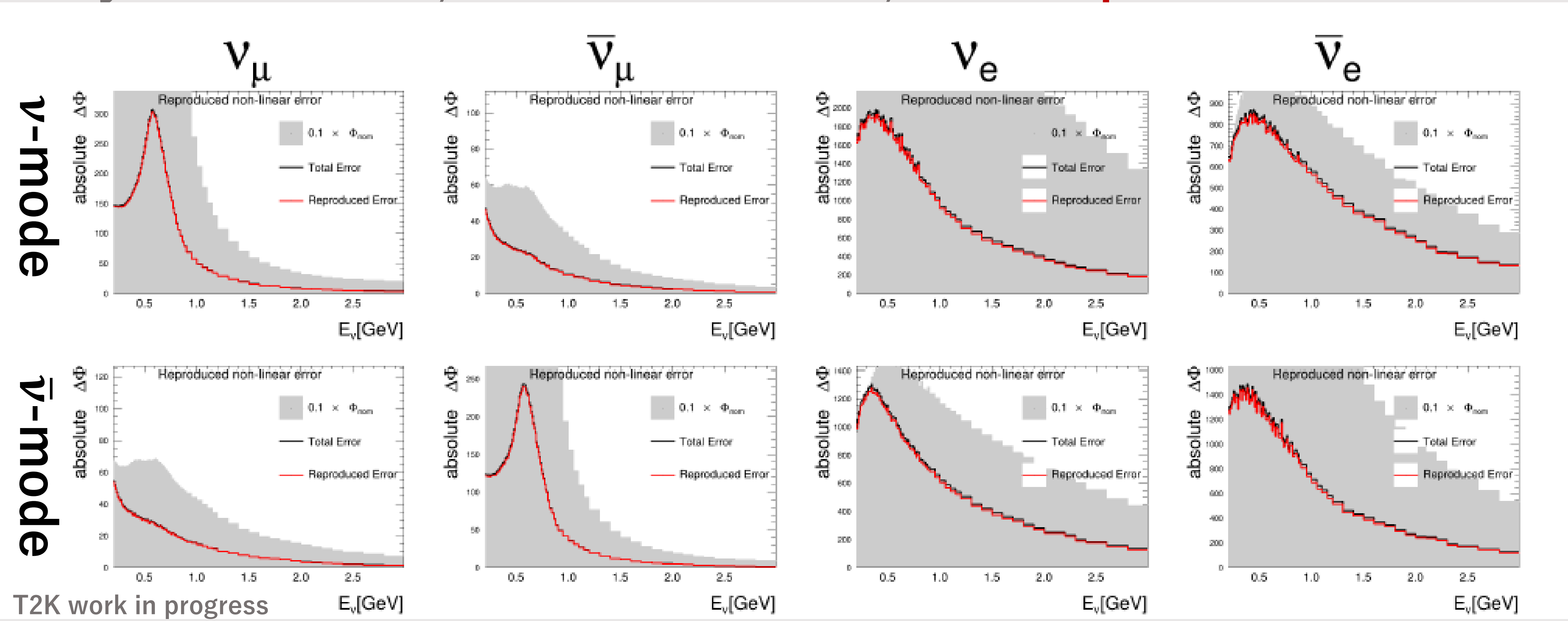
More non-linear samples in “X to X” scattering mode

The other modes change particle
→ Need 3rd scattering at least
→ Fewer non-linear samples

Reproduced with 5 parameters corresponding to the largest eigen vector of 5 “X to X” modes

Reproduced error with 44 linear + 5 non-linear parameters

Grey: Flux $(\times 0.1)$, Black: Total error, Red: Reproduced



Modes not chosen:

$$\pi^\pm \to K^{\pm,\mp},\ K_L^0 \to \pi^\pm,\ K^+ \to \pi^-, K^-,\ K^- \to \pi^\pm, K^+,\ n \to K_L^0, K^-,\ p \to K^\pm$$

Summary and plans

- We need a new method to handle flux correlations between multiple detection points with reduced number of parameters
- Energy bin-based flux parameters → Original systematic parameters But the number of parameters for hadron interaction is large
- Reduced NA61 Replica parameters (2609→11) and Unconstrained Interactions (420→49)

Plans:

1. Complete works of new parameterization of flux covariance and reduction of parameters for hadron interaction
2. Development for Off-Axis Angles≠2.5° (including Off-Axis Angle interpolation)
3. Implementation to the oscillation analysis